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An optimization model for aircraft maintenance scheduling and re-assignment

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Abstract

The aircraft maintenance scheduling is one among the major decisions an airline has to make during its operation. Though maintenance scheduling comes as an end stage in an airline operation, it has potential for cost savings. Maintenance scheduling is an easily understood but difficult to solve problem. Given a flight schedule with aircraft assigned to it, the aircraft maintenance-scheduling problem is to determine which aircraft should fly which segment and when and where each aircraft should undergo different levels of maintenance check required by the Federal Aviation Administration. The objective is to minimize the maintenance cost and any costs incurred during the re-assignment of aircraft to the flight segments.

This paper provides a complete formulation for maintenance scheduling and a heuristic approach to solve the problem. The heuristic procedure provides good solutions in reasonable computation time. This model can be used by mid-sized airline corporations to optimize their maintenance costs.

Keywords: Aircraft; Maintenance; Scheduling; Optimization; Heuristic

1. Introduction

1.1. Background

Consideration of maintenance constraints has long been recognized to be a cornerstone in aircraft scheduling. The development of aircraft maintenance schedule is a complicated task involving the synthesis of a range of economic, political, legal and technical factors. Demand for

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service, aircraft utilization and operational cost of aircraft are the principal drivers. The goal is to achieve a balanced pattern of flights that results in a timetable consistent with the FAA regulations and airline policies. The major airlines have witnessed significant changes in their operating environment after the airline deregulation act of 1978. As a result of fierce competition the airlines had to cut their prices down and this led to more passengers flying than ever before. More than 80% of passengers are now traveling on tickets priced at less than base fare. This accompanying downward pressure on revenues has led many carriers to focus their attention on controlling maintenance and personnel costs.

From an operating point of view, the demand for service sets the daily flight schedule and determines which type of aircraft will be flown on a given route. This is the primary constraint faced by the maintenance planners who must schedule inspection for each plane in the fleet in compliance with the FAA regulations. The possibility of assigning individual aircraft to different routes throughout the day offers the flexibility needed to meet this requirement.

In this paper we consider the problem faced by an airline needing to construct a 7-day planning horizon cyclic schedule with maintenance constraints for a heterogeneous fleet of aircraft. The maintenance checks on aircraft are to be scheduled based on the given flight schedule. The flight schedule consists of a sequence of flight legs to be carried out by an aircraft. So the maintenance-scheduling problem is solved after the aircraft are assigned to the flight legs. Solving the maintenance-scheduling problem to optimality may cause a re-assignment of aircraft to the flight legs. Different aircraft assignments lead to different costs and revenues for an airline. For example a flight leg that can be flown by two aircraft of different capacities can result in a loss of revenue if the smaller aircraft is chosen when the demand for the leg exceeds the smaller aircraft capacity. In contrast, a flight leg that can be flown by two aircraft of different capacity can result in higher operation cost if the larger aircraft is chosen when the demand for the leg is lower than the capacity of the larger aircraft. So the shuffling (re-assignment) of aircraft would result in loss of revenue or increase in operating cost. This cost is taken into account by penalizing the assignment of inappropriate aircraft to the flight legs.

Since most of the maintenance checks are done during night the problem of concern is where the aircraft spends the night each day in a 7-day cyclic schedule rather than intermediate stops. So a sequence of flight legs to which an aircraft is assigned for any given day can be considered as one trip. In the course of re-assignment an aircraft is assigned to a trip rather than a single flight leg. The penalty cost for a trip would be the sum of the penalties for all flight legs that constitute the trip. Hereafter a sequence of flight legs that make a trip will be identified as an Origin Destination (OD) pair.

1.2. Airline maintenance requirement

Aircraft maintenance takes place in a series of checks of increasing diligence with the exception of unscheduled fixes. The frequency of these checks depends on the combination of flight hours and number of take-off and landing cycles, and may be performed at any site appropriately equipped. Because each aircraft type has different inventory requirement, little savings can be achieved by combining facilities for different fleets.

To be in compliance with the Federal Aviation Administration constraints, some companies have adopted maintenance policies that call for routine inspections at least every four days. There are four major types of check mandated by the FAA that each aircraft has to undergo. These vary in scope, duration, and frequency (Clarke et al., 1997).

1.2.1. Type A check

The first major check (denoted as Type A) actually mandated by the FAA occurs at every 65 flight-hours, or about once a week. Type A checks involve inspection of all major systems such as landing gear, engines and control surfaces.

1.2.2. Type B check

The second major check (designated as Type B) is performed every 300–600 flight-hours, and entails a thorough visual inspection plus lubrication of all moving parts such as horizontal stabilizers and ailerons.

1.2.3. Type C and Type D check

The very major checks designated as Type C and D are done about once every one to four years, respectively, and require taking the aircraft out of service for up to a month at a time.

Because Type C and D checks are spaced at relatively large intervals and because of the dynamic nature of the market, these two checks need not be taken into account in maintenance scheduling. The principal concern of the airlines is in meeting the Type A and B checks requirements through their self-imposed 4-day inspection and maintenance policy. Unless there are exceptional circumstances, inspections and repairs take place at night.

1.3. Objectives and scope

The objective of this paper is to present an innovative mathematical formulation and an effective methodology to solve the aircraft maintenance-scheduling problem. The formulation and the solution method are to assign aircraft to the OD pairs to minimize the maintenance cost. The scope of the problem discussed here is limited to weekly domestic flight schedules. During the period of flight inactivity that is usually in the late evening to early morning the maintenance is routinely performed.

Given a flight schedule, the aircraft re-assignment and maintenance-scheduling problem is to determine which aircraft should fly which OD pair and when and where each aircraft should undergo maintenance checks of Type A and B. The Objective is to minimize the maintenance cost and the penalty incurred during the re-assignment of aircraft to the OD pairs.

This problem can be solved at different levels. A typical one would consider two important levels of maintenance requirement (Type A and B), heterogeneity in the fleet, the location of maintenance base facility for different aircraft types, and the cyclical schedule of an *n*-day planning horizon. The scope of the problem with which we are dealing in this research is as follows:

1. Only domestic airline operation is considered.
2. Aircraft assignment is made before maintenance scheduling.
3. Only Type A and B maintenance constraints are being considered.
4. Only the existing maintenance bases are considered and no recommendations are made for the construction of new feasible and optimal location of maintenance bases.
5. Unexpected maintenance requirements are not being considered.

This paper is organized as follows. A literature review is provided in Section 2. An integer program formulation is discussed Section 3. The proposed heuristic to solve large size problems is discussed in detail in Section 4. The performance of the heuristic is discussed in Section 5. The results and conclusions are given in Section 6.

2. Literature review

In this section we provide a brief review of some of the existing models in airline flight and crew scheduling, fleet assignment, and maintenance scheduling.

Until 1970 the focus of the researchers was on demand forecasting, aircraft fleet scheduling and routing. Work that has been done during this period on these topics was limited because of computationally intensive nature of the problem when optimal solutions were sought. The principal method of attack was dynamic programming, which usually involves a very large state space for any real-life transportation system. The scheduling and fleet routing problem was formulated as an integer linear program by Levin (1971). This was the first stage of improvement in the formulation from a dynamic programming model to an integer programming model. No heuristic or approximation methods were used in this paper to solve large size problem, as the aim of the paper was to formulate and obtain a optimal solution to the real world problem. The methods used in this paper to solve the problem are branch and bound methods (Levin, 1971).

Desaulniers (1997) presented two models for the aircraft routing and scheduling problem. One is a set partitioning type formulation and another one is a time constrained multi-commodity network flow formulation. A column generation technique is used to solve linear relaxation of the first model and a Dantzig–Wolfe decomposition approach is used to solve the linear relaxation of the second model.

Given a flight schedule, the airline crew scheduling problem is to assign the flight crews to the flight schedule optimally while satisfying restrictions dictated by the labor agreements and the Federal Aviation Administration. The problem is modeled as a set-partitioning problem in most cases. The objective of the crew scheduling problem is to find a minimum cost assignment of flight crews to a given flight schedule. Since in most cases crews are differentiated based on which aircraft they can fly, usually the flight schedule considered includes only the flights that have been assigned to a single aircraft type. So the crew assignment is solved for each aircraft type separately. The crews to be assigned are all qualified to fly this particular aircraft type and thus treated identically.

The assignment of aircraft types to the flight legs, or fleet assignment, is completed before the crew scheduling decision. Models and algorithm for fleet assignment are given in Abara (1989), Hane et al. (1995), and Subramanian et al. (1994). The flight attendant scheduling problem is also similar to crew scheduling problem, but it tends to be much larger because a flight attendant may be qualified to serve in more than one type of aircraft. Usually they are qualified to serve in almost all aircraft types. A more recent survey on crew scheduling can be found in Etschmaier and Mathaisel (1985).

Most recent approaches to crew scheduling centered on the set-partitioning problem given in Vance et al. (1997). The large number of variables in the model given in Vance et al. (1997) leads to billions of pairings even for a problem with a few hundreds of flights. Because of this explosive nature of the problem, a local optimization approach, as discussed by Anbil (1991) and Gershkoff

(1989), is often used. Graves et al. (1993) describe the crew scheduling optimization system used by United Airlines. Other approaches have used constrained shortest path methods on specially structured networks to price out attractive pairings. Examples can be seen in Lavoie et al. (1988). Hoffman and Padberg (1993) found optimal integer solutions to the problems with a maximum of 300,000 pairings using a branch and cut algorithm. In their approach, crew base constraints were explicitly considered.

Clarke et al. (1996) consider the constraints related to crew in their fleet assignment formulation. Given a flight schedule and set of aircraft, the fleet assignment problem is to determine which type of aircraft should fly each flight segment. Mostly, this problem is formulated as large multi-commodity flow problem with side constraints defined on a time-expanded network. These problems are often severely degenerate, which leads to poor performance of standard linear programming (LP) techniques. Also the large number of integer variables can make finding optimal integer solutions difficult and time consuming. The methods used to attack this problem include an interior-point algorithm, dual steepest edge, simplex, cost perturbation, model aggregation, branching on set-partitioning constraints and prioritizing the order of branching.

The fleet assignment solution must satisfy balance constraints that force the aircraft to circulate through the network of flights. The balance of aircraft is enforced by the conservation of flow equations for a time expanded multi-commodity network. A typical mathematical formulation is given in the paper by Hane et al. (1995), which presents the formulation and discusses various methods to decrease the size of the problem.

While a great deal of work has been devoted to flight scheduling, crew scheduling, equipment selection and usage, and passenger-mix optimization, very few optimization models exist for solving maintenance-scheduling problems. A large-scale mixed integer programming formulation is given in Feo and Bard (1989). Though this approach is a significant one, even a LP relaxation of this formulation is likely to be too large to solve. Also, in this formulation a cyclic constraint is not considered though this is very important in the case of weekly schedules and a homogeneous fleet is considered for simplicity rather than a heterogeneous fleet.

Few research papers have considered the human involvement in obtaining an optimal solution. A human interaction system was introduced to solve the maintenance-scheduling problem by Brio (1992). "Finally if, nothing helps, the best partial solution obtained so far is produced for the human planner to try his hand" (Ritcher, 1989). "Few optimization models exist for solving maintenance related problems" (Feo and Bard, 1989). These approaches were innovative, but more emphasis is placed in human judgment.

Hane et al. (1995) formulated a basic fleet assignment problem that considered maintenance and crew constraints. In this paper only maintenance checks of short duration are considered. The time frame of this fleet assignment problem is one day. For maintenance that occurs every x days, the number of aircraft that are to undergo maintenance check is calculated as $100/x\%$ of the planes in the fleet. This approach works well for daily schedule, but not for the weekly schedule.

"If fleeting and rotation were combined into one problem, maintenance scheduling could be handled in one step, but a model that included both would be computationally intractable" (Hane et al., 1995). The cyclic constraint and the heterogeneity of the fleet, would make the maintenance-scheduling problem much more difficult to solve. This led many of the researchers to avoid considering at least these two factors together. The basis of all maintenance-scheduling problems given by Feo and Bard (1989), does not consider the heterogeneity of the fleet and the cyclic constraint is not considered for any particular given period. This means that the solution of the problem would decide the period of cycle, which in reality determined by the demand in the market. Other work related to maintenance scheduling is reported in Clarke et al. (1997), Kabbani and Patty (1992), Talluri (1996), and Barnhart et al. (1998).

In order to minimize the maintenance cost, the aircraft have to be assigned to the appropriate maintenance bases. This might involve change in flight schedule. The change in flight schedule might result in loss of revenue or increase in operation cost. Therefore there is a tradeoff between assignment of aircraft to the appropriate maintenance bases and maintaining the flight schedule. In the past several people have worked on this problem, but most of them have focused on solving the problem without considering cyclic constraints, heterogeneity in fleet and Type B maintenance constraints. The very first attempt has been made in this paper to consider the two important maintenance requirements with cyclic constraints and heterogeneous fleet. Since most of the flight schedules are weekly schedules, cyclic constraints are one of the most important constraints to be considered.

From the above list we can see that there is not much change in the formulation of maintenance-scheduling problem. Most of the researchers formulated the problem as a multi-commodity network flow model. But there is a vast difference in computational approach. Each paper differs from the others just in the approach to solving the problem. It is very evident from the above list that the challenge the researchers are facing in aircraft scheduling, routing and maintenance scheduling is not in formulating the problems but rather in solving the formulated models.

It is also observed that an important Type B maintenance check was not considered in any of the formulations found in the literature. This paper attempts to fill the gap of considering the maintenance Type B check, and suggest a good heuristic approach to solve medium size problem.

3. Problem formulation

The maintenance-scheduling problem is most naturally modeled as a closed loop network. Using OD schedule as input, the maintenance-scheduling problem is formulated as a min-cost multi-commodity network flow model with integer restrictions on the variables, as such each plane represents a separate commodity. Usually fleet of different aircraft is considered rather than individual aircraft in scheduling, but since maintenance requirement is considered in this scheduling problem, each aircraft should be considered as separate commodity. Each OD pair segment has an upper and lower capacity of one unit of flow. Now define n_d as the planning horizon and n_c as the number of cities in the OD schedule. The total number of nodes in the underlying graph is $n_d n_c$. But not necessarily each node should have input and output arcs. If n_p is the total number of planes in the fleet, then the total number of arcs is $n_d n_p$. This is because each aircraft is assigned to exactly one OD segment, and each OD segment is covered by exactly one aircraft. This produces a large graph.

Origin is the airport where an aircraft leaves in the early morning after spending previous night. Destination is the airport where the aircraft spends the night for that day. Each arc in the network represents a unique OD trip assigned to an aircraft in the flight schedule. In case of two aircrafts having the same Origin and Destination for any given day, then each trip of those aircraft for that day is identified by the subscript r. For example an aircraft is assigned to a trip, which start, from

Washington to Austin to Las Vegas to Los Angeles and another aircraft is assigned to a trip from Washington to Chicago to Arizona to Los Angeles in the original flight schedule. In this case for both trips Washington is the Origin and Los Angeles is the Destination. Since the proposed formulation only needs to consider the Origin and Destination (intermediate stops are not considered), to make these two trips unique subscript r is introduced. In this formulation we do not need to consider the intermediate stops because they have no effect on the maintenance schedule. The flight schedule including the sequence of flight legs that are to be flown by an aircraft during a day is an input to the problem and we refer to it as an OD trips. The proposed formulation is only concerned with assignment of aircrafts to these OD trips.

A three cities, three planes, 7-day network is shown in Fig. 1. Although three disjoint paths are indicated they are by no means unique. A close look at the network given in Fig. 1 reveals that there are four nodes (1(2), 2(5), 1(6) & 3(7)) that are different from the rest of the nodes. These four nodes have more than one incoming and outgoing arcs. Therefore, only at these nodes a decision must be made regarding which incoming aircraft should be assigned to which outgoing arc. Over the planning horizon, aircraft can be assigned a variety of routes depending on their maintenance needs and the availability of facilities. Next consider the cyclic network depicted in Fig. 2 obtained from Fig. 1 by contracting nodes 2(8) and 3(8). In effect, the planning horizon is balanced. That is, the in-degree and out-degree for each node that represents a city on a particular day is the same (the number of a particular type of aircraft that are going into a node is the same as those going out of that node). This implies that the graph is Eulerian so removing any cycle leaves the Eulerian property.

Among the factors considered in maintenance scheduling are passenger demand, revenue, seating capacity, fuel costs, crew size and maintenance costs. Many of these factors are captured in the objective coefficients of the decision variables and others are captured by the constraints. For example, the potential revenue generated by flights can be determined by forecasting the demand for seats on those flights and multiplying the minimum of the forecasted demand and the seat capacity by the average fare. This model satisfies balance constraints that force the aircraft to circulate through the network of flights.

The assumptions that are made in the proposed formulation are as follows:

1. Aircraft maintenance checks are performed only during the night.
2. There is no aircraft operation during the night.
3. The maintenance bases are located at the airport.

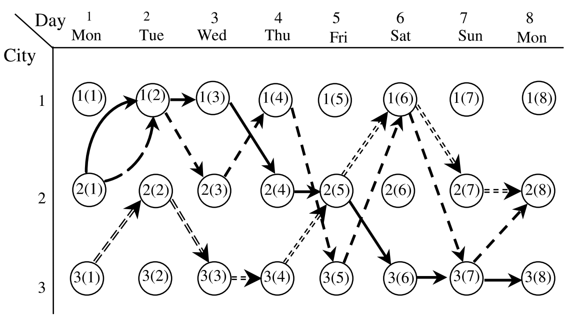

Fig. 1. City–Day network with 7-day planning horizon.

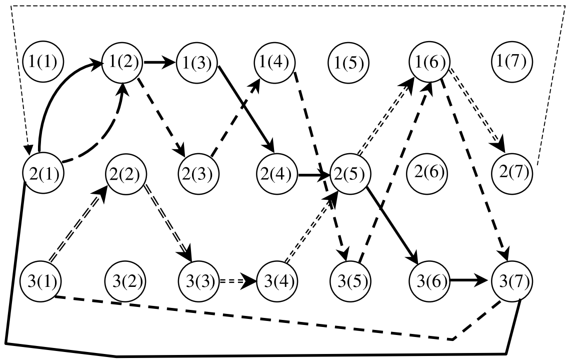

Fig. 2. City–Day network with infinite planning horizon.

The following notation is used in the model:

n_p : number of planes in the fleet.
n_c : number of cities in the network.
n_d : number of days in the planning horizon.
i : index for planes $i = 1, 2, \dots, n_p$.
j, k : index for cities; $j, k = 1, \dots, n_c$.
d : index for days; $d = 1, 2, 3, \dots, n_d$. $n_d + 1 = 1$, $n_d + 2 = 2$, $n_d + 3 = 3$, because of the cyclic schedule. Like the same one day before day $d = 1$ is n_d, and 2 days before the day $d = 1$ is $n_d - 1$.
$j(d)$: city j on day d (a node in a network).
$j(d-1)k(d)r$: link Connecting the city j on day $(d-1)$ to city k on day d through route r.
$G(k(d))$: the set of nodes that are connected to the node $k(d)$.
$F(k(d))$: the set of nodes that the node $k(d)$ is connected to.
L : set of all edges in the graph defined by OD schedule.
N : set of all nodes in the graph defined by OD schedule.
g_{ij} : the Type A maintenance cost for aircraft i at city j.
h_{ij} : a fraction of the Type B maintenance cost for aircraft i at city j. The reason for taking a fraction of the cost rather than the cost is explained later in this section where constraints are explained.
p_j : the number of aircraft that can take Type A maintenance check at city j.

The following decision variables are used in the formulation:

$w_{ij(d)}$ = 1, if aircraft i takes the maintenance check of Type A at city j on day d, 0 otherwise.
$Z_{ij(d)}$ = 1, if aircraft i takes the maintenance check of Type B at city j on day d, 0 otherwise.
$C_{ij(d-1)k(d)r}$ = the cost for assigning the aircraft i to the link $j(d-1), k(d)r$.
$x_{ij(d-1)k(d)r}$ = 1, if aircraft i is assigned to the link $j(d-1)k(d)r$, 0 otherwise.
y_{id} = the number of days left until the due date of aircraft i's Type A maintenance check.

The mathematical formulation is as follows:

$$\min \sum_{i=1}^{n_p} \sum_{j=1}^{n_c} \sum_{d=1}^{n_d} (g_{ij} w_{ij(d)} + h_{ij} Z_{ij(d)}) + \sum_{i=1}^{n_p} \sum_{j(d-1)k(d)r \in L} C_{ij(d-1)k(d)r} x_{ij(d-1)k(d)r} \tag{3.1}$$

Subject to :

$$\sum\nolimits_{j(d-1)r} x_{ij(d-1)k(d)r} - \sum_{j(d+1)r} x_{ik(d)j(d+1)r} = 0 \tag{3.2}$$

$$j(d-1)r \in G(k(d)), j(d+1)r \in F(k(d)),\ \forall i\ \&\ k(d), \quad \text{where } d = 1,2,\ldots,n_d$$

$$\sum_{i=1}^{n_p} x_{ij(d)k(d+1)r} = 1 (j(d)k(d+1)) \in L, \quad \text{where } d = 1,2,\ldots,n_d \tag{3.3}$$

$$\sum_{i=1}^{n_p} w_{ij(d)} \leqslant p_j \qquad \forall j(d), \quad \text{where } d = 1,2,\ldots,n_d \tag{3.4}$$

$$W_{ij(d)} - \sum_{k(d-1)r} x_{ij(d-1)j(d)r} \leqslant 0, \quad k(d-1)r \in G(j(d)),\ \forall j(d), \quad \text{where } d = 1,2,\ldots,n_d \tag{3.5}$$

$$\sum_{j=1}^{n_c} \sum_{d=m}^{m+3} w_{ij(d)} \geqslant 2\ \forall i, \quad m = 1,2,\ldots,n_d \tag{3.6}$$

$$\sum_j (d) Z_{ij(d)} = 1 \quad j(d) \in N,\ \forall i, \tag{3.7}$$

$$Z_{ij(d)} - \sum_{k(d-1)r} x_{ik(d-1)j(d)r} \leqslant 0, \quad k(d-1)r \in G(j(d)), \forall j(d), \quad \text{where } d = 1,2,\ldots,n_d \tag{3.8}$$

$$\sum_{j(2)r} x_{ij(2)k(3)r} = 1 \qquad \forall i\ \&\ k(3), j(2)r \in G(K(3)) \tag{3.9}$$

$$x_{ij(d)k(d+1)}, w_{ij(d)}, Z_{ij(d)} = 0,1 \text{ binary variables}$$

The Objective function (3.1) consists of three components. The first one is the total cost of Type A maintenance check. The second one is the total cost of Type B maintenance check and the third one is the penalty for assigning inappropriate aircraft to the OD trips.

Constraints (3.2) ensure that the aircraft that come into the city k on day d should leave city k on the next day. Constraints (3.3) ensure that any OD trip could be served by one and only one aircraft. Constraints (3.4) are node capacity constraints for Type A maintenance checks that ensure that each maintenance base can serve only a certain number of aircraft at a time. Typically the capacity is based on the number of available mechanics and the inventory of various parts of the aircraft available. Constraints (3.5) are availability constraints of aircraft i at city j on day d for Type A maintenance check. These constraints ensure that aircraft i is assigned for maintenance check of Type A at city j on day d only if it is available at city j on day d. Constraints (3.6) force the aircraft to undergo Type A maintenance check once in 4 days.

Constraints (3.7) ensure that the aircraft undergo their Type B maintenance check once in the given cycle. Type B maintenance check is usually done once in two months, so if the given cycle is say seven days (weekly schedule), it does not necessarily mean that the aircraft has to take the Type B maintenance check once a week. Since the schedule is repeated every week, the Type B maintenance can be done once in every eight sweeks at the optimal city and day. That is why a fraction of the Type B maintenance check cost is used as a cost coefficient rather than the actual cost. So in case of the scenario explained before, one eighth of the actual cost would be used as the cost coefficient for Type B maintenance check.

Constraints (3.8) are availability constraints of aircraft i at city j on day d for Type B maintenance check. These constraints ensure that aircraft i is assigned for maintenance check of Type B at city j on day d only if it is available at city j on day d.

Constraints (3.9) are to make sure that the same aircraft is not used for two cyclic schedules. In other words, the same aircraft should not be assigned to more than one link between any two given consecutive days. Since we have the conservation of flow constraint, any one of the two consecutive days can be chosen to restrict the assignment of one aircraft to only one link. In the formulation given above days 2 and 3 are chosen as two consecutive days.

The important difference between the new formulation and the existing one by Feo and Bard (1989) are:

1. Inclusion of a penalty for assigning inappropriate aircraft to a link. This penalty was not considered in the existing formulation, which would lead to assigning small aircraft to the links that require large ones. This would result in loss of revenue. Conversely if the large aircraft were assigned to the links where small ones are enough, it would result in higher operating cost.
2. Consideration of maintenance Type B check.
3. Simple and efficiently constructed constraints, which force the aircraft to undergo Type A maintenance check once in four days.

Fortunately, a number of simplifications to this problem are possible due to the economics of maintenance base operations. The fact that spare parts are often not interchangeable implies that a significant fraction of the costs defining the coefficients in (3.1) can be separated out by plane type. The only remaining issue is labor, and while important in other contexts, it is not a critical factor here.

The second simplification relates to capacity. Because an ample supply of space and labor are usually available, the capacity constraint (3.4) is rarely binding, and can be eliminated. But even with both of these reductions, there is little possibility of solving the LP relaxation of the resultant min-cost, multi-commodity integral flow problem. So it is almost impossible to find a global optimal solution by solving the LP problem. A heuristic method is suggested in this paper and tested for ample number of cases to prove that the solutions obtained by the heuristic are as close as 5% to the global optimal solutions. Although these simplifications are possible, it should be noted that in all test problems presented in this paper we did not implement these simplifications. In other words, the problems that are solved optimally include all variables and constraints presented in the original formulation. Also, the heuristic solutions obtained for these problems using the proposed method are all feasible for the integer programming problem formulated in (3.1)–(3.9).

The maintenance-scheduling problem can also be formulated based on aircraft flight hours rather than based on number of days between two consecutive maintenance checks of a particular type. This kind of formulation would be more appropriate if maintenance scheduling is done strictly by flight hours. In this type of formulation, however, the full planning horizon has to be taken into account as opposed to the cyclic schedule. This would tend to increase the problem size significantly. A formulation considering flight hours as maintenance criteria is given below, though no attempt was made to solve the problem exactly or to suggest any heuristic procedure for its solution.

In presenting this formulation, we use the same notation that was used in the previous formulation with some added new definitions. The new variables are explained below:

$Y_{i(d)}$ = Total flight hours from recent Type A maintenance check till day d.
$V_{i(d)}$ = Total flight hours from recent Type B maintenance check till day d.
F_{A} = Maximum flight hours allowed by Federal Aviation Administration between two consecutive Type A maintenance checks.
F_{B} = Maximum flight hours allowed by Federal Aviation Administration between two consecutive Type B maintenance checks.
H_{A} = An arbitrary number greater than $F_{\mathrm{A}}+24$.
H_{B} = An arbitrary number greater than $F_{\mathrm{B}}+24$.
ndp = Number of days in the planning horizon.
$t_{j(d)k(d+1)r}$ = Flight hours to cover the OD pair $j(d)k(d+1)r$.

The mathematical formulation for maintenance scheduling based on flight hours is as follows:

$$\min \sum_{i=1}^{n_{\mathrm{p}}}\sum_{j=1}^{n_{\mathrm{c}}}\sum_{d=1}^{n_{\mathrm{d}}}(g_{ij}w_{ij(d)}+h_{ij}Z_{ij(d)})+\sum_{i=1}^{n_{\mathrm{p}}}\sum_{j(d-1)k(d)r\in L}C_{ij(d-1)k(d)r}x_{ij(d-1)k(d)r} \tag{3.10}$$

Subject to :

$$\sum_{j(d-1)r}x_{ij(d-1)k(d)r}-\sum_{j(d+1)r}x_{ik(d)j(d+1)r}=0 \quad j(d-1)r\in G(k(d)),j(d+1)r\in F(k(d)),\forall i\ \&\ k(d),\quad \text{where } d=2,3,4,\ldots,ndp-1 \tag{3.11}$$

$$\sum_{i=1}^{n_{\mathrm{p}}}x_{ij(d)k(d+1)r}=1\quad \forall(j(d)k(d+1))\in L,\quad \text{where } d=1,2,\ldots,ndp-1 \tag{3.12}$$

$$\sum_{i=1}^{n_{\mathrm{p}}} w_{ij(d)}\leqslant p_j\quad j(d),\quad \forall d, \tag{3.13}$$

$$w_{ij(d)}-\sum_{k(d-1)r}x_{ik(d-1)j(d)r}\leqslant 0,\quad k(d-1)r\in \mathrm{G}(j(d)),\forall j(d),\quad \text{where } d=2,3,\ldots,ndp \tag{3.14}$$

$$Y_{i(d+1)}\geqslant Y_{i(d)}+\sum_{j(d)k(d+1)r}\left(x_{ij(d)k(d+1)r}\times t_{j(d)k(d+1)r}\right)-H_{\mathrm{A}}\sum_{j(d+1)}w_{ij(d)}\quad j(d),\forall i,\quad \text{where } d=1,2,\ldots,ndp-1 \tag{3.15}$$

$$V_{i(d+1)}\geqslant V_{\mathrm{i(d)}}+\sum_{j(d)k(d+1)r}\left(x_{ij(d)k(d+1)r}\times t_{j(d)k(d+1)r}\right)-H_{\mathrm{B}}\sum_{j(d+1)}Z_{ij(d)}\quad j(d),\forall i,\quad \text{where } d=1,2,\ldots,ndp-1 \tag{3.16}$$

$$Z_{ij(d)} - \sum_{k(d-1)r} x_{ik(d-1)j(d)r} \leqslant 0, \quad k(d-1)r \in \mathrm{G}(j(d)), \forall d \ \&\ j(d)$$

$$\text{where } d = 2, 3, \ldots, ndp \tag{3.17}$$

$$\sum_{j(2)r} x_{ij(2)k(3)r} = 1 \quad j(2)r \in \mathrm{G}(k(3)), \forall i \ \&\ k(3) \tag{3.18}$$

$$x_{ij(d)k(d+1)}, w_{ij(d)}, Z_{ij(d)} = 0, 1 \text{ binary variables}, \quad Y_{i(d)} < F_{\mathrm{A}} \qquad \forall d \ \&\ i,$$

$$\text{where } d = 2, 3, \ldots, ndp \tag{3.19}$$

$$V_{i(d)} < F_{\mathrm{B}} \qquad \forall d \ \&\ i, \quad \text{where } d = 2, 3, \ldots, ndp \tag{3.20}$$

$$Y_{i(1)} = V_{i(1)} = 0 \qquad \forall i \tag{3.21}$$

The objective function (3.10) is the same as the objective function in the previous formulation that was expressed in Eq. (3.1). Also constraints (3.11) through (3.14) are the same as constraints (3.2) through (3.5) except that the former is repeated for "$d = 2, 3, 4, \ldots, ndp - 1$" and the later for "$d = 1, 2, \ldots, n_{\mathrm{d}}$". Similarly, constraints (3.17) and (3.18) are the same as those expressed in (3.8) and (3.9) except that the former is repeated for "$d = 2, 3, 4, \ldots, ndp - 1$" and the later for "$d = 1, 2, \ldots, n_{\mathrm{d}}$".

Constraints (3.15) along with constraints (3.19) ensure that the aircraft have maintenance check Type A at least once after flying F_A consecutive hours. Constraints (3.16) along with constraints (3.20) ensure that the aircraft will not fly in excess of F_B hours before completing a maintenance Type B check. Constraints (3.21) are initializing the Y and V variables.

As mentioned before this formulation results in a very large integer programming problem that cannot be solved exactly in reasonable computation time. Development of heuristic procedures for solving this problem is left for future research. In the next section we focus on presenting an efficient solution procedure for the original formulation of the problem presented in (3.1)–(3.9) that is based on the limitations on the number of days between two consecutive maintenance checks. This formulation is more appropriate because airlines normally impose upon themselves restrictions on the number of days between consecutive maintenance checks that are generally more stringent than those mandated by the FAA.

4. Solution approach

This search technique is a combination of depth first search and random search. First a list of nodes in a given day is made. The first aircraft and the first node are chosen from the respective list. Then an exhaustive depth first search is performed from the node to find the best cyclic schedule for the chosen aircraft. Then the assigned links are removed from the network. Now the second aircraft is chosen from the list, if the number of outgoing arcs in the first node is not equal to zero, again a depth first search is performed to find the best cyclic schedule for the second aircraft. If the number of outgoing arcs is equal to zero, then the second node from the node list is chosen and an exhaustive search is performed from this node to find the best cyclic schedule for the aircraft. This procedure is repeated until there is no more aircraft or nodes in corresponding list.

After processing all the aircraft in the above-mentioned procedure, a feasible schedule and the objective function value is found. This objective value is compared with the objective value obtained from the previous iteration and the better solution is saved. For the next iteration the node list and aircraft list are perturbed (shuffled). The procedure is performed for the new list of nodes and aircraft. The steps of the algorithm are as follows:

Step 0: Make a list of aircraft in any order. Make another list of nodes (cities) for any given day. Initialize number of iterations = 1.
Step 1: Let $n = 1$.
Step 2: Pick the nth aircraft from the list of aircraft.
Step 3: Let $K = 1$.
Step 4: Pick the Kth node from the list of nodes.
Step 5: If there are no more nodes available for the allocation in the Kth node, let $K = K + 1$ and go to step 4, otherwise go to step 6.
Step 6: Do depth first search to find the best possible cyclic schedule for the nth aircraft. If a feasible cyclic schedule exists go to step 7, otherwise let $K = K + 1$, go to step 5.
Step 7: Add the route to the schedule. Delete the arcs from the network that are assigned to the nth aircraft.
Step 8: If n = number of aircraft, (a) reconstruct the network (in step 7 arcs are removed from the network, for new iteration, these arcs need to be placed back in the network), (b) perturb the aircraft list randomly (construct a list by choosing each aircraft in random from the list of aircraft), (c) perturb the node list randomly (construct a list by choosing each node in random from the list of nodes that belongs to any given day). If a feasible solution was found in the previous iterations, compare the current solution with the existing one and if the current one has a lower objective function value save the current solution and delete the previous solution. Otherwise let $n = n + 1$ and go to step 2.
Step 9: If the number of iterations is less than the maximum number of iterations, increment the number of iterations and go to step 2, otherwise stop.

Fig. 3 shows the flow chart of this heuristic procedure. In all test problems, the maximum number of iterations is set at 5000.

5. Performance analysis

This section presents the computation results obtained with the heuristic approach and those obtained from directly solving the IP formulation. CPLEX optimization software is used to obtain the exact solutions for the IP formulations of the test problems. The combinatorial nature of the maintenance-scheduling problem increases the computation time astronomically even for a moderate size problem. It is quite difficult to solve the IP formulation in a reasonable execution time using any commercial IP solver software, such as LINDO and CPLEX. Therefore, we were not able to have exact solutions for all test problems. This computation study is intended to verify the effectiveness of the proposed heuristic algorithm. The proposed heuristic algorithm is coded in the C++ language.

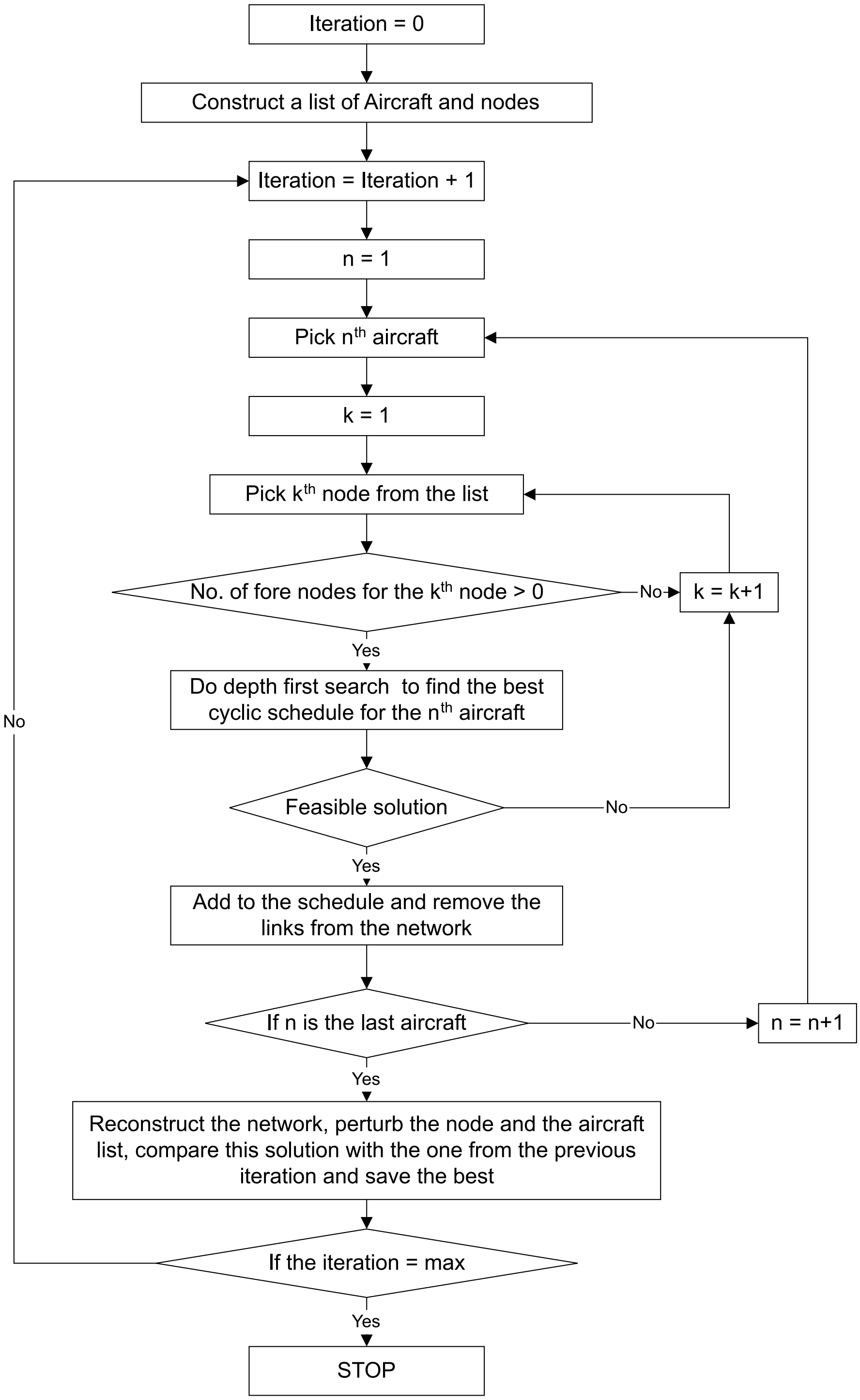

Fig. 3. Heuristic solution flow chart.

Nine sets of problem were solved in this study. Each set of problems is identified by the number of cities considered in the problems in that set. A program written in the C language randomly generated all of them. This program takes the number of cities and the number of aircraft as input and generates a random network with cyclic constraint being satisfied. This program assigns randomly the link costs and the maintenance Type A check costs and Type B check costs for the aircraft at different cities. In each set of problems the number of cities have been kept constant. Since the commercial IP solver (CPLEX) could not solve even medium size problem, the first category of problems generated were small size problems to compare the results with the CPLEX solution. In these problems the ratio of the cities to aircraft in most cases was kept high for the same reason. There are five problem sets in this category with 7, 8, 9, 10, and 11 cities respectively. 18 test problems were created in these problem sets (1 through 18 respectively).

Next, a set of problems with 20 cities was generated. These are larger problems with smaller number of aircraft. There are eight problems in this set (19 through 26 respectively). We managed to solve these problems using the heuristic procedure as well as CPLEX. These problems stretched CPLEX almost to its limits in terms of the computation time. Solving one of the problems (number 26 with 20 cities and 13 aircraft) using CPLEX took 16,000 s or almost 4.5 h! Finally three other problem sets were generated with much larger number of cities and aircraft. These were problem sets with 60, 65, and 75 cities (problems 27 through 44 respectively). These problems were all solved using the heuristic procedure. Because of the size of these problems, it is impractical to solve them optimally and therefore we did not use CPLEX to solve them.

As an example, we discuss problem number 8 in more detail. Fig. 4 shows the network generated by the program for problem number 8. This is the network that represents seven cities and a 7-day planning horizon. In problem number 8, eight aircraft are to be assigned to this network. The cost matrix is also shown in Table 1. This network was modeled as an IP problem and solved using CPLEX. The heuristic approach was also used to solve the same problem. The program written to implement the heuristic stores the assignments of aircraft in a text form. Table 2 shows the results obtained from heuristic. In Table 2 each row represents an aircraft. Columns Day 1 through Day 7 represent the city in which the aircraft stays for the night.

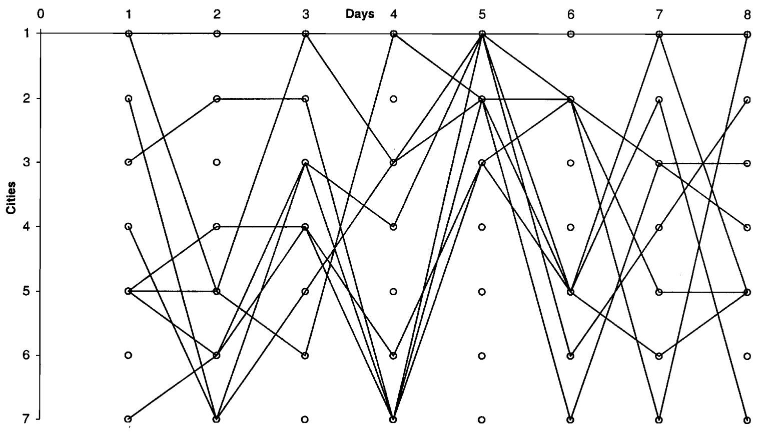

Fig. 4. Randomly generated network.

Table 1
The cost for maintenance check A for each aircraft in the fleet

Aircraft	Maintenance Type A cost in						
	City 1	City 2	City 3	City 4	City 5	City 6	City 7
1	2	5	10	10	4	3	1
2	4	6	1	1	4	5	9
3	3	7	3	8	7	9	10
4	7	10	3	7	10	4	8
5	10	7	2	2	5	2	9
6	4	1	6	10	9	6	9
7	7	1	4	4	1	4	4
8	1	6	7	5	4	5	10

Table 2
The aircraft assignment made by the heuristic

Aircraft	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7	Day 1
1	1	2	3	4	7	5	3	1
2	2	5	6	5	5	1	3	2
3	1	5	1	5	6	3	4	1
4	2	7	3	3	2	2	7	2
5	1	6	4	2	7	3	7	1
6	2	2	7	1	5	6	1	2
7	3	2	5	5	4	4	7	3
8	3	5	2	7	6	4	6	3

Table 3 shows the aircraft maintenance assignment. Each row represents the maintenance schedule for an aircraft in the fleet. "Type Check A Day 1" and "Type Check A Day 2" columns represent the days on which the aircraft undergoes Type A maintenance check and "Type B Check" column represent the day on which the aircraft undergoes Type B maintenance check.

Table 4 shows the comparison of the results obtained from the heuristic procedure and those of CPLEX for the problems 1 through 18. As this table indicates, the heuristic solution procedure

Table 3
The maintenance schedule generated by heuristic

Aircraft	Type Check A		Type Check B
	Day 1	Day 2	
1	1	5	5
2	4	7	3
3	3	6	1
4	3	7	1
5	2	6	1
6	1	4	6
7	1	4	1
8	2	6	6

Table 4
Performance of heuristic in several small size problems

Problem no.	No. of cities	No. of flights	Objective value		CPU time		% Difference
			Heuristic	Global	Heuristics	CPLEX	
8	7	8	213	208	21	30	2.4
9	7	9	229	224	32	35	2.2
10	7	10	251	240	47	160	4.6
11	7	11	271	259	59	650	4.6
4	8	8	222	215	20	8	3.3
27	8	9	221	219	33	30	0.9
5	8	10	277	265	45	758	4.5
28	9	8	200	200	21	3	0.0
13	9	9	234	228	31	15	2.6
15	9	11	247	238	63	500	3.8
16	9	12	270	259	78	1750	4.2
29	10	8	199	199	26	3	0.0
30	10	9	211	204	35	26	3.4
17	10	11	243	236	62	145	3.0
20	10	14	335	332	100	3250	0.9
31	11	8	197	197	16	4	0.0
32	11	9	210	208	33	17	1.0
33	11	10	236	233	49	319	1.3

produces solutions that are very good. In three cases the heuristic procedure produces the optimal solution. In all other cases the heuristic procedure produces solutions that are at worse 4.6% from the optimal solution in terms of the objective function value. Table 5 compares the performance of the heuristic with that of CPLEX for the test problem with 20 cities. The results in these problem sets are even better. In 4 out of 8 cases the heuristic solution is the optimal solution. In the other 4 cases where the procedure does not obtain the optimal solution, the objective functions only differ by at most 2.7%. However, when comparing the time required for obtaining the solution the results are extremely encouraging. As Table 5 shows the heuristic procedure produces solutions in a matter of seconds while for larger problems CPLEX require up to 4.5 h to solve the problem.

Table 5
Comparison of performance of the heuristic and CPLEX solutions

Problem No.	No. of cities	No. of aircraft	Objective value		CPU time (s)		% Difference
			Heuristic	Global	Heuristic	CPLEX	
19	20	6	154	154	1	20	0.0
20	20	7	182	182	3	22	0.0
21	20	8	192	192	17	35	0.0
22	20	9	215	214	25	80	0.5
23	20	10	241	241	38	290	0.0
24	20	11	277	270	49	2700	2.6
25	20	12	304	301	60	4600	1.0
26	20	13	342	333	72	16000	2.7

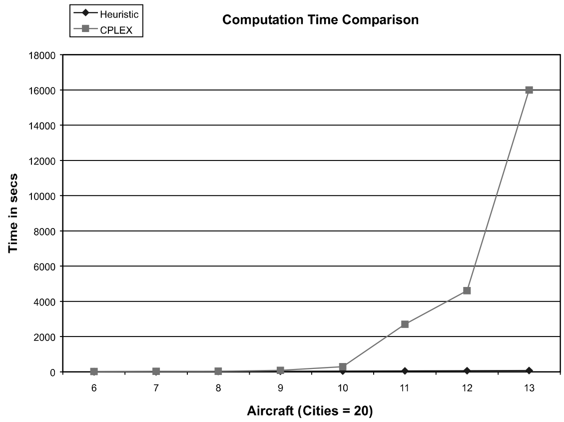

Fig. 5. Comparison of performance of heuristic II and CPLEX solutions.

Fig. 5 shows the comparison of the solution times versus the size of the problem for the two approaches in a graph.

Table 6 shows the results obtained for larger size problems using the heuristic. It indicates the objective function value and the time required for obtaining the solution. While we do not have a comparison with the optimal solutions because we could not solve these problems exactly, we are confident that the heuristic approach has produced good solutions for these problems given the results discussed above and based on numerous other tests that we have performed.

Table 6
Performance of heuristic in larger size problems

Problem No.	No. of cities	No. of flights	Objective value	CPU time (min:s)
			Heuristic	Heuristic
27	60	35	812	2:10
28	60	36	935	2:08
29	60	37	857	2:03
30	60	38	938	1:32
31	60	39	914	2:15
32	60	40	966	2:12
33	65	41	1036	2:20
34	65	42	1015	2:45
35	65	43	1022	2:56
36	65	44	1048	3:01
37	65	45	1100	3:53
38	75	46	1084	4:30
39	75	47	1033	4:21
40	75	48	1102	3:56
41	75	49	1098	4:00
42	75	50	1150	4:06
43	75	55	1101	4:14
44	75	58	1166	5:00

6. Conclusions

This paper presented an innovative formulation for the aircraft re-assignment and maintenance scheduling and a heuristic method to solve the problem efficiently and quickly. Since the solution time for solving larger integer-programming model gets into astronomical units, a heuristic was proposed in this paper to solve larger problems. This heuristic is a hybrid of random search and depth first search. The closeness of the solution produced by this heuristic to the global optimal solution largely depends on how many different combinations of orders of aircraft and orders of nodes were explored.

The consideration of constraints for typical maintenance checks in an aircraft maintenance scheduling has been shown to be a critical one for any airline operation. The key to solve a large scale integer programming problem is development of efficient heuristic procedures. A ample number of problems were tested for comparing the performance of the heuristic against the commercial software CPLEX. The results show that the heuristic consistently produces good solutions, with objective values within 5% of the global optimal solutions. The heuristic presented also solves the problem in a very reasonable amount of time. We also considered whether the most popular random search method, namely, genetic algorithms could be used to solve the large size problem. But, too many constraints involved in this problem would make the search inefficient and computationally intensive for a genetic algorithm.

The way Type B maintenance check is taken into account in the formulation given in this paper may not be an efficient one. Another way of considering Type B check would involve scheduling for longer planning horizon. This would certainly result in a tremendous increase in the number of variables and constraints that is certainly not favorable in an integer program formulation.

There are a number of improvements that can be made to the above-mentioned formulation and the heuristic approach presented in this paper. One such improvement could be in considering the flight legs rather than the OD pairs. This would increase the size of the problem enormously but an efficiently designed heuristic could handle the problem. It is not clear whether this type of formulation would result in a significant improvement over the one presented in this paper. Another area for research could be considering the linking of two consecutive OD segments. This would increase the number of constraints in the problem, but still may not have a significant impact on the computation. The scope of the model can be increased with respect to the length of the decision horizon (may be bi-weekly or monthly schedule). Developing a procedure to generate a lower bound on the value of the objective function is yet another fruitful area for future research. Such lower bounds are the best measures of the quality of the solutions obtained using the heuristic procedures especially in large size problems where exact solutions cannot be obtained.

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