Dynamic Density Algorithms Used by The Target Generation Facility

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Introduction

The FAA is currently examining and evaluating various metrics for quantification of air traffic complexity and control difficultly as part of a research effort to develop more accurate predictors of potential conflicts. These metrics under development are known as dynamic density. As of 7/21/06, there are variations between the numbers generated by the Target Generation Facility using certain metrics and by the numbers generated by previous implementations. Consequently, at the request of Parimal Kopardekar of the NASA Ames Research Center, and Sherri Magyarits and Albert Schwartz of the FAA William J. Hughes Technical Center, this document was created to highlight differences in the implementation of metrics utilized by the Target Generation Facility and a source document from MITRE.

In an effort to clarify differences, alterations to the algorithms are colored in red in the subsequent tables. The following sections are of special interest:

- TGF2b – This is a new definition introduced to provide an alternate means of quantifying the volume in a sector occupied by aircraft. This definition is necessary to realistically model sparse sectors since the original definition overestimates the sector volume occupied in such a situation.
- TGF12, TGF13 – In both metrics, the original definition of W_{ij} includes counting the distance between an aircraft and itself. The purpose of this formula is to weight the distances between two different aircraft. The distance between an aircraft and itself is zero and does not correspond to a distance between two unique aircraft.
- TGF 12 - 17 – The vertical and horizontal caps for minimum separation were recently corrected to resolve inconsistencies discovered during the preparation of this document.
- TGF 13, TGF 15, TGF 17 – 20, TGF 23 – While reviewing definitions and equations in the document “Detailed Algorithm Descriptions,” 2002, MITRE Corporation, possible errors merit further exploration.

Change Log for Dynamic Density

10/17/2006	Corrected error in calculation of J
11/29/2006	Corrected error in TGF40, removed minimum.
12/04/2006	Added mod 90 to TGF50. Fixed error in TGF in v2.
12/05/2006	Added absolute value to the heading difference in TGF50.
02/05/2007	Change in desription for TGF46. Headvar modified for TGF48.
03/14/2007	Added TGF 51.
07/16/2007	Fixed typos in TGF18-20,23 –"(c)oordinates". Added Appendix A: Variables. Minor formating changes.
10/31/2007	Changed range of TGF28 from 10-15 nm to 5-10 nm.
11/01/2007	Changed names and formulas of TGF27-32 to reflect number of pairs of aircraft instead of number of aircraft.

Metric	Description	Equations	Variables	Notes
TGF1	Sector count	$C_1 = N$	N = number of aircraft	
TGF2a AD1	Aircraft Density	If $N > 2$ $C_{2a} = N/V_{hull}$ If $N = 1$, $C_{2a} = C_{2b}$ If $N = 2$ $C_{2a} = N/V_*$ $V_* = r(r + d_{12})(h_{12} + \Delta h)$	N = number of aircraft V_{hull} = Volume of the bounding polyhedron formed by the aircraft inside the sector Δh = vertical neighborhood parameter r = horizontal neighborhood parameter d_{12} = horizontal separation h_{12} = vertical separation	Bounding polyhedron created by using algorithm to find the convex hull. Δh = 2000 ft and is two times the minimum vertical separation. $r = \begin{cases} 10\text{ nm, if } \textit{en route} \\ 6\text{ nm, if terminal} \end{cases}$ r is based on if the aircraft is en route or in the sector of its destination. In units of 1/nm
TGF2b	Aircraft Density TGF	$C_{2b} = \dfrac{N}{\sum_{1 \le i \le N} V_j}$ $V_j = \Delta h r^2$	N = number of aircraft V = volume of the box formed by the minimal separation parameters Δh = vertical neighborhood parameter r = horizontal neighborhood parameter	Δh = 2000 ft and is two times the minimum vertical separation. $r = \begin{cases} 10\text{ nm, if } \textit{en route} \\ 6\text{ nm, if terminal} \end{cases}$ r is based on if the aircraft is en route or in the sector of its destination. In units of 1/nm
TGF3 AD2	Aircraft Density by Sector	$C_3 = N/V_{sector}$	N = number of aircraft V_{sector} = Volume of the sector	In units of 1/nm
TGF4 CRI	Convergence Recognition Index	-	-	See large algorithm chart
TGF5 SCI	Separation Criticality Index	-	-	See large algorithm chart

Metric	Description	Equations	Variables	Notes
TGF6 DOFI	Degrees of Freedom Index	-	-	See large algorithm chart
TGF7 CTI1	Coordination Taskload Index 1	$C_7 = \sum_{i \in F} (10 - c_i)^2$	c_i = time for aircraft i to be within 5 nm of the sector boundary, in minutes F = the set of all indices of aircraft that are predicted to be within 5 nm of the sector boundary within 10 minutes	In units of min^2
TGF8 CTI2	Coordination Taskload Index 2	$C_8 = \sum_{1 \le i \le N} \left(c_i + (b_i - c_i)^2\right)^{-1}$	N = number of aircraft c_i = time for aircraft i to be within 5 nm of the sector boundary, in minutes b_i = time for aircraft i to reach the sector boundary, in minutes	
TGF9 NASA1.2	Fraction of flights that are climbing	$C_2 = N_{cl} / N$	N = number of aircraft N_{cl} = number of aircraft with climb rate > 200 ft/min	
TGF10 NASA1.3	Fraction of flights that are cruising	$C_3 = N_{lv} / N$	N = number of aircraft N_{lv} = number of aircraft with altitude rate < 200 ft/min in magnitude	

Metric	Description	Equations	Variables	Notes
TGF11 NASA1.4	Fraction of flights that are descending	$C_4 = N_{ds} / N$	N = number of aircraft N_{lv} = number of aircraft with descent rate greater than 200 ft/min in magnitude	
TGF12 NASA1.5	Inverse weighted mean of horizontal separation	$C_5 = \frac{N}{\sum_{1\le i\le N}\left(\frac{\sum_{1\le j\le N} W_{ij} d_{ij}}{\sum_{1\le j\le N} W_{ij}}\right)}$ $W_{ij} = \begin{cases} i \ne j, \left(d_{ij}^2 + S_h^2 h_{ij}^2\right)^{-1} \\ i = j, 0 \end{cases}$	N = number of aircraft W_{ij} = weighting factor d_{ij} = distance between aircraft i and aircraft j h_{ij} = vertical separation between aircraft i and aircraft j S_h = scaling factor	$S_h = 0.0025$ nm/ft when h_{ij} is in feet. W_{ij} is altered so that an aircraft is not compared with itself. Consequently d_{ij} and h_{ij} do not require minimums. In units of 1/nm
TGF13 NASA1.6	Inverse weighted mean of vertical separation	$C_6 = \frac{N}{\sum_{1\le i\le N}\left(\frac{\sum_{1\le j\le N} W_{ij} h_{ij}}{\sum_{1\le j\le N} W_{ij}}\right) S_h}$ $W_{ij} = \begin{cases} i \ne j, \left(d_{ij}^2 + S_h^2 h_{ij}^2\right)^{-1} \\ i = j, 0 \end{cases}$	N = number of aircraft W_{ij} = weighting factor d_{ij} = lateral distance between aircraft i and aircraft j h_{ij} = vertical separation between aircraft i and aircraft j S_h = scaling factor	$S_h = 0.0025$ nm/ft when h_{ij} is in feet. W_{ij} is altered so that an aircraft is not compared with itself. Consequently d_{ij} and h_{ij} do not require minimums. In units of 1/nm

Metric	Description	Equations	Variables	Notes
TGF14 NASA1.7	Inverse average minimum horizontal separation	$C_7 = \dfrac{\sum_{1\le i\le N}[j \in J_i]}{\sum_{1\le i\le N} \min_{j\in J_i}\{d_{ij}\}}$ $J_i = \{j \mid h_i - \Delta h/2 \le h_j \le h_i + \Delta h/2;\ j \ne i\}$	d_{ij} = lateral distance between aircraft i and aircraft j Δh = vertical neighborhood parameter h_i = altitude of aircraft i S_h = scaling factor	$\Delta h = \begin{cases} 4000\,\text{ft}, \min\{h_i, h_j\} > 29000\,\text{ft} \\ 2000\,\text{ft}, \text{else} \end{cases}$ $S_h = 0.0025$ nm/ft when h_i is in feet. J_i = set of aircraft indices such that their corresponing altitudes are within the vertical neighborhood of aircraft i h_{ij}, d_{ij} have no minimums due to changes in W_{ij} (see TGF12 notes) In units of 1/nm
TGF15 NASA1.8	Inverse average minimum vertical separation	$C_8 = \dfrac{\sum_{1\le i\le N}[j \in K_i]}{S_h \sum_{1\le i\le N} \min_{j\in K_i}\{h_{ij}\}}$ $K_i = \{j \mid d_{ij} \le r;\ j \ne i\}$	S_h = scaling factor r = horizontal neighborhood parameter d_{ij} = lateral distance between aircraft i and aircraft j h_{ij} = vertical separation between aircraft i and aircraft j	$r = 10$ nm $S_h = 0.0025$ nm/ft when h_{ij} is in feet. $K_i =$ set of aircraft indices such that heir corresponding a horizontal separations are within the horizontal neighborhood of aircraft i h_{ij}, d_{ij} have no minimums due to changes in W_{ij} (see TGF12 notes) In units 1/nm

Metric	Description	Equations	Variables	Notes
TGF16 NASA1.9	Inverse of minimum horizontal separation in same vertical neighborhood	$C_{16} = \left(\min_{1 \le i \le N} \left\{ \min_{j \in J_i} \{ d_{ij} \} \right\} \right)^{-1}$ $J_i = \{ j \mid h_i - \Delta h/2 \le h_j \le h_i + \Delta h/2 ; j \ne i \}$	d_{ij} = lateral distance between aircraft i and aircraft j Δh = vertical neighborhood parameter h_i = altitude of aircraft i	$\Delta h = \begin{cases} 4000\text{ft}, \min\{h_i, h_j\} > 29000\text{ft} \\ 2000\text{ft}, \text{else} \end{cases}$ J_i = set of aircraft indices such that their corresponding altitudes are within the vertical neighborhood of aircraft i In units 1/nm
TGF17 NASA1.10	Inverse of minimum vertical separation in same horizontal neighborhood	$C_{17} = \frac{1}{S_h} \left(\min_{1 \le i \le N} \left\{ \min_{j \in K_i} \{ h_{ij} \} \right\} \right)^{-1}$ $K_i = \{ j \mid d_{ij} \le r ; j \ne i \}$	S_h = scaling factor r = horizontal neighborhood parameter d_{ij} = lateral distance between aircraft i and aircraft j	$r = 10$ nm $S_h = 0.0025$ nm/ft when h_{ij} is in feet. $K_i =$ set of aircraft indices such that their corresponding a horizontal separations are within the horizontal neighborhood of aircraft i d_{ij} has no minimum due to changes in W_{ij} (see TGF12 notes) In units 1/nm

Metric	Description	Equations	Variables	Notes
TGF18 NASA1.11	Fraction of aircraft with time-to-go to conflict less than 600s	$C_{18} = \frac{\sum_{1 \le i \le N} \sum_{j \in T_i, J_i} 1}{2N}$ $J_i = \{j \mid h_i - \Delta h/2 \le h_j \le h_i + \Delta h/2; j \ne i\}$ $T_i = \{j \mid 0 \le t_{ij} \le \Delta t; j \ne i\}$ $t_{ij} = -\frac{d_{ij}}{\dot{d}_{ij}}$ $\dot{d}_{ij} = \frac{(d_{xij} V_{xij} + d_{yij} V_{yij})}{d_{ij}}$	N = number of aircraft h_i = altitude of aircraft i Δh = vertical neighborhood parameter Δt = time-to-go threshhold t_{ij} = time-to-go $\dot{d}_{ij}$ = range rate d_{xij}, d_{yij} = distance coordinates V_{xij}, V_{yij} = velocity coordinates	$\Delta h = \begin{cases} 4000\,\text{ft}, \min\{h_i, h_j\} > 29000\,\text{ft} \\ 2000\,\text{ft}, \text{else} \end{cases}$ $\Delta t = 600$ s J_i = set of aircraft indices such that their corresponding altitudes are within the vertical neighborhood of aircraft i T_i = indices with time-to-go less than the threshhold value
TGF19 NASA1.12	Inverse minimum time to go to conflict with time to go to conflict less than 600s	$C_{19} = \frac{\sum_{1 \le i \le N} \lfloor j \in T_{i,J_i} \rfloor}{\sum_{1 \le i \le N} \min_{j \in T_i} \{t_{ij}\}}$ $J_i = \{j \mid h_i - \Delta h/2 \le h_j \le h_i + \Delta h/2; j \ne i\}$ $T_i = \{j \mid 0 \le t_{ij} \le \Delta t; j \ne i\}$ $t_{ij} = -\frac{d_{ij}}{\dot{d}_{ij}}$ $\dot{d}_{ij} = \frac{(d_{xij} V_{xij} + d_{yij} V_{yij})}{d_{ij}}$	h_i = altitude of aircraft i Δh = vertical neighborhood parameter Δt = time-to-go threshhold t_{ij} = time-to-go $\dot{d}_{ij}$ = range rate d_{xij}, d_{yij} = distance coordinates V_{xij}, V_{yij} = velocity coordinates	J_i = set of aircraft indices such that their corresponding altitudes are within the vertical neighborhood of aircraft i T_i = indices with time-to-go less than the threshhold value

Metric	Description	Equations	Variables	Notes
TGF20 NASA1.13	Inverse of smallest time to go to conflict for aircraft pairs with time to go to conflict less than 600s	$C_{20} = \left(\min_{1 \le i \le N} \left\{ \min_{j \in T_i, J_i} \left\{ t_{ij} \right\} \right\} \right)^{-1}$ $J_i = \left\{ j \mid h_i - \Delta h/2 \le h_j \le h_i + \Delta h/2;\ j \ne i \right\}$ $T_i = \left\{ j \mid 0 \le t_{ij} \le \Delta t;\ j \ne i \right\}$ $t_{ij} = -\frac{d_{ij}}{\dot{d}_{ij}}$ $\dot{d}_{ij} = \frac{\left(d_{xij} V_{xij} + d_{yij} V_{yij} \right)}{d_{ij}}$	N = number of aircraft h_i = altitude of aircraft i h_{ij} = vertical separation between aircraft i and aircraft j Δh = vertical neighborhood parameter Δt = time-to-go threshhold t_{ij} = time-to-go $\dot{d}_{ij}$ = range rate d_{xij}, d_{yij} = distance coordinates V_{xij}, V_{yij} = velocity coordinates	$\Delta h = \begin{cases} 4000\,\text{ft}, \min\left\{ h_i, h_j \right\} > 29000\,\text{ft} \\ 2000\,\text{ft}, \text{else} \end{cases}$ $\Delta t = 600$ s J_i = set of aircraft indices such that their corresponding altitudes are within the vertical neighborhood of aircraft i T_i = indices with time-to-go less than the threshhold value
TGF21 NASA1.14	Variance of groundspeed	$C_{21} = \sigma_{vg}^2$ $\sigma_{vg}^2 = \frac{\sum_{1 \le i \le N} \left(V_i - \bar{V} \right)^2}{(N-1)}$ $\bar{V} = \frac{1}{N} \sum_{1 \le i \le N} V_i$	N = number of aircraft σ_{vg}^2 = variance of groundspeed V_i = groundspeed of aircraft i $\bar{V}$ = mean of groundspeed	V_i is in nm/hr σ_{vg}^2 is in nm^2/hr^2
TGF22 NASA1.15	Ration of standard deviation of ground speed to mean of ground speed	$C_{22} = \frac{C_{21}}{\bar{V}}$ $\bar{V} = \frac{1}{N} \sum_{1 \le i \le N} V_i$,	N = number of aircraft σ_{vg}^2 = variance of groundspeed V_i = groundspeed of aircraft i $\bar{V}$ = mean of groundspeed	V_i is in nm/hr σ_{vg}^2 is in nm^2/hr^2

Metric	Description	Equations	Variables	Notes
TGF23 NASA1.16	Mean conflict resolution difficulty	$C_{23} = \frac{\sum_{1\le i\le N} \sum_{j\in T_i, J_i} \varpi_{\varepsilon_{ij}} n_f}{2N}$ $\varepsilon_{ij} = \min\{\lvert\chi_{ij}\rvert, 2\pi - \lvert\chi_{ij}\rvert\}$ $\chi_i = \tan^{-1}(V_{yi}/V_{xi})$ $J_i = \{j \mid h_i - \Delta h/2 \le h_{ij} \le h_i + \Delta h/2;\, j \ne i\}$ $T_i = \{j \mid 0 \le t_{ij} \le \Delta t;\, j \ne i\}$ $t_{ij} = -\frac{d_{ij}}{\dot{d}_{ij}}$ $\dot{d}_{ij} = \frac{(d_{xij}V_{xij} + d_{yij}V_{yij})}{d_{ij}}$	N = number of aircraft h_i = altitude of aircraft i h_{ij} = vertical separation between aircraft i and aircraft j Δh = vertical neighborhood parameter Δt = time-to-go threshhold t_{ij} = time-to-go $\dot{d}_{ij}$ = range rate d_{xij}, d_{yij} = distance coordinates V_{xij}, V_{yij} = velocity coordinates χ_i = heading angle $\varpi_{\varepsilon_{ij}}$ = level of resolution difficulty n_f = normalization factor V_{xi}, V_{yi} = horizontal speeds	$\Delta h = \begin{cases} 2500\,\text{ft}, \min\{h_i, h_j\} > 29000\,\text{ft} \\ 1500\,\text{ft}, \text{else} \end{cases}$ $\Delta t = 900$ s J_i = set of aircraft indices such that their corresponding altitudes are within the vertical neighborhood of aircraft i T_i = indices with time-to-go less than the threshhold value $n_f = 3208.2$ $\varpi_{\varepsilon_{ij}}$ is available as a lookup table as Table 1
TGF24 HC	Number of aircraft with heading change greater than 15 degrees	$C_{24} = \sum_{1\le j\le N} [j \in S_{24}]$ $S_{24} = \{i \mid s_{i24} \ge 15^o\}$	s_{i24} = heading change of aircraft i in degrees from beginning of interval	Created over an arbitrary fixed interval, in this case 10 minutes
TGF25 SC	Number of aircraft with Speed Change greater than 10 knots or 0.002 M	$C_{25} = \sum_{1\le j\le N} [j \in S_{25}]$ $S_{25} = \{i \mid s_{i25} \ge 10\}$	s_{i25} = difference in speed of aircraft i in knots from beginning of interval	Created over an arbitrary fixed interval, in this case 10 minutes

Metric	Description	Equations	Variables	Notes
TGF26 AC	Number of aircraft with Altitude Change greater than 750ft	$C_{26} = \sum_{1\le j\le N}[j \in S_{26}]$ $S_{26} = \{i \mid s_{i26} \ge 750\}$	s_{i26} = difference in altitude of aircraft i in knots from beginning of interval	Created over an arbitrary fixed interval, in this case 10 minutes
TGF27 MD5	No. of aircraft pairs with 3-D Euclidean Distance between 0-5nm	$C_{27} = \sum_{1\le i\le N}\sum_{i<j\le N}[D_{ij} < 5]$	D_{ij} = radial distance between aircraft i and j in nm.	
TGF28 MD10	No. of aircraft pairs with 3-D Euclidean Distance between 5-10nm	$C_{28} = \sum_{1\le i\le N}\sum_{i<j\le N}[5 < D_{ij} < 10]$	D_{ij} = radial distance between aircraft i and j in nm	
TGF29 CP25	No. of aircraft pairs with lateral distance between 0-25 nm and vertical separation less than 2000/1000ft above/below 29000ft	$C_{29} = \sum_{1\le i\le N}\sum_{i<j\le N}[j \in J_i \cap K_i]$ $J_i = \{j \mid h_i - \Delta h/2 \le h_j \le h_i + \Delta h/2;\ j \ne i\}$ $K_i = \{j \mid d_{ij} < 25;\ j \ne i\}$	d_{ij} = lateral distance between aircraft i and aircraft j h_i = altitude of aircraft i Δh = vertical neighborhood parameter	$\Delta h = \begin{cases} 4000\text{ft}, \min\{h_i, h_j\} > 29000\text{ft} \\ 2000\text{ft}, \text{else} \end{cases}$
TGF30 CP45	No. of aircraft pairs with lateral distance between 25-40 nm and vertical separation less than 2000/1000ft above/below 29000ft	$C_{30} = \sum_{1\le i\le N}\sum_{i<j\le N}[j \in J_i \cap K_i]$ $J_i = \{j \mid h_i - \Delta h/2 \le h_j \le h_i + \Delta h/2;\ j \ne i\}$ $K_i = \{j \mid 25 < d_{ij} < 40;\ j \ne i\}$	d_{ij} = lateral distance between aircraft i and aircraft j h_i = altitude of aircraft i Δh = vertical neighborhood parameter	$\Delta h = \begin{cases} 4000\text{ft}, \min\{h_i, h_j\} > 29000\text{ft} \\ 2000\text{ft}, \text{else} \end{cases}$

Metric	Description	Equations	Variables	Notes
TGF31 CP70	No. of aircraft pairs with lateral distance between 40-70 nm and vertical separation less than 2000/1000ft above/below 29000ft	$C_{31} = \sum_{1 \le i \le N} \sum_{i < j \le N} [j \in J_i \cap K_i]$ $J_i = \{j \mid h_i - \Delta h/2 \le h_j \le h_i + \Delta h/2;\ j \ne i\}$ $K_i = \{j \mid 40 < d_{ij} < 70;\ j \ne i\}$	d_{ij} = lateral distance between aircraft i and aircraft j h_i = altitude of aircraft i Δh = vertical neighborhood parameter	$\Delta h = \begin{cases} 4000\,ft, \min\{h_i, h_j\} > 29000\,ft \\ 2000\,ft, else \end{cases}$
TGF32	No. of aircraft pairs with horizontal separation under 8 nm	$C_{32} = \sum_{1 \le i \le N} \sum_{i < j \le N} [d_{ij} < 8]$	d_{ij} = lateral distance between aircraft i and aircraft j	
TGF33	Convergence angle of conflicting aircraft (average)	$C_{33} = \frac{\sum_{1 \le i \le N} \sum_{j \in K_i \cap J_i} \varepsilon_{ij}}{N(N-1)}$ $\varepsilon_{ij} = \min\{\lvert\chi_{ij}\rvert, 2\pi - \lvert\chi_{ij}\rvert\}$ $J_i = \{j \mid h_i - \Delta h/2 \le h_j \le h_i + \Delta h/2;\ j \ne i\}$ $K_i = \{j \mid d_{ij} \le r;\ j \ne i\}$	χ_{ij} = difference in heading angle between aircraft i and j h_i = altitude of aircraft i h_i = altitude of aircraft i Δh = vertical neighborhood r = horizontal neighborhood d_{ij} = lateral distance	$\Delta h = 2000\,ft$ $r = 10$ nm
TGF34	Proximity Count	$C_{34} = \sum_{1 \le i \le N} \sum_{j \in J_i, K_i} 1$ $J_i = \{j \mid h_i - \Delta h/2 \le h_j \le h_i + \Delta h/2;\ j \ne i\}$ $K_i = \{j \mid d_{ij} \le r;\ j \ne i\}$	h_i = altitude of aircraft i h_i = altitude of aircraft i Δh = vertical neighborhood r = horizontal neighborhood d_{ij} = lateral distance	$\Delta h = 2000\,ft$ $r = 10$ nm

Metric	Description	Equations	Variables	Notes
TGF35	Conflict Count	$C_{35} = \sum_{1 \le i \le N} \sum_{j \in J_i, K_i} [(i, j) \in S_{34}]$ $K_i = \{j \mid d_{ij} \le r; j \ne i\}$ $J_i = \{j \mid h_i - \Delta h/2 \le h_j \le h_i + \Delta h/2; j \ne i\}$ $S_{26} = \{(i, j) \mid s_{i35} \ge 10 \vee s_{j35} \ge 10\}$	h_i = altitude of aircraft i Δh = vertical neighborhood r = horizontal neighborhood d_{ij} = lateral distance s_{i35} = distance to boundary	Counts conflicts that are also withing 10 nm of a sector boundary $\Delta h = 2000\,ft$ $r = 10$ nm
TGF36	Altitude Variation	$C_{36} = \sigma^2_{altitude}$	-	In units of ft^2
TGF37	Aircraft Heading variation	$C_{37} = \sigma^2_{\chi}$	χ_i = heading angle	
TGF38	Number of aircraft close to sector boundary	$C_{38} = \sum_{1 \le j \le N} [j \in S_{38}]$ $S_{38} = \{i \mid s_{i38} < 10\}$	s_{i38} = difference in speed of aircraft i in knots from beginning of interval	Counts number of aircraft within 10nm of sector boundary
TGF39	Aircraft-Axis Heading Variation	$C_{39} = C_{40} \sum_{1 \le i \le N} (\chi_i - \chi)^2$ $\chi = \chi_{cV_1}$	*C* = center of sector's bounding rectangle *V1* = subsector vertex furthest from C χ = heading from *C* to *V1* χ_i = heading angle	
TGF40	Aspect Ratio	$C_{40} = \frac{d_1}{d_{2,}}$	*C* = center of sector's bounding rectangle d_1 = distance from C to furtherst subsector vector d_2 = distance from C to nearest subsector vector	

Metric	Description	Equations	Variables	Notes
TGF41 WACT	A measure of the aircraft count	$C_{41} = 0.5055 N_*$	N_* = maximum aircraft in sector over a time period of 10 minutes	Accumulated over 10 mins
TGF42 WDEN	A measure of the aircraft density per sector	$C_{42} = 20497.5 N_* / A_{sector}$	N_* = maximum aircraft in sector over a time period of 10 minutes A_{sector} = area of sector via bounding polygon	In units of $1/nm^2$ Accumulated over 10 mins
TGF43 WCLAP	A measure of the number of aircraft pairs with less than 8 or 13 nautical miles horizontal distance between them	$C_{43} = 0.17158 \sum_{i,j} cl_{ij}$ $cl_{ij} = \begin{cases} 1, d_{ij} < 8\,\text{nm} \\ 0.5, d_{ij} \in [8\,\text{nm}, 13\,\text{nm}] \\ 0, \text{ else} \end{cases}$	d_{ij} = lateral distance between aircraft i and aircraft j	Accumulated over 15 mins Checked every 2 mins
TGF44 WCONVANG	A measure of the covergence angle for aircraft pairs which are within 13 nm of each other	$C_{44} = 0.1070 \sum_{i,j} co_{ij}$ $co_{ij} = \begin{cases} 0, d_{ij} \geq 13\,\text{nm} \\ \dfrac{180 - \lvert \chi_i - \chi_j \rvert}{360} \end{cases}$	d_{ij} = lateral distance between aircraft i and aircraft j χ_i = heading angle for aircraft i in degrees.	Accumulated over 15 mins Checked every 2 mins

Metric	Description	Equations	Variables	Notes
TGF45 WCONFLICTBRS wmbr	A measure of the number of aircraft in the neighborhood of an aircraft pair projected to be in conflict	$C_{45} = 0.2316\sum_{i,j} cn_{ij}$ $cn_{ij} = \begin{cases} 0, d_{ij} \geq 13\,\text{nm} \\ n10_{ij} + \frac{1}{2} n15_{ij} \end{cases}$ $n10_{ij} = \left\lvert \{k \mid d_{ik} < 10nm \vee d_{jk} < 10nm; j \neq k; j \neq i\} \right\rvert$ $n15_{ij} = \left\lvert \{k \mid d_{ik} \in [10,15] \vee d_{jk} \in [10,15]; j \neq k; j \neq i\} \right\rvert$	d_{ij} = lateral distance between aircraft i and aircraft j	$n10_{ij}$ = number of other flights within 10nm of i or j $n15_{ij}$ = number of other flights within 10nm to 15nm of i or j Accumulated over 15 mins Checked every 2 mins
TGF46 WCOFBOUND wcbou	A measure of the number of aircraft pairs which are in conflict with each other and are close to a subsector boundary.	$C_{46} = 0.378523\sum_{i,j} cb_{ij}$ $cb_{ij} = \begin{cases} 0, d_{ij} \geq 13\,\text{nm} \\ 1, d_{ij} < 8\,\text{nm} \wedge db_{ij} < 10\,\text{nm} \\ 0.5, d_{ij} < 8\,\text{nm} \wedge db_{ij} < 20\,\text{nm} \\ 0.5, d_{ij} \in [8,13) \wedge db_{ij} < 10\,\text{nm} \\ 0.25, d_{ij} \in [8,13) \wedge db_{ij} < 20\,\text{nm} \end{cases}$	d_{ij} = lateral distance between aircraft i and aircraft j db_{ij} = the minimum of the distances from i or j to their respective nearest sector boundary in nm	Accumulated over 15 mins Checked every 2 mins
TGF47 WALC	Number of aircraft with an altitude change greater than 500ft per minute	$C_{47} = 1.60355\sum_{10\min s}\sum_{1 \leq i \leq N} al_i$	N = number of aircraft al_i = 1 if climb rate $\geq$500ft/min, 0 otherwise	Accumulated over 15 mins Checked every 2 mins
TGF48 WHEADVAR wdvr	Measure of the variation in heading	$C_{48} = 0.001208\max_{10\min}\{hv\}$ $\chi_{ij} = \chi_i - \chi_j$ $hv = \frac{\sum \chi_{ij}^2}{N(N-1)}$	N = number of aircraft χ_i = heading angle for aircraft i in degrees. χ_j = heading angle for aircraft j in degrees.	Accumulated over 10 mins Checked every 2 mins

Metric	Description	Equations	Variables	Notes
TGF49 WBPROX	Measure of the number of aircraft close to a subsector boundary	$C_{49} = 0.04055 \sum_{1 \le i \le N} bp_i$ $bp_i = \begin{cases} 0, \text{else} \\ 1, db_i < 10\,\text{nm} \\ 0.5, db_i \in [10,15] \end{cases}$	N = number of aircraft db_i = distance from aircraft i to nearest sector boundary in nm	Accumulated over 10 mins Checked every 2 mins
TGF50 WASP	A measure of airspace structure and the distribution of aircraft within a sector	$C_{50} = 0.45024 \max_{10\,\text{min}\,s} \left\{ \sum_{1 \le i \le N} ar_i \right\}$ $M_i = \lvert \chi_i - \chi \rvert \bmod 90$ $\chi = \chi_{cV_1}$ $ar_i = \begin{cases} 0, M_i < 10^0 \\ C_{40} M_i / 90 \end{cases}$	N = number of aircraft χ_i = heading angle for aircraft i in degrees. C = center of sector's bounding rectangle *V1* = subsector vertex furthest from C χ = heading from C to *V1*	Accumulated over 10 mins Checked every 2 mins
TGF51 WASP_MODIFIED	An alternative measure of airspace structure and the distribution of aircraft within a sector	$C_{51} = 0.45024 \max_{10\,\text{min}\,s} \left\{ \sum_{1 \le i \le N} ar_i \right\}$ $hd_i = \lvert \sin(\lvert \chi - \chi_i \rvert) \rvert * (3.14156/180)$ $ar_i = C_{40} hd_i$ $\chi = \chi_{cV_1}$	N = number of aircraft χ_i = heading angle for aircraft i in degrees. C = center of sector's bounding rectangle *V1* = subsector vertex furthest from C χ = heading from C to V	Accumulated over 10 mins Checked every 2 mins 3.14156 is not π, $\pi = 3.14159...$ $\forall x \in \Re, \lvert \sin(\lvert x \rvert) \rvert = \lvert \sin(x) \rvert$

Changes from Mitre's "Detailed Algorithm Descriptions" are in red.

LARGE ALGORITHM CHART

Metric	Algorithm
TGF4 CRI	$C_4 = \sum CRI_{ij}^2$ Calculation of CRI for i,j: If aircraft not within VERT_SEP (1000 ft) and CRI_MAX_DISTANCE (50 nm), CRI =0 If absolute difference of the headings of i,j is less than 15 degrees and for i or j the heading differs from the heading of the connecting segment by less than 30 degree, CRI = 0 If the absolute difference in headings of i,j is less than 30 degrees, linearly extrapolate i,j's position for 125nm. From this , estimate the time each aircraft will reach the extrapolated point. If the distance between flights at the smallest of these times is less than 10 nm, use $CRI_{ij} = \left[7 - \frac{Adjusted\ Convergence\ Angle}{30}\right]^2$ See Table 2 for Adjusted Convergence Angles
TGF5 SCI	$C_5 = \max_{15\,\min} \sum (3 - SI)^2$ If en route or in terminal domain: $SI = \frac{SIV + SIH}{2} = \frac{\frac{\Delta Z}{SEP\_VERTICAL} + \frac{\sqrt{(\Delta X^2 + \Delta Y^2)}}{SEP\_LATERAL}}{2}$ If in Oceanic domain: $SI = \frac{SILO + SIVA + SIV}{3} = \frac{\frac{\Delta Y}{Longitudnal\_Minima} + \frac{\Delta X}{Lateral\_Minima} + \frac{\Delta Z}{Vertical\_Minima}}{3}$ Critical separations caluclated iff SIH <4 and SIV <2

Metric	Algorithm
TGF6 DOFI	$C_6 = \sum (12 - DOF_{ij})^2$ Initial value: $DOF_{ij} = 12$ Iff SIH <4 and SIV <2, subtract two from available DOF if within vertical (within 1000ft) or horizontal (within 5 nm) neightboorhoods Subtract one DOF for each aircraft that is too close to the sector boundary (less than 5 nm). See Table 3.

Table 1

crossing angle	resolution difficulty	crossing angle	resolution difficulty	crossing angle	resolution difficulty
<=4	1	46	0.15178	112	0.096038
5	0.842176	48	0.146407	114	0.095366
6	0.742109	50	0.141034	116	0.095366
7	0.674278	52	0.137005	118	0.094694
8	0.626595	54	0.132975	120	0.094694
9	0.591001	56	0.129617	122	0.094694
10	0.564809	56	0.129617	124	0.094023
11	0.544661	58	0.126931	126	0.094023
12	0.528543	60	0.123573	128	0.093351
13	0.515782	62	0.121558	130	0.093351
14	0.505709	64	0.118872	132	0.093351
15	0.496978	66	0.116857	134	0.093351
16	0.490262	68	0.114842	136	0.09268
17	0.484889	70	0.113499	138	0.09268
18	0.480188	72	0.111484	140	0.09268
19	0.476158	74	0.110141	142	0.092008
20	0.442579	76	0.108798	144	0.092008
21	0.408999	78	0.107455	146	0.092008
22	0.380121	80	0.106783	148	0.092008
23	0.355272	82	0.10544	150	0.092008
24	0.333109	84	0.104768	152	0.092008
25	0.313633	86	0.103425	154	0.091336
26	0.296172	88	0.102754	156	0.091336
27	0.280725	90	0.102082	158	0.091336
28	0.266622	92	0.10141	160	0.091336
29	0.254533	94	0.100739	162	0.091336
30	0.243116	96	0.100067	164	0.091336
32	0.22364	98	0.099396	166	0.091336
34	0.208193	100	0.098724	168	0.091336
36	0.194762	102	0.098052	170	0.091336
38	0.183345	104	0.098052	172	0.091336
40	0.173271	106	0.097381	174	0.091336
42	0.165212	108	0.096709	176	0.091336
44	0.157824	110	0.096709	>=178	0.091336

Table 2

Convergence Recognition Index (CRI)		
Convergence Angle	Adjusted Convergence Angle	CRI
1	30	36.00
10	30	36.00
20	30	36.00
25	30	36.00
30	30	36.00
40	40	32.11
50	50	28.44
60	60	25.00
70	70	21.78
80	80	18.78
90	90	16.00
100	100	13.44
110	110	11.11
120	120	9.00
130	130	7.11
140	140	5.44
150	150	4.00
160	160	2.78
170	170	1.78
180	180	1.00
0	180	1.00

Table 3

Degrees of Freedom Index (DOFI)		
Available DOF		DOFI
0		144
1		121
2		100
3		81
4		64
5		49
6		36
7		25
8		16
9		9
10		4
11		1
12		0

APPENDIX A: Variables

TGF DD (Dynamic Density) calculations are based on a list of Java objects. In other words, the input file for DD calculations is a binary file which consists of Java objects recorded from a simulation. Dynamic object's (e.g. AIRCRAFT) are recorded every second while static object's (e.g. SECTOR) are written to the DD input file before the start of a simulation.

Static:

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1. SECTOR:
- MAX SECTOR ALTITUDE;
- MIN SECTOR ALTITUDE;
- SECTOR TYPE (Oceanic, En-route, Terminal);
- SECTOR FPA (Part of sector and consist of Fpa Modules);
- SECTOR FPA MODULES (Fpa Module – 3D Polygon(Prism) with Node list);
- LIST OF FPA MODULES NODES IN SEQUENTIAL ORDER;

3D Polygon (Prism) needed for sector volume and sector area.

2. FPA MODULE NODES:
- COORDINATES (Lat., Long.);
- ALTITUDE;

3. ROUTE:
- LIST OF ROUTE NODES IN SEQUENTIAL ORDER

4. ROUTE NODE:
- COORDINATES (Lat., Long.);
- ALTITUDE;

5. POINT OF TANGENCY (Used as origin for coordinate system of calculations to convert Lat. Long. to XY):
- COORDINATES (Lat., Long.);

Dynamic:

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6. AIRCRAFT:
- ID;
- COORDINATES (Lat., Long.);
- ALTITUDE;
- HEADING (True and Ground Truck);
- SIMULATION TIME;
- SPEED (Indicated);