Low-Dimensional Modeling and Uncertainty Quantification for Airfoil Icing

Anthony DeGennaro
Clarence W. Rowley III
Luigi Martinelli
Princeton University

FAA JUP Fall 2015 Quarterly Meeting
Cambridge, MA
November 2015

Outline

1. Motivation/Background
2. Methodology
3. Simulation Dataset
4. Computational-Based UQ

Topic

1 Motivation/Background

2 Methodology

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4 Computational-Based UQ

Introduction

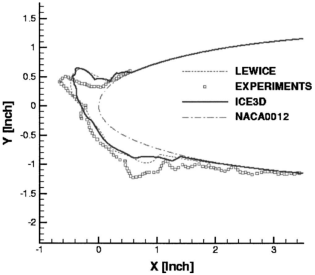
Motivation

- Wing icing is a serious issue for pilots
  - Massive flow separation, lower lift + higher drag
  - Unpredictable stall
- Wing ice shapes exhibit wide variation, sensitivity to physical parameters
  - Complex physics (coupled airflow-thermodynamics)
  - Uncertainty in physical parameters

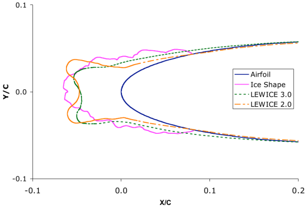
Research Goals

- How can observed variations in ice shape be modeled efficiently with a low number of parameters?
- How does uncertainty in the ice shape create uncertainty in aerodynamic performance?

Experimental/Computational Variation in Ice Shape



(a) Habashi, 2006



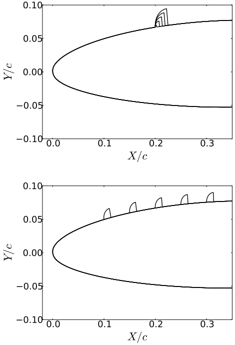
(b) Wright, 2004

- Wide variation in experimental/computational ice shapes[1, 2]
- Suggests sensitivity to perturbations in underlying physical processes
- *UQ approach:* parameterize the shape variation and study its effects on aerodynamics

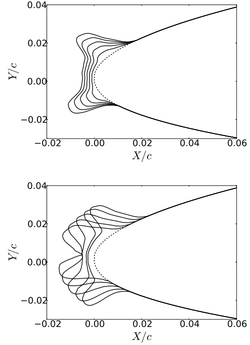
[1]Beaugendre H., Morency M., and Habashi W.G. *Development of a Second Generation in-Flight Icing Simulation Code*. Journal of Fluids Engineering, ASME, 2006.

[2]Wright W. and Potapczuk, M.G. *Semi-Empirical Modeling of SLD Physics*, AIAA 2004-412. 42nd AIAA Aerospace Sciences Meeting, Reno, NV, 2004.

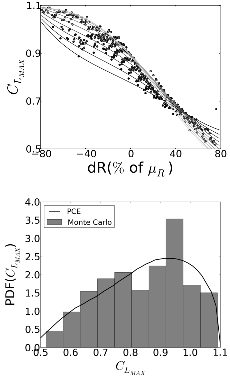
Relation to Previous Work



Ridge



Horn



Statistics

- Previous study examined parameterized ridge and horn ice shapes[3]
- Approach was heuristic, not directly based on observed shape variations
- Parameter space was low dimensional; no low-dimensional modeling

[3] DeGennaro A., Rowley C.W., and Martinelli, L. *Uncertainty Quantification for Airfoil Icing using Polynomial Chaos Expansions*. To appear in Journal of Aircraft, 2015.

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Low-Dimensional Modeling

- The space of all ice shapes is infinite dimensional
- Consider small number of parameters that describe *likely* shapes
- Analyze database of shapes from experiments/simulations
- **Proper Orthogonal Decomposition** [4]
  - Assume database of M ice shapes
  - Each individual ice shape can be represented by a vector $\mathbf{x} \in \mathbb{R}^N$
  - Approximate $\mathbf{x}$ using some basis vectors ψ_i:

$$\mathbf{x} \approx \sum_{i=1}^{P} a_i \psi_i$$

  - Choose basis vectors to be the eigenvectors of the dataset covariance matrix

$$\mathcal{R}\psi_k = \lambda_k \psi_k \text{ where:}$$

$$\mathcal{R} = \frac{1}{M}\mathbf{X}\mathbf{X}^T \text{ and: } \mathbf{X} = \begin{bmatrix} | & & | \\ x_1 & \cdots & x_M \\ | & & | \end{bmatrix}$$

[4] Holmes P. et. al. *Turbulence, Coherent Structures, Dynamical Systems and Symmetry*, Cambridge University Press, New York, 2012.

Polynomial Chaos Expansions (PCE)

- **Polynomial Chaos Framework** [5]
  - Let $\mathbf{Z} = (Z_1 \ldots Z_d)$ be d random variables with PDF $\rho(\mathbf{Z})$ that parameterize ice
  - Let $\{\Phi_k\}$ denote the set of polynomials which are orthogonal w.r.t. $\rho(\mathbf{Z})$
  - Let $y(\mathbf{Z})$ denote the mapping from $\mathbf{Z}$ to an aerodynamic performance metric
- **Probabilistic Collocation Method:**
  - *Representation*

$$y(\mathbf{Z}) \approx \sum_{|i|=0}^{N} y_i \Phi_i(\mathbf{Z})$$

  - *Orthonormality*

$$\langle f, g \rangle = \int_\Gamma f(\mathbf{z}) g(\mathbf{z}) \rho(\mathbf{z}) d\mathbf{z}$$

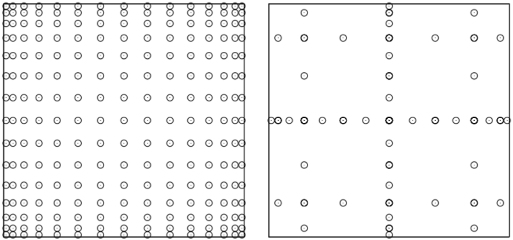
$$\langle \Phi_i, \Phi_j \rangle = \delta_{ij}$$

  - *Quadrature*

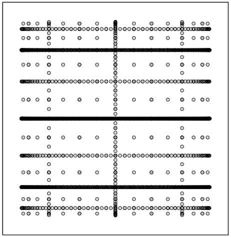
$$y_k = \langle y, \Phi_k \rangle \approx \sum_{i=0}^{Q} y(\mathbf{Z}^{(k)}) \Phi_k(\mathbf{Z}^{(k)}) w_k$$

[5] Xiu D. *Numerical Methods for Stochastic Computations: A Spectral Method Approach*. Princeton University Press, 2010.

PCE with Sparse Grids



Full Tensor Product vs. Sparse Grid



Anisotropic Grid

- Sparse Grids[6]
  - *Efficient*: Only a subset of the full quadrature grid is used
    - For $d >> 1$, number of samples scales as $2^N \binom{N+d}{d} << (N+1)^d$
  - *Adaptive*: Start with a coarse mesh, adaptively refine until achieve desired resolution
  - *Anisotropic*: Refine grid in most "important" directions
  - Implemented in DAKOTA[7] (open-source UQ code)

[6] LeMaitre O. *Spectral Methods for Uncertainty Quantification*. Springer, 2010.

[7] Adams et. al. *DAKOTA, A Multilevel Parallel Object-Oriented Framework for Design Optimization...* V. 5.3 User's Manual. SAND2010-2183.

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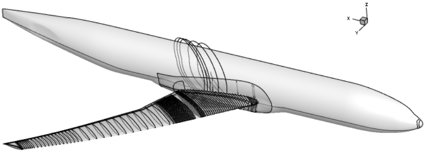
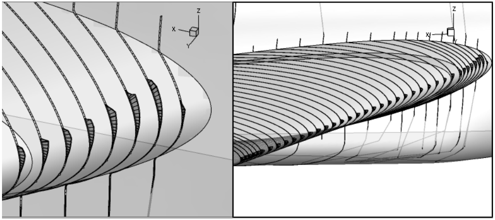
Dataset Description

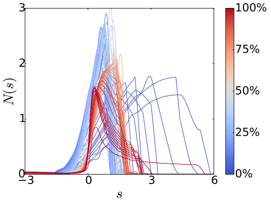




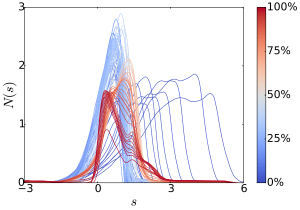
- NASA Common Research Model (CRM), 65% scale[8]
  - 45 min accretion time, altitude = 10,000 ft, velocity = 232 knots, temperature = -4° C, MVD = 20 μm, LWC = 0.55 g/m^3

[8] Broeren A. et. al. *Swept-Wing Ice Accretion Characterization and Aerodynamics*, AIAA 2013-2824.

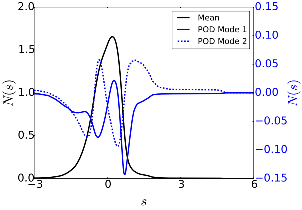
Low-Dimensional Modeling of Dataset



Original Data



POD Reconstruction

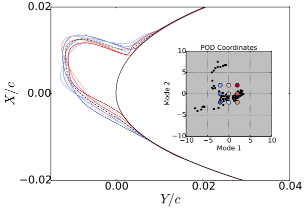


POD Modes

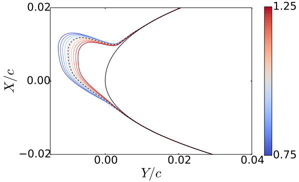
$$N(s) = h\{\bar{N}(as+b) + \sum_{i=1}^{2} c_i \Phi_i(as+b)\}$$

- h, a, b are scaling parameters
- c_1, c_2 are POD coefficients
- This collapses 100 different snapshots into 5 parameters

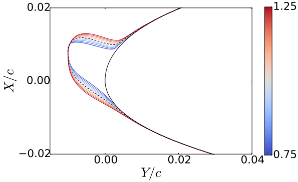
5-Dimensional UQ Study



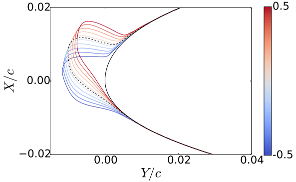
POD Modes



Height



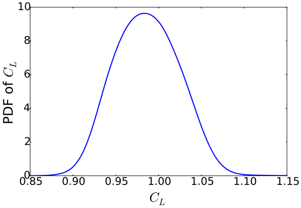
Width

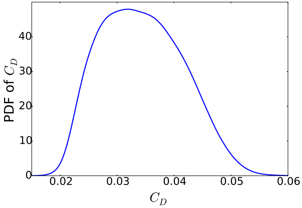


Position

- 2 POD coefficients (*shape*) + width, height, position parameters (*scaling*)

Statistics





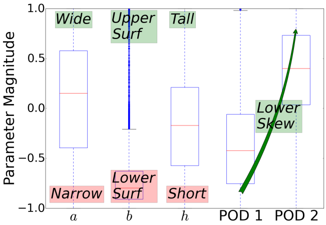
C_L Statistics **C_D Statistics**

	Width	Position	Height	POD 1	POD 2
T (C_L)	0.03	0.69	0.15	0.11	0.14

- Our surrogate is an explicit polynomial function of the input variables, making statistical inference easy/quick
- PCE surrogate computed using 1,103 sparse grid points
- Sobol index $T_i = \frac{\mathbb{E}\left[Var(Y|Z_{-i})\right]}{Var(Y)}$ is a measure of how much Z_i contributes to the total variance of $Y(\mathbf{Z})$
- For our parameter ranges, position perturbation accounts for most of the statistical variation

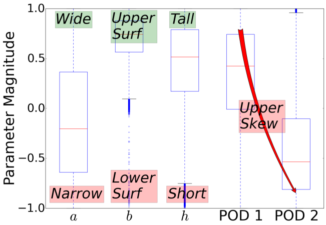
Statistical Inference

- Analyze statistical clustering of horns that produce bottom and top 10% of C_L variation



Favorable Horns

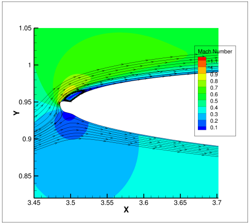
- Wider/rounded
- Lower surface
- Shorter
- Gentle downward skew



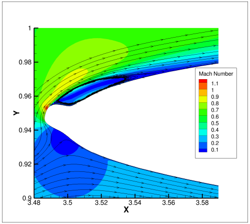
Unfavorable Horns

- Sharper/narrower
- Upper surface
- Taller
- Sharp, upper skew shape

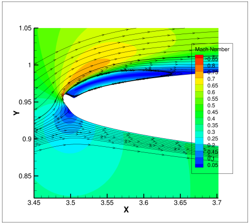
Flow Solutions



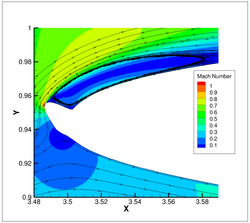
Favorable Position



Favorable shape skew



Unfavorable Position



Unfavorable shape skew

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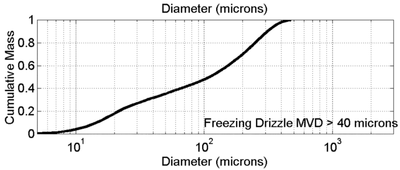
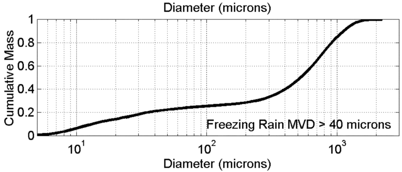
Motivation

- **Investigate uncertainty in the physical process of icing**
  - Distribution of droplet diameters affects collection efficiency
    - How sensitive is collection efficiency to perturbations in MVD distribution?
  - Temperature, LWC, and convective heat transfer strongly affect icing conditions
    - Glaze: warmer temperature, lower convective heat transfer, higher LWC –> horn shapes
    - Rime: colder temperature, higher convective heat transfer, lower LWC –> "blob" shapes
    - Convective heat transfer is strongly affected by surface roughness (difficult to model accurately)

Droplet Diameter Distribution



Freezing Drizzle MVD PDF

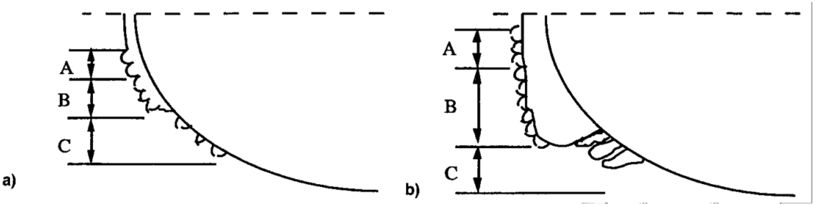


Freezing Rain MVD PDF

- Several MVD distributions exist for different flight conditions [9]
- Each gives a different collection efficiency
- How sensitive are collection efficiency and ice shape to perturbations in MVD distribution?

[9] Airplane and Engine Certification Requirements in Supercooled Large Drop, Mixed Phase, and Ice Crystal Icing Conditions; Final Rule. Federal Register, Vol. 79, No. 213.

Roughness Variations

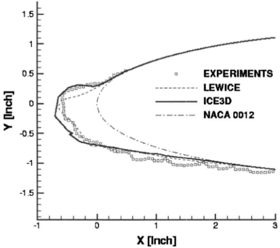


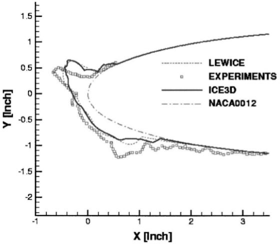
Roughness Growth

- **Surface roughness varies with parameters** [10]
  - Roughness height increases with temperature and LWC
  - Beginning of roughness varies with temperature, speed, LWC
- **Surface roughness affects shape/aerodynamics** [12]
  - Roughness elements probably protrude out of boundary layer and cause transition
  - Irregularity of shape should be calculated by ice accretion code, not treated as part of roughness model

[10] Shin, J. Characteristics of Surface Roughness Associated with Leading-Edge Ice Accretion. Journal of Aircraft, Vol. 33, No.2, April 1996.

Glaze vs. Rime Accretion



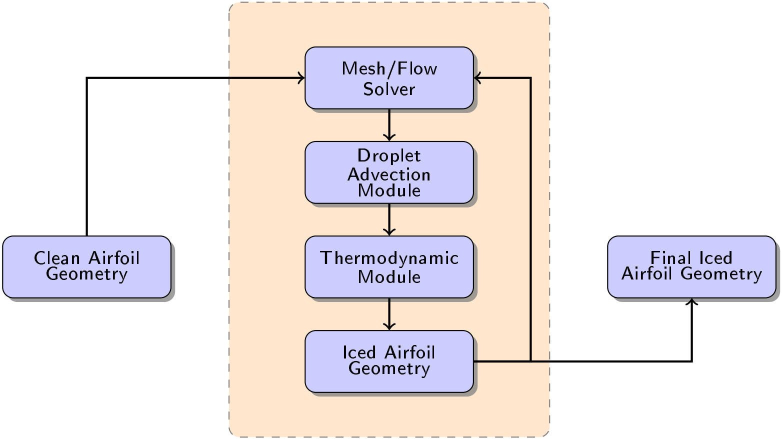


Rime (left) vs. Glaze (right) Accretion

- **Glaze Ice** [11]
  - Warmer temperature, higher LWC Ice shape follows distribution of convective heat transfer over airfoil surface
  - "Horn" shapes
- **Rime Ice**
  - Colder temperature, lower LWC
  - Ice shape follows distribution of incoming water (collection efficiency)
  - "Blob" shapes

[11] Beaugendre H., Morency M., and Habashi W.G. *Development of a Second Generation in-Flight Icing Simulation Code*. Journal of Fluids Engineering, ASME, 2006.

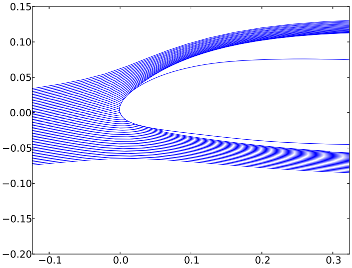
Airfoil Icing Code Flowchart



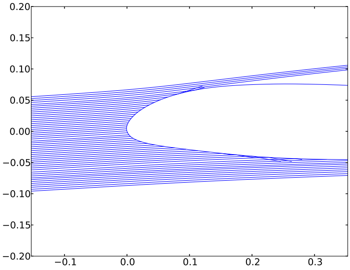
Droplet Advection

$$\frac{d\mathbf{x}}{dt} = \mathbf{v}$$

$$m\frac{d\mathbf{v}}{dt} = \frac{1}{2}\rho_g C_D \pi r^2 ||\mathbf{v_g} - \mathbf{v}||(\mathbf{v_g} - \mathbf{v}) + m\mathbf{g}$$

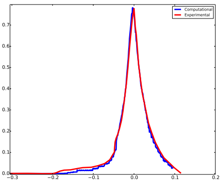


$\mathbf{R} = \mathbf{10}\mu\mathbf{m}$

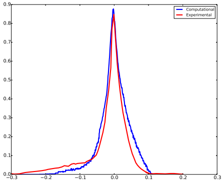


$\mathbf{R} = \mathbf{100}\mu\mathbf{m}$

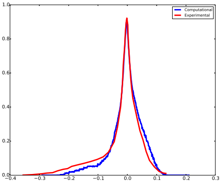
Preliminary Intermediate Results: Mass Flux



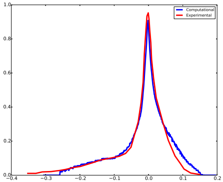
MVD 52



MVD 111



MVD 154



MVD 236

- Collection efficiency is the ratio of surface to free-stream water flux

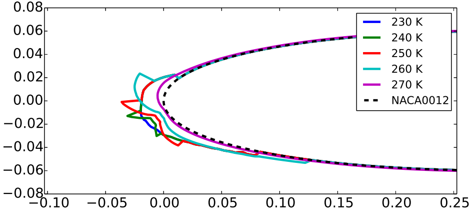
Thermodynamics

$$\rho_w \left\{ \frac{\partial h_f}{\partial t} + \nabla \cdot (\mathbf{u}_f h_f) \right\} = \dot{m}_{imp} - \dot{m}_{evap} - \dot{m}_{ice}$$

$$\rho_w \left\{ \frac{\partial (h_f c_W T)}{\partial t} + \nabla \cdot (\mathbf{u}_f h_f c_W T) \right\} = \left[c_W T_d + \frac{u_d^2}{2} \right] \dot{m}_{imp}$$
$$-0.5(L_{evap} + L_{sub})\dot{m}_{evap}$$
$$+ (L_{fus} - c_{ice} T)\dot{m}_{ice}$$
$$+\epsilon\sigma(T_\infty^4 - T^4)$$
$$+ c_H(T_\infty - T)$$

- **Mass**
  - Enters through impinging droplets
  - Exits via evaporation/sublimation and freezing
- **Energy**
  - Enters through impinging droplets, freezing of ice
  - Exits via evaporation/sublimation, radiation, convection
- Solved explicitly using finite volume discretization with Roe scheme upwinding

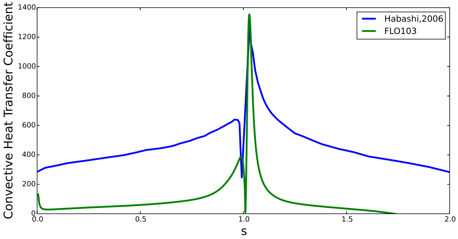
Preliminary Intermediate Results: Ice Shapes



- NACA0012, $\alpha = 4^{\circ}$, $U_{\infty} = 103$ m/s, MVD $= 20\ \mu m$, LWC $= 0.55$ g/m^3, Re $= 4.14$ million, T $= 7$ min
- Low temperatures: convective heat transfer high enough to freeze all incoming droplets instantly (rime)
- High temperatures: low amount of ice accretion with liquid film on top (glaze)
- Intermediate temperatures (glaze/rime mix)
  - Convective heat transfer is minimal at stagnation point and then rises sharply before decaying
  - Results in familiar horn shapes

Sources of Error

- **Convective Heat Transfer**
  - Convective heat transfer calculation is lower than published benchmark test results
  - Habashi,2006 uses rough-wall modification of Spalart-Almaras turbulence model (specifically developed for icing applications)
  - Ice shapes are very sensitive to heat transfer by convection



- **Neglected Thermodynamic Mechanisms**
  - Neglected mass/energy transfer via evaporation/sublimation and radiation

Work In-Progress

- Implement rough-wall extension in Spalart-Almaras turbulence model
- Implement neglected mass/energy transfer mechanisms
- Verify icing calculations against published results
- Perform UQ studies, investigate sensitivity to physical parameters
  - Temperature, convective heat transfer coefficient, Reynolds number, MVD, LWC, angle of attack, etc.

Conclusions/Future Work

Conclusions

- Airfoil icing is a process subject to much uncertainty
  - Wide variation in ice shapes
  - Sensitivity to perturbations in physical conditions
- We have briefly demonstrated three approaches to quantifying uncertainty in this problem
  - Heuristic parameterization
  - Data-based parameterization
  - Computational-based UQ

Future Work

- Parameterized UQ
  - Investigate effect of more shape parameters
  - Extend efforts to 3D wing icing
- Computational modeling
  - Continue development and testing of icing code
  - Use icing code to investigate statistical variation of ice shape