Wide Area Multilateration for Alternate Position, Navigation, and Timing (APNT)

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ABSTRACT

As the National Airspace System (NAS) is modernized, the Global Positioning System (GPS) is playing a more central role in providing means to position, navigation and timing. While many aircraft operators still file flight plans that are based on airways defined by Very High Frequency (VHF) Omnidirectional Range (VOR) navigational aids, most aircraft are flying those routes using GPS navigation. This trend will continue until GPS is the primary electronic navigation system for all aircraft in the NAS. The Automatic Dependent Surveillance Broadcast (ADS-B), which derives its position from GPS, is planned to become the main surveillance system in the future. Many of the Next Generation Air Traffic Control System (NextGen) operational improvements (OI) planned for the future depend on GPS. However, GPS has a very weak signal that is easily denied by intentional or un-intentional interference. The NAS must be robust enough to continue to operate safely during periods of interference detrimental to the GPS signal spectrum. Specifically air carriers must be able to continue conducting operations through a GPS interference area, including dispatching to and from an airport without access to the GPS signal. Small general aviation (GA) aircraft must have an option for a safe landing during GPS interference in instrument meteorological conditions (IMC). However, it may not be economically feasible for either the FAA to maintain a system that allows all GA aircraft to depart into IMC without GPS in the future, or to always get to the airport originally planned when a GPS interference event occurs unexpectedly during the flight.

The Alternate, Position, Navigation and Timing (APNT) project's goal is to provide a backup means of navigation and surveillance during a localized GPS interference event. The current backup for navigation utilizes legacy VOR and Distance Measuring Equipment (DME) navigational aids (navaids), while the current near-term surveillance backup once ADS-B is available is Secondary Surveillance Radar (SSR). There is a number of reasons to consider an alternative to the legacy navaid system: (1) VOR based navigation does not provide the area navigation (RNAV) desired by many NextGen OIs, (2) The existing VOR navaids are very dated and will be expensive to replace, and (3) Additional user benefits of modern replacement system. The negatives of a new APNT system include: (1) Near 100% user equipage of VOR radios for instrument rated aircraft, (2) Development and acceptance cost of a new system.

The APNT program considered a wide spectrum of technologies. A low-frequency, high power ground-based system, such as LORAN provided the perfect contrast to high-frequency, low power space-based GPS. However, the team concluded that sufficient research had already been performed on this option and decided to focus its research efforts on three other candidates: (1) Improving DME performance, (2) Wide-Area Multilateration (WAM), and (3) Ground based Pseudolites. This paper describes this second option.

INTRODUCTION

As the NAS is modernized, GPS is playing a more central role in providing means to position, navigation and timing. While many aircraft operators still file flight plans that are based on airways defined by Very High Frequency (VHF) Omnidirectional Range (VOR) navigational aids, most aircraft are flying those routes using GPS navigation. This trend will continue until GPS is the primary electronic navigation system for all aircraft in the NAS. The Automatic Dependent Surveillance Broadcast (ADS-B), which derives its position from GPS, is planned to become the main surveillance system in the future. Many of the NextGen operational improvements (OI) planned for the future depend on GPS. However, GPS has a very weak signal that is easily denied by intentional or un-intentional interference. The NAS must be robust enough to continue to operate safely during periods of interference detrimental to the GPS signal spectrum. Specifically air carriers must be able to continue conducting operations through a GPS interference area, including dispatching to and from an airport without access to the GPS signal. Small general aviation (GA) aircraft must have an option for a safe landing during GPS interference in instrument meteorological conditions (IMC). However, it may not be economically feasible for either the FAA to maintain a system that allows all GA aircraft to depart into IMC without GPS in the future, or to always get to the airport originally planned when a GPS interference event occurs unexpectedly during the flight. However, the APNT system must insure a safe landing for all aircraft during GPS interference.

THE APNT ENVIRONMENT

The APNT project's goal is to provide a backup means of navigation and surveillance during a localized GPS interference event. The current backup for navigation utilizes legacy VOR and Distance Measuring Equipment (DME) navigational aids (navaids), while the current near-term surveillance backup once ADS-B is available is Secondary Surveillance Radar (SSR). There is a number of reasons to consider an alternative to the legacy navaid system: (1) VOR based navigation does not provide the area navigation (RNAV) desired by many NextGen OIs, (2) The existing VOR navaids are very dated and will be expensive to replace, and (3) Additional user benefits of modern replacement system. The negatives of a new APNT system include: (1) Near 100% user equipage of VOR radios for instrument rated aircraft, and (2) Development and acceptance cost of a new system.

Multilateration (MLAT) is a concept of determining the position of an emitter (e.g., aircraft transponder) by measuring the time-difference of arrival (TDOA) of a signal between several known and carefully surveyed observation points (e.g., MLAT sensors.)

REQUIREMENTS

The summary of the APNT surveillance and navigation requirements is shown in Table 1. The parameters used for these requirements are: horizontal position error (HPE) for accuracy, horizontal protection level (HPL) for integrity, and horizontal dilution of precision (HDOP) for geometry.

	Requirements/ Targets for Surveillance	Requirements/ Targets for Navigation
Accuracy	HPE≤0.05nmi = 92.6m (NAC_p = 8) [1, 2]	0.3nmi
Integrity	HPL≤0.2nmi = 92.6m (NIC=7, SIL=3, $P_{FA}=1x10^{-6}$) [1, 2]	?
Geometry	HDOP≤2√2= 2.8284 [3]	HDOP≤2√2= 2.8284 [3]
Time to Alarm (TTA)	Terminal: 10 secs, Enroute: 15 secs	Terminal: 10 secs, Enroute: 15 secs

Table 1: Summary of APNT Surveillance and Navigation Requirements

COVERAGE AREA

The APNT Project currently defines three zones as shown in Figure 1. Zone 1 is aligned with Class A airspace over the Contiguous United States (CONUS), namely from 18,000 ft mean sea level (MSL) to Flight Level (FL) 600 (FL600 is 60,000 feet pressure altitude.) Zone 2 is from 5,000 ft Above Ground Level (AGL) to the bottom of Zone 1. Zone 3 consists of a truncated conical section of flat constant altitude surface at 1000 ft height above airport (HAA) from the Airport Reference Point (ARP) out to 10 nmi. From this 10 nmi point the surface slopes up as the distance from the airport grows at a two degree angle up to the bottom of Zone 1. Zone 3 is present at the 135 busiest airports as shown in Figure 1. Zone 3 was established to capture the air carrier traffic arriving and departing from these busy airports.

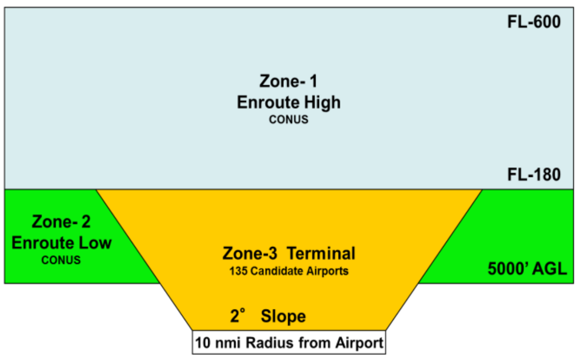

Figure 1: APNT Service Volume defined by 3 Zones.

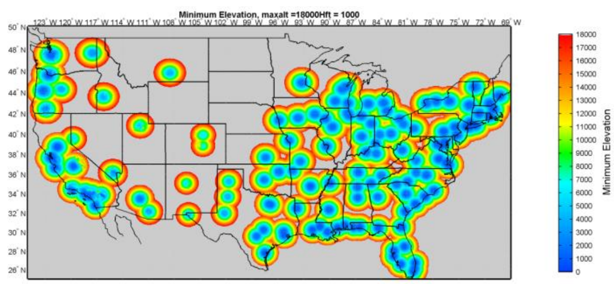

Figure 2: APNT Zone 3.

Passive MLAT requires at least three ground stations to calculate the position of the aircraft using TDOA and four for integrity. ADS-B MLAT stations have a range of 60 nmi. This requires an enormous number of stations to cover a large area, especially at low altitude. The current ADS-B WAM Specification only requires coverage out to 60 nmi from the airport. This corresponds to an APNT Zone 3 altitude of 10,600 feet. While this is insufficient for the current APNT requirement of the full Zone 3, this lower central part of Zone 3 is the most critical part of this coverage area.

Zone 1 and 3 were designed to provide coverage primarily for the air carrier aircraft, but also include turbine GA, turbine Part 135 operators and most cargo operators. Zone 2 was designed for the piston GA aircraft. Navigation service for Zone 1 is currently served by the DME-DME using existing DME performance. Currently, and in all known future FAA plans for surveillance, Zone 1 (En-route) will have complete coverage from SSR. The benefits of Zone 1 coverage using MLATs would be limited without removal of En-route DME or SSR. WAM could make a good backup for Zone 2 navigation service for GA aircraft, although, initial analysis indicates that it would take many (possibly thousands) receiver stations to provide complete coverage over the entire Zone 2 service volume. However, the current SBS plan includes good SSR coverage throughout Zone 2 (Figure 3). This current ADS-B plan is to provide small pockets of WAM service where SSR is removed for terminal coverage of medium-to-small Part 139 airports and in the Gulf of Mexico. The busiest 44 airports in the NAS will keep SSR coverage; thus these airports would be unlikely to be provided with WAM coverage in lieu of an APNT requirement. There are currently 500 Part 139 airports in CONUS, 345 of which are Index I, which means large (greater than 30 passenger) aircraft may have scheduled operations to that field. These medium to small Part 139 airports with future WAM service which would also provide limited areas of Zone 2 coverage could be a part of larger APNT plan for GA. If GA aircraft in these areas were equipped with a navigation unit that could use own-ship traffic information services broadcast (TIS-B) message to navigate, a backup form of both surveillance and navigation would be available to these aircraft (Table 2). GA aircraft in areas of SSR coverage would be able to navigate based on the own-ship message of the ground broadcast of aircraft position TIS-B message derived from the SSR. (Figure 3).

APNT Zones	Surveillance	Navigation
1	SSR	DME/DME
2	SSR/WAM	?
3	SSR/WAM	?

Table 2: Summary of Alternate Surveillance and Navigation Sources

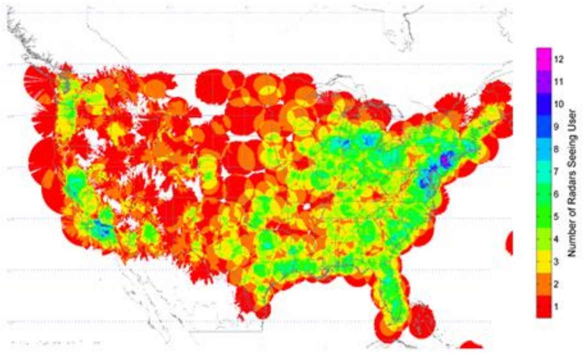

Figure 3: Expected 5,000 ft AGL SSR Coverage through 2025

OVERVIEW OF CANDIDATE TECHNOLOGY

Any future APNT system needs to backup not only navigation, but also surveillance. Any of these candidate systems not only needs to satisfy an accuracy requirement, but also integrity and time-to-alarm (TTA) requirements.

One of the three principal methods considered for APNT is Wide Area Multilateration (WAM) for improved surveillance combined with Traffic Information Service-Broadcast (TIS-B) for navigation. Multilateration (MLAT) is the concept of determining the position of an emitter (i.e., aircraft transponder) by measuring the time-difference of arrival (TDOA) of a signal between several known and carefully surveyed observation points (e.g., MLAT sensors). The most common use of MLAT in aviation today relies on the 1090 MHz reply of an aircraft transponder to an interrogation by an SSR or MLAT active sensor. An MLAT system that involves widely spaced sensors to cover a large area is often referred to as WAM. This paper does not attempt to discriminate between the two terms MLAT and WAM.
Automatic Dependent Surveillance Broadcast (ADS-B) is a key technology for NextGen. The FAA has already included MLAT as a contract option for the ADS-B program. Multilateration can provide a backup and/or replacement for SSR. This makes MLAT a major contender for the APNT program. The principle challenge for the APNT-MLAT is making this system a navigation backup as well.

SURVEILLANCE BACKGROUND

Aircraft surveillance for IFR flight has historically had both main and backup systems for determining aircraft position by the ground system. Currently the main system used for surveillance is SSR. The backup system is primary radar. Primary radar is the traditional "skin paint" radar that involves the transmitter sending a strong signal out and timing the reflected portion of the signal to determine the range. The azimuth is provided by the rotation of the antenna assembly. The speed of an aircraft can be estimated by tracking the target over several successive measurements. SSR utilizes a transponder on board the aircraft to respond to interrogations. SSR transmits its interrogation on 1030 MHz and the aircraft transponder replies on 1090 MHz. The older transponder system still used on smaller General Aviation aircraft is the Air Traffic Control Radio Beacon System (ATCRBS) (Mode A/C). Higher-end GA and transport category aircraft use Mode S transponders. One advantage of Mode S is that each aircraft has a unique code associated with its registration, while Mode A/C receivers only have a four digit octal code assigned by ATC. Another advantage of Mode S is that each aircraft can be "roll called" individually, while Mode A/C receivers only respond to "all call" interrogations.

With ADS-B becoming the new main surveillance system for beyond 2020, the role of SSR will transition to serve as a backup to ADS-B in the event of a GPS outage in en-route and high density terminal areas. Currently the backup surveillance system is SSR. Primary radar systems will continue to be retained where they are currently used for aviation safety, weather and security purposes. If a different, lower-cost backup system could be made available then significant saving may be achieved. WAM could be that backup surveillance system. The ADS-B mandate for 2020 dictates that all aircraft flying above 10,000 ft MSL (excluded 2,500 ft AGL or below), within 30 nmi of a Class B airport and within Class C airspace must be broadcasting an ADS-B signal. ADS-B consists of two different signals that may be used to satisfy this mandate. Aircraft that will exclusively fly below 18,000 ft may use a TSO-C154c, Universal Access Transceiver (UAT) ADS-B equipment operating on the frequency of 978 MHz. Aircraft flying above 18,000 ft MSL must use TSO-C166b, Mode S extended squitter (ES) ADS-B equipment operating on the radio frequency of 1090 MHz.

There has been a steady growth in the number of operating aviation multilateration systems. These systems consist of a number of 1090 MHz receivers (i.e., passive sensors) spread throughout the service area, with some of the stations also having 1030 MHz interrogators (i.e., active sensors) as depicted in Figure 4. A passive MLAT station is one that only receives transmissions from the aircraft. An active MLAT station will also interrogate the aircraft in the area. A WAM system will have mostly passive stations with some active stations to interrogate ATCRBS aircraft. The ITT Inc., WAM system is designed to limit interrogation based on the aircraft in the area. As we approach the 2020 mandate for ADS-B equipage it is expected that it will become a mostly passive system.

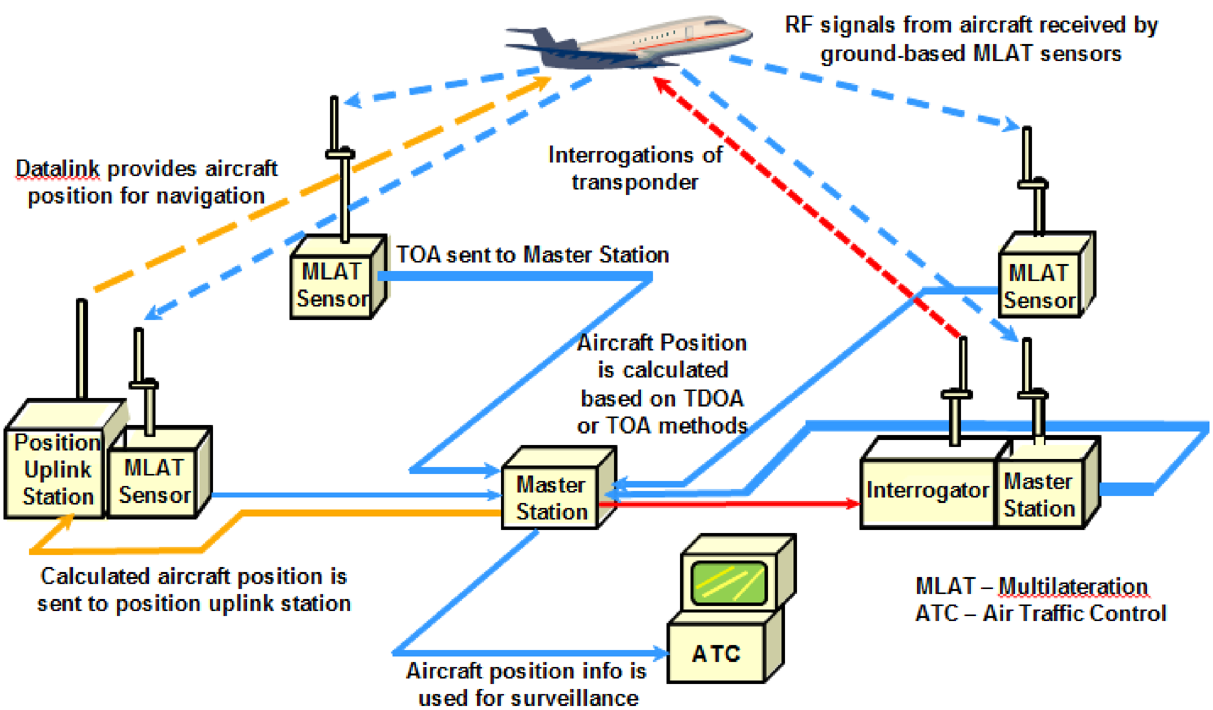

Figure 4: Passive MLAT Surveillance and Navigation System

OVERVIEW OF TECHNOLOGY DESIGN

By measuring the TDOA of a unique aircraft's transponder or ADS-B transmission between various known locations on the ground, the location of the transmission (e.g., aircraft) can be determined. Because the speed of light is a constant in all reference frames, the time that it takes a signal to travel from the aircraft to the ground is directly proportional to the distance between the transmitter and the receiver. Given a fixed difference of arrival of a signal to two ground receivers the possible locations of the transmitter form a hyperbola. (Note: the method of multilateration is also known as hyperbolic positioning.) Given a second time difference to a third receiver, another hyperbola can be formed that will intersect the first hyperbola at one or two locations. One of these intersection points is the location of the transmitter (See Figure 5). Three ground stations may not be enough to determine the correct location as in Figure 5. The image on the left has good geometry while the image on the right has bad geometry. In the case of bad geometry additional sites would be required. If the geometry is good and only three ground stations are receiving the transmission, then the correct location may need to be determined by the motion of these intersections over time. Geometry is measured by dilution of precision (DOP), whereby low DOP values represent strong geometry. Only one set of intersections will move in a way that makes physical sense. The better situation is to have four or more ground stations in view. With four or more stations in view, not only can the location of the transmitter be determined instantaneously, the extra station(s) can be used as an integrity check on the solution. In Figure 5, the first time difference between stations one and two form a locus of points that define a hyperbola. The TDOA of station 3 and station 1 form a second hyperbola. The intersections of these two hyperbolas reveal the location of the transmitter. If a fourth station is available it can be used as an integrity check to verify that none of the reception stations are faulted.

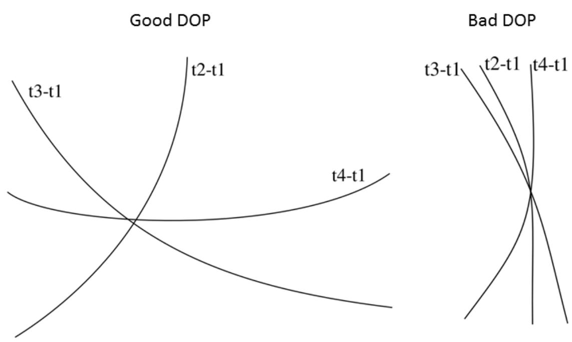

Figure 5: Hyperbolic Intersections of MLAT

NAVIGATION CONCEPT

Navigation could be added to the system by providing means for the aircraft to receive its own position through the TIS-B message broadcast from the ground stations. A smaller general aviation aircraft would likely have a single combined display that provides both navigation and a cockpit display of traffic information (CDTI). This system could have a fall back mode when GPS is lost and use its own ship TIS-B location for navigation. For larger aircraft that have a flight management system (FMS) for navigation, a new connection from the CDTI to the FMS would need to be established and the FMS would need to be modified to allow this "own ship" position to be used as a degraded mode of navigation. Therefore, adding TIS-B-based backup navigation would likely be a simpler and less expensive change for small GA aircraft than for large air transport aircraft. The own ship SSR or MLAT position would be uploaded to the aircraft via TIS-B and then forwarded to the navigation system. This TIS-B navigation modification would work independent of the surveillance source (WAM or SSR). However, for SSR the update rate may be as low as 1/12 Hz, which will introduce excessive latency. Although there is considerable experience in using MLAT for surveillance, there is no experience using TIS-B for navigation.

The estimated delay or lag of this navigation system between pilot control input and resulting course deviation indicator (CDI) needs to be determined. A timing budget would include the time for the following:

- aircraft transmission to reach the ground receivers,
- the signal to go from the receiver to the master processing unit,
- computation of the solution,
- transmission of this solution to the TIS-B transmitter,
- TIS-B to wait for a transmission slot if using Time Division Multiple Access (TDMA) UAT link,
- the TIS-B transmission to be sent back to the aircraft,
- the aircraft to receive and decode the message,
- the message to be forwarded to the navigation system, and
- the navigation system to show a course deviation.

Existing ITT TIS-B deployments have been shown to report traffic from SSR in less than one second.

An additional issue is that of system loading of 1090 MHz. As more and more aircraft occupy the same area the frequency will become congested. At first, this will result in tracking delay. At some point traffic may result in loss of track of some aircraft. It may not be technically feasible to implement TIS-B backup navigation using the 1090 MHz link; the UAT link may need to be used instead. Also, computation of all the aircraft positions may become a limiting factor. Careful analysis of the system capacity will need to be performed to determine maximum number of aircraft a given area can track.

MLAT SURVEILLANCE MATURITY

MLAT is already being used by the FAA as part of the Airport Surface Detection Equipment, Model X (ASDE-X) program. MLAT is currently deployed as a surveillance alternative to SSR in a number of countries around the world. All existing operating installations currently listen to 1090 MHz transmitters. ITT is in the midst of deployment of a MLAT system that listens to both 1090 MHz transponders and 978 MHz UAT broadcasts at Montrose, Colorado (MTJ). The schedule of the Montrose system has initial operating capability beginning in March 2012, followed by final operating capability in June 2012. The system will be expanded to cover the nearby airports of Gunnison (GUC), Telluride (TEX), and Durango (DRO). ITT has shown that they can meet the WAM specified accuracy of 128 meters, can tolerate a DOP of 8, and can achieve a ranging timing accuracy of 30 ns. MLAT has also been installed as part of the Precision Runway Monitor (PRM-A) system for parallel approaches operations at Washington Dulles airport. These numerous existing installations make the maturity level of this APNT solution high for surveillance.

AUTHENTICATION

Using TIS-B as a data link for navigation is a completely new concept. TIS-B has no authentication. There is no way for an airborne user to know if the TIS-B information received is from a trusted source. Because a principal risk we are addressing with APNT is intentional GPS interference, one should seriously consider the fact that a nefarious agent could both jam the GPS signal and broadcast misleading TIS-B. This would completely defeat this APNT solution as a navigation system. TIS-B is only authorized for use as a situational awareness tool, not as a traffic avoidance system. These issues are challenges for aircraft certification of TIS-B based navigation. Mitigations could include limiting TIS-B (its) use to emergency backup navigation during GPS interference events for piston aircraft (mostly Part 91 operators).

Adding authentication to TIS-B would require a relatively major change to the existing ADS-B Minimum Operational Performance Standards (MOPS) for both UAT and 1090ES receivers. Existing ADS-B equipment already in the field would have to be modified or replaced. Authentication would likely involve standard public key cryptography methods. The TIS-B message would be signed by a secure FAA private key and a public key stored in the avionics would be able to verify the message as authentic. This modification would add some overhead to each TIS-B message. Authentication was considered during the ADS-B design phase, but was rejected due to the additional datalink overhead.

Another option is to use a new datalink other than TIS-B. Frequency spectrum would need to be reserved. If the frequency used is outside the DME band it will likely also require an additional antenna, thus increasing cost. Dedicated avionics could be created or the datalink could be added as an additional optional service of the ADS-B system. This new service would provide the same information as TIS-B in an authenticated channel.

TIS-B POLICY ISSUES

The current TIS-B policy is to only broadcast non-ADS-B traffic (via ground stations) that comes within 15 nmi and ± 3500 ft altitude of an ADS-B participating aircraft. Therefore, in an area of GPS interference where none of the aircraft are able to broadcast their position, no TIS-B traffic would be available to them. This policy would need to be modified to allow aircraft to receive their own-ship position at a minimum when the system detected a possible GPS interference event. This ADS-B GPS interference event detection algorithm could also be extremely helpful if the output could be forwarded to air traffic control to visualize approximate boundaries of the event. The TIS-B service is also intended to only be a transition service. If TIS-B becomes a key part of APNT then plans would need to be modified to make it a permanent service of the ADS-B program.

TIMING

A critical aspect of a working MLAT system is precise synchronization of the ground stations. It is important to understand that these stations do NOT need to be synchronized with coordinated universal time (UTC), only amongst each other. The typical way this is done with small-area MLAT is with either direct fiber optical cables or microwave links back to a master station. A few systems also use GPS or an alternative line-of-sight (LOS) system from the master station. Another method of synchronization is called transponder synchronized system which is a LOS system. In this method, one of the MLAT active sensors sends an interrogation signal to the other passive sensors. The time of arrivals (and then the TDOAs) will be used to synchronize the whole system in this region. ITT's current plans call for synchronization via existing broadcast messages from the airport master station with a few stations outside of line-of-sight relying on GPS timing. The current ITT system allows for the system to coast without GPS timing to these remote ground stations for one hour. A more robust timing source would be required for full MLAT coverage to remain viable for longer than this one hour minimum. This could be an atomic clock at these locations or a beam steering antenna to see through the interference.

ANALYSIS APPROACH AND INITIAL RESULTS

The values of HPE, HPL, and HDOP are generated for points on a user cone. (The equations for calculating HPL, HPE, and HDOP are included in Appendices B and C) The cone represents a worst case condition since aircraft are generally flying within (above) the cone. The user cone begins 1000 feet above the ARP with a radius and slope determined for each airport. Even though currently these parameters are specific to an airport, in the future they will be aligned with the APNT requirements of a 2º slope and a radius of 10 nmi. A not-to-scale representation of a user cone for a typical terminal area is shown in Figure 6.

The resolution of the user cone can be controlled by the number of user points generated on it. Currently, a user cone extending to 18,000 feet over terrain slightly above sea level may have upwards of 500 points.

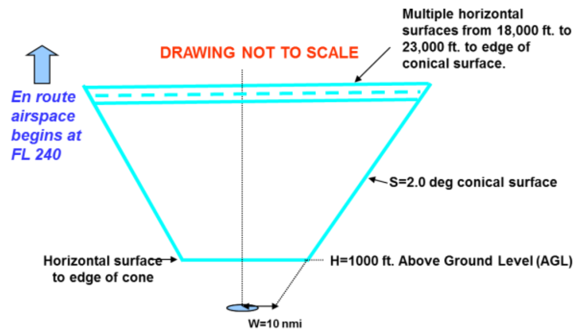

Figure 6:. Typical Analysis Surface for Terminal Area DME/DME RNAV Coverage [4]

Attention is also given to determining obstructions from local terrain. For points within range, *nothing* can so seriously degrade performance as loss of line-of-sight. Any point on the user cone which violates the requirement that an aircraft avoid terrain by at least 2,000 feet vertically and 4 nmi horizontally is raised in elevation to satisfy the requirement [5]. During the calculation of HPE, HPL, and HDOP, all lines-of-sight are subjected to terrain scrutiny and any site not directly visible from a user point is not used. Since the analysis uses passive MLAT with Receiver Autonomous Integrity Monitoring (RAIM), there must be at least 4 sites visible from a user point in order to make a calculation. RAIM uses a redundant measurement to check for erroneous ranges.

DETERMINATION OF SITES

A principal focus of this effort is to determine the minimum number of sites that can satisfy the above requirements. In practical terms, this means satisfying the HPL requirement since it is usually the most difficult to satisfy. The approach is to assemble a large number of sites from different sources which in some ways are already vetted in that they are all acceptable site candidates for MLAT sensors. This group of sites is made large enough so that it can easily satisfy the requirements for an ARP and can then be systematically reduced to the point where any further reduction of sites would not satisfy the requirements.

The initial set of sites for CONUS has been assembled from five sources: (1.) current ground based transceivers (GBT), (2.) public-use airports (APT) sourced from NFDC, (3.) DMEs, (4.) NextGen DME sites identified in [4], and (5.) sites proposed by ITT for future GBTs. These sites have been examined in the above order and any site found to be within 10 nmi of a site already accepted has been removed from the database.

The resulting set of sites currently totals 4967 and consists of:

- 404 GBT
- 4154 APT
- 185 DME
- 10 NextGen DME
- 214 ITT future GBTs.

Most of the original NextGen DME sites have been eliminated due to their proximity to public-use airports or to their being used as GBT sites.

The second phase of site selection involves the systematic elimination of sites to a minimal set which can still satisfy the requirements. MITRE has developed an algorithm to do this based on the well-known Voronoi method for solving the nearest neighbor problem. The Voronoi method constructs cells around each data point (site) so that any position inside that cell is closer to that data point than any other point in the database (See Figure 7). Where the set of data points exhibits high density, the cell area surrounding each point will become smaller. Thus a good metric for site density is 1/(Voronoi cell area) for each site data point [6]. Since we are trying to eliminate as many points as possible, the best candidates are those points with the highest Voronoi density. For any given ARP, the algorithm in its simplest form can be described as follows:

1. From the database of sites described above, determine the set of sites around an ARP in a circle extending 60 nmi beyond the outer edge of the cone (since user points on the cone edge can still see 60 nmi farther out). Mark all GBT sites as "frozen" which means they cannot be eliminated.
2. Construct the Voronoi tessellation for the new subset of points, determine the cell areas, and calculate the site densities as stated above.
3. Sort the sites according to declining density.
4. Select the highest density site that is *not* frozen.
5. Temporarily eliminate it from the set of points and examine all users within 60 nmi of the eliminated site (since these are the only users that can be affected). If *any* user in this set fails the HPL requirement (i.e., HPL $\leq$ 0.2 nmi), then the site *cannot* be eliminated. It is frozen and we return to Step 4. However, if *every* user within that set passes the HPL test, the site *can* be eliminated permanently. A new set of sites which does not include the eliminated site is determined and we return to Step 2.
6. When all sites from the original set surrounding the ARP are either eliminated or marked frozen, the process stops.

The result will be a minimal set of sites such that every user point on the cone still satisfies the HPL requirement. This set of sites is not necessarily optimal but is quite reasonable.

Figure 7 shows on the left side the Voronoi tessellation initially constructed in Step 2. The right side of Figure 7 shows the final Voronoi tessellation constructed at the end of the above process. The x and y axes denote respectively longitude and latitude. The ARP location is depicted by the star in the center. Note the larger cells denoting lower density. Also note that the final set of sites is more uniformly distributed.

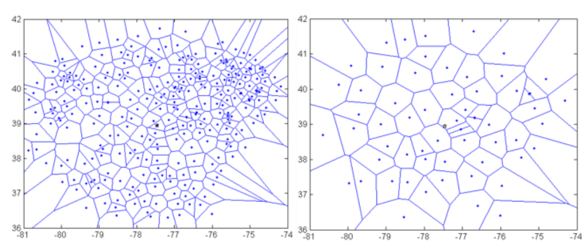

Figure 7: Voronoi Tessellations Showing Initial and Final Set of Sites for IAD

Table 7 shows the initial and final number of sites for the three airports in the metropolitan Washington D.C. area after the final site selection described above: Washington Dulles International Airport (IAD), Ronald Reagan Washington National Airport (DCA), and Baltimore-Washington International Airport (BWI). Both the initial and final number of sites are for the expanded circle around the ARP (60 nmi farther than the edge of the cone).
Table 3 shows the initial and final number of sites at IAD, DCA, and BWI before and after using this process, respectively.

ARP	Initial Number of Sites	Final Number of Sites
IAD	273	61
DCA	281	67
BWI	279	70

Table 3. Initial and Final Number of Sites at IAD, DCA, and BWI

The number of sites common to all three ARPs is 47. The union of all sites numbers 89. If these airports were separated from any other airport, the average number of sites required would be around 60-70. But when combined with others (in this case 3), the average number approximates 30. In crowded areas with many airports, the average number of sites per ARP can fall even more.

In these examples the cone rose to 18000 feet MSL and the edge of the cone extended beyond 100 nmi. It is worth noting that the final APNT radius around an ARP may be 60 nmi with the cone rising to about 11000 feet AGL. In this case, the number of sites required would fall dramatically.

SAMPLE COVERAGE RESULTS

Terrain can produce markedly different results for line-of-sight calculations. The three airports mentioned above were examined for HPE, HPL, and HDOP. All of these airports are in an area with relatively flat terrain and thus represent a best case. In areas which are more mountainous, the challenge is greater.

The assumptions used in this analysis are shown in Appendix A. The MLAT method used requires a minimum of four sites for integrity using RAIM (see appendix B). Figures 8 and 9 show respectively accuracy and integrity results as calculated by HPE and HPL. These results are shown for each of the three airports. Figure 10 shows HDOP. In all of these figures, green circles denote GBT sites, cyan diamonds denote APT sites, black circles denote DME sites, black squares denote NextGen suggested sites, and magenta circles denote ITT suggested sites. The outer ring denotes the farthest extension of the cone. The inner ring denotes the beginning 1000 feet above the ARP.

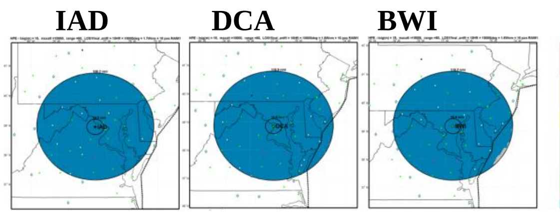

Figure 8: Accuracy (HPE ≤ 92.6 m = 0.05 nmi, NAC_p=8, Max. Alt.=18,000ft MSL, Time Error =50ns (15 m), Max. Range=60 nmi)

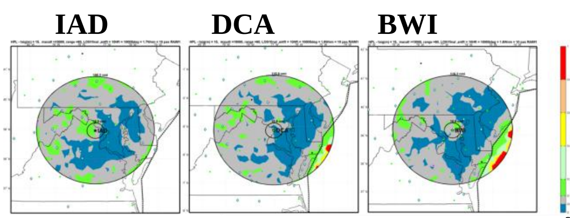

Figure 9: Integrity (HPL ≤ 0.2 nmi, NIC=7, SIL=3, P_{FA}=1x10^{-6}, Max. Alt.=18,000ft MSL, Time Errpr=50 ns (15 m), Max. Range=60 nmi)

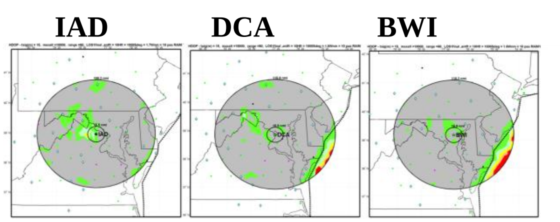

Figure 10: HDOP (≤ 2√2=2.8284), Max. Alt.=18,000ft MSL, Max. Range=60 nmi

CONCLUSION

Given the complexity of equipping large turbojet aircraft with a TIS-B based navigation system, the fact that Zone 1 already has excellent DME-DME coverage and the busiest airports will retain both SSR and ILS capabilities, MLAT seems to have a limited role for this aircraft segment. However, piston GA aircraft with a unified display and navigation system could be easily designed in the future to accommodate using the own-ship TIS-B message for navigation. This would provide a backup form of navigation independent of the choice of surveillance systems between MLAT and SSR. Accepting an unauthenticated traffic datalink is probably a stakeholder consideration that needs to be assessed and decided upon.

Where existing terminal SSR coverage should be replaced with MLAT is an FAA business decision. To cover an entire airport Zone 3 coverage area for BWI, IAD or DCA required 60 to 70 sites, however in metroplexes such as the Baltimore/Washington area presented here, overlap will reduce the total number of sites required (in this case down to 89 sites to support all three airports). In the previous three figures, it can be seen that Martinsburg Airport, WV (MRB) is used as a site for IAD and BWI, but not for DCA. This is due to the fact that the technique presented analyzed these three airports one at a time. A new approach of using a grid and analyzing all airports in a given area all at once is currently under development. This will provide consistent results for those regions where Zone 3 areas overlap. It may also provide an additional reduction in the total number of sites required in metroplex areas.

Independent of where MLAT is deployed the FAA has committed to providing surveillance coverage everywhere it is available today. The real opportunity is where WAM coverage can provide backup surveillance coverage where it is not available today. This process has already begun with ADS-B in the Gulf of Mexico and WAM in Colorado.

ADDITIONAL STEPS:

Assess Accuracy

1. Measure MLAT signal to assess performance and signal quality.
2. Work with providers (ITT, etc.) to obtain data and analysis on performance of different implementations.
3. Work with providers to collect and analyze long term data from multiple locations.

Integrity

4. Work with providers to understand safety/integrity case.
5. Within program team/FAA, determine safety/integrity concern of re-transmission of position for navigation (en-route, approach).

Coverage

6. Continue coverage analysis to estimate the number of stations needed for coverage of each zone/all zones.

Capacity

7. Determine major capacity concerns and potential mitigations from capacity study.
8. Conduct measurements/field tests to validate capacity study and mitigations.

Continuity/Availability

9. Determine station performance for availability, reliability comparing actual data to specification, have N passive sensors visible to the aircraft, where the position of each one is known and located at $(x_i, y_i, z_i)\ i = 1,2,\ldots,N$. Assume also that the aircraft transponder transmits its signal at time τ_u, and the time of arrival (TOA) at the i[th] sensor is $\tau_i,\ i = 1,2,\ldots,N$.

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Appendix A: Assumptions

- Passive sensors
- Miss Detection Probability = $P_{md} = 1 \text{ x } 10^{-7}$
- False Alarm probability = $P_{fa} = 1 \text{ x } 10^{-6}$
- For each user node, All in View sites within the max range and radio horizon:
  - N ≥ 4 (one used as reference; additional site for RAIM [7, 8])
  - Max Range Limit 60 nmi nominal (future variation from 60 to 120)
  - Radio Horizon (m) = 1609.3 ($\sqrt{}$(2*site ant. ht (ft)) + $\sqrt{}$(2*a/c ht (ft)))
- Terrain Model
  - Resolution = 3 minutes or approx. 3 nmi (0.05º in lat and lon)
  - Used to check line of sight (LOS) obstruction from terrain
  - Also used to determine any terrain interference with conical surface
- Time accuracy: ≤ 50 ns (15m, 1-sigma)
- Vertical Accuracy (Mode-C): TSO-C129 (1-sigma) [9] has little effect on horizontal accuracy ($\square_{baro}$ = standard deviation of pressure altitude error)

Table A-1. Standard Deviation of Pressure Altitude Error

Geometric Altitude (ft)	$\square_{baro}$ (m)
18,000	477
10,000	290
5,000	165
1,000	34
500	19
200	12

- Stations come from four sources and are filtered (needs to be updated):

  (1.) 404 GBT sites that currently exist

  (2.) 4154 APT public-use airports

  (3.) 185 DME

  (4.) 10 NextGen DME Stations added to airports in [4]

  (5.) 214 ADS-B sites proposed by ITT

- Elevations of all stations and ARPs are made consistent by assigning new elevations from the terrain database.
- Conical surface extends to 18,000 ft MSL
- Conical Surface defined by four parameters (see next slide):

  - W, the radius of the circle formed by the intersection of the conical surface and a plane tangent to the Earth at the ARP
  - H, height of the floor of the conical surface above the ARP
  - S, the slant (degrees) of the conical surface
  - T, the top elevation (MSL) of the cone formed from the intersection of the en route airspace plane with conical surface
- Analysis for points on conical surface
- All points on conical surface are checked with the Terrain Model to adjust terrain elevations when violations exist by making a new user height by adding 2000 feet to terrain at user position:
  - If user point on cone is less than terrain elevation
  - If user point is within 4nmi horizontally and 2000 feet vertically of terrain (future work will restrict this to designated mountainous areas and 1000 ft everywhere else) [5, 10]. This determination is not made until user is more than 2000 ft higher than the ARP elevation.

Appendix B

Summary of Solution Separation Method (RAIM) [7, 8]

1. Calculate test statistic (TS = distance between fullset solution and a subset solution.
2. Estimate threshold (D_n):

$$D_n = K_{ff,fa} d_{\text{major,fs-ss}}$$

$$K_{ff,fa} = \left| Q^{-1}\left(\frac{P_{fa}}{2N}\right) \right|, \; Q(x) = \frac{1}{2\pi}\int_x^\infty e^{-\frac{t^2}{2}} dt$$

$$d_{\text{major,fs-ss}} = \text{square root of the maximum eigenvalue of the covariance matrix } C_H$$

$$C_H = \begin{bmatrix} C(1,1) & C(1,2) \\ C(2,1) & C(2,2) \end{bmatrix}$$

$$C = (G^T W G)^{-1}$$

$$P_{fa} = \text{False Alarm Probability} = 1.0 \times 10^{-6}$$

$$N = \text{Number of tests}$$

$$G = \text{Obervation matrix}$$

$$W = \text{Weighting matrix}$$

3. If $TS \leq D_n$ for all subsets, then declare there is no fault.
4. Calculate the Horizontal Protection Level (HPL):

$$HPL = \max(D_n + a_n) \text{ over } n = 1,\ldots, N$$

$$a_n = K_{md} d_{\text{major,ss}},$$

$$K_{md} = \left| Q^{-1}(P_{md}) \right|$$

$$P_{md} = \text{miss detection probability} = 1.0 \times 10^{-7}.$$

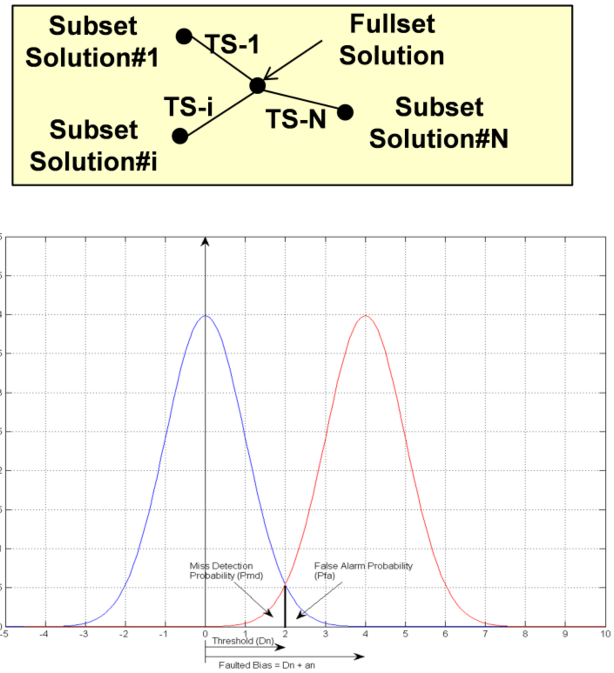

Figure B-1. A Schematic Showing the Solution Separation Method

Appendix C

Calculation of the Aircraft Position Using Passive MLAT Sensors

Assume the unknown aircraft position is at (x_u, y_u, z_u), and we have N passive sensors visible to the aircraft, where the position of each one is known and located at (x_i, y_i, z_i), $i = 1,2,\ldots,N$. Assume also that the aircraft transponder transmits its signal at time τ_u, and the time of arrival (TOA) at the ith sensor is τ_i, $i = 1,2,\ldots,N$. The unknown pseudorange between the aircraft and the ith sensor is given by [11]:

$$\rho_i = c(\tau_i - \tau_u) + \varepsilon_i = \sqrt{(x_i - x_u)^2 + (y_i - y_u)^2 + (z_i - z_u)^2} = f_i(x_u, y_u, z_u),\ i = 1,2,\ldots,N \qquad \text{(C-1)}$$

where c is the speed of light, and ε_i is the measurement error which is assumed to be a zero-mean normal distribution and its variance = $(\sigma_\tau)^2$.

These are N nonlinear equations in 3 unknowns (x_u, y_u, z_u), where N ≥ 3, which can be solved by linearizing them around an approximate solution $(\hat{x}_u, \hat{y}_u, \hat{z}_u)$ using Taylor series. Then an approximate pseudorange can be calculated using:

$$\hat{\rho}_i = \sqrt{(x_i - \hat{x}_u)^2 + (y_i - \hat{y}_u)^2 + (z_i - \hat{z}_u)^2} = f_i(\hat{x}_u, \hat{y}_u, \hat{z}_u),\ i = 1,2,\ldots,N \qquad \text{(C-2)}$$

The aircraft position (x_u, y_u, z_u) consists of an approximate position component $(\hat{x}_u, \hat{y}_u, \hat{z}_u)$ and an incremental component $(\Delta x_u, \Delta y_u, \Delta z_u)$:

$$x_u = \hat{x}_u + \Delta x_u,\ y_u = \hat{y}_u + \Delta y_u,\ z_u = \hat{z}_u + \Delta z_u \qquad \text{(C-3)}$$

Therefore we can write:

$$\begin{aligned} f_i(x_u, y_u, z_u) &= f_i(\hat{x}_u + \Delta x_u, \hat{y}_u + \Delta y_u, \hat{z}_u + \Delta z_u) \\ &\cong f_i(\hat{x}_u, \hat{y}_u, \hat{z}_u) + \frac{\partial f_i(\hat{x}_u, \hat{y}_u, \hat{z}_u)}{\partial \hat{x}_u}\Delta x_u + \frac{\partial f_i(\hat{x}_u, \hat{y}_u, \hat{z}_u)}{\partial \hat{y}_u}\Delta y_u + \frac{\partial f_i(\hat{x}_u, \hat{y}_u, \hat{z}_u)}{\partial \hat{z}_u}\Delta z_u \\ &= f_i(\hat{x}_u, \hat{y}_u, \hat{z}_u) - \left(\frac{x_i - \hat{x}_u}{\hat{\rho}_i}\right)\Delta x_u - \left(\frac{y_i - \hat{y}_u}{\hat{\rho}_i}\right)\Delta y_u - \left(\frac{z_i - \hat{z}_u}{\hat{\rho}_i}\right)\Delta z_u \end{aligned} \qquad \text{(C-4)}$$

Therefore:

$$\rho_i = \hat{\rho}_i - \left(\frac{x_i - \hat{x}_u}{\hat{\rho}_i}\right)\Delta x_u - \left(\frac{y_i - \hat{y}_u}{\hat{\rho}_i}\right)\Delta y_u - \left(\frac{z_i - \hat{z}_u}{\hat{\rho}_i}\right)\Delta z_u + \varepsilon_i \qquad \text{(C-5)}$$

Rearranging (C-5), therefore:

$$\hat{\rho}_i - \rho_i = \left(\frac{x_i - \hat{x}_u}{\hat{\rho}_i}\right)\Delta x_u + \left(\frac{y_i - \hat{y}_u}{\hat{\rho}_i}\right)\Delta y_u + \left(\frac{z_i - \hat{z}_u}{\hat{\rho}_i}\right)\Delta z_u \text{, or}$$

$$\Delta\rho_i = a_{xi}\Delta x_u + a_{yi}\Delta y_u + a_{zi}\Delta z_u,\ i = 1,2,\ldots,N \qquad \text{(C-6)}$$

Where:

$$\Delta\rho_i = \hat{\rho}_i - \rho_i,\ a_{xi} = \left(\frac{x_i - \hat{x}_u}{\hat{\rho}_i}\right),\ a_{yi} = \left(\frac{y_i - \hat{y}_u}{\hat{\rho}_i}\right),\ a_{zi} = \left(\frac{z_i - \hat{z}_u}{\hat{\rho}_i}\right) \qquad \text{(C-7)}$$

Equation (C-6) can be put in a matrix form as follows:

$$\begin{bmatrix} \Delta\rho_1 \\ \Delta\rho_2 \\ \ldots \\ \Delta\rho_N \end{bmatrix} = \begin{bmatrix} a_{x1} & a_{y1} & a_{z1} \\ a_{x2} & a_{y2} & a_{z2} \\ \ldots & \ldots & \ldots \\ a_{xN} & a_{yN} & a_{zN} \end{bmatrix} \begin{bmatrix} \Delta x_u \\ \Delta y_u \\ \Delta z_u \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \ldots \\ \varepsilon_N \end{bmatrix} \qquad \text{(C-8)}$$

For passive MLAT, one of the sensors is used as a reference, and the time difference of arrivals (TDOAs) are calculated from the TOA measurements. If we assume the first sensor as a reference, therefore Equation (C-8) becomes:

$$\begin{bmatrix} \Delta\rho_1 - \Delta\rho_2 \\ \Delta\rho_1 - \Delta\rho_3 \\ \ldots \\ \Delta\rho_1 - \Delta\rho_N \end{bmatrix} = \begin{bmatrix} a_{x1} - a_{x2} & a_{y1} - a_{y2} & a_{z1} - a_{z2} \\ a_{x1} - a_{x3} & a_{y1} - a_{y3} & a_{z1} - a_{z3} \\ \ldots & \ldots & \ldots \\ a_{x1} - a_{xN} & a_{y1} - a_{yN} & a_{z1} - a_{zN} \end{bmatrix} \begin{bmatrix} \Delta x_u \\ \Delta y_u \\ \Delta z_u \end{bmatrix} + \begin{bmatrix} \varepsilon_1 - \varepsilon_2 \\ \varepsilon_1 - \varepsilon_3 \\ \ldots \\ \varepsilon_1 - \varepsilon_N \end{bmatrix}$$
$$= \begin{bmatrix} 1 & -1 & 0 & \ldots & 0 \\ 1 & 0 & -1 & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ 1 & 0 & 0 & \ldots & -1 \end{bmatrix} \begin{bmatrix} a_{x1} & a_{y1} & a_{z1} \\ a_{x2} & a_{y2} & a_{z2} \\ \ldots & \ldots & \ldots \\ a_{xN} & a_{yN} & a_{zN} \end{bmatrix} \begin{bmatrix} \Delta x_u \\ \Delta y_u \\ \Delta z_u \end{bmatrix} + \begin{bmatrix} \varepsilon_1 - \varepsilon_2 \\ \varepsilon_1 - \varepsilon_3 \\ \ldots \\ \varepsilon_1 - \varepsilon_N \end{bmatrix} \qquad \text{(C-9)}$$

Putting (C-9) in a matrix form, therefore:

$$\underset{(N-1)\times 1}{\Delta R} = \underset{(N-1)\times N}{T}\ \underset{N\times 3}{H}\ \underset{3\times 1}{\Delta X_u} + \underset{(N-1)\times 1}{\Delta E} = \underset{(N-1)\times 3}{G}\ \underset{3\times 1}{\Delta X_u} + \underset{(N-1)\times 1}{\Delta E} \qquad \text{(C-10)}$$

Where:

$$\underset{(N-1)\times 1}{\Delta R} = \begin{bmatrix} \Delta\rho_1 - \Delta\rho_2 \\ \Delta\rho_1 - \Delta\rho_3 \\ \dots \\ \Delta\rho_1 - \Delta\rho_N \end{bmatrix}, \quad \underset{(N-1)\times 3}{G} = \underset{(N-1)\times N}{T} \underset{N\times 3}{H} = \begin{bmatrix} a_{x1} - a_{x2} & a_{y1} - a_{y2} & a_{z1} - a_{z2} \\ a_{x1} - a_{x3} & a_{y1} - a_{y3} & a_{z1} - a_{z3} \\ \dots & \dots & \dots \\ a_{x1} - a_{xN} & a_{y1} - a_{yN} & a_{z1} - a_{zN} \end{bmatrix},$$

$$\underset{3\times 1}{\Delta X_u} = \begin{bmatrix} \Delta x_u \\ \Delta y_u \\ \Delta z_u \end{bmatrix}, \quad \underset{(N-1)\times 1}{\Delta E} = \begin{bmatrix} \varepsilon_1 - \varepsilon_2 \\ \varepsilon_1 - \varepsilon_3 \\ \dots \\ \varepsilon_1 - \varepsilon_N \end{bmatrix},$$

$$\underset{(N-1)\times N}{T} = \begin{bmatrix} 1 & -1 & 0 & \dots & 0 \\ 1 & 0 & -1 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 1 & 0 & 0 & \dots & -1 \end{bmatrix}, \quad \underset{N\times 3}{H} = \begin{bmatrix} a_{x1} & a_{y1} & a_{z1} \\ a_{x2} & a_{y2} & a_{z2} \\ \dots & \dots & \dots \\ a_{xN} & a_{yN} & a_{zN} \end{bmatrix}$$

Solving (C-10) using weighted least squares (WLS) solution, then:

$$\underset{3\times 1}{\Delta \hat{X}_u} = (G^T W G)^{-1} G^T W \Delta R = (H^T T^T W T H)^{-1} H^T T^T W \Delta R \qquad \text{(C-11)}$$

The covariance matrix is given by [12]:

$$Cov\underset{(3\times 3)}{(\Delta \hat{X}_u)} = \left(\underset{3\times(N-1)}{G^T} \underset{(N-1)\times(N-1)}{W} \underset{(N-1)\times 3}{G} \right)^{-1} = (H^T T^T W T H)^{-1}, \text{ where}$$

$$\underset{(N-1)\times(N-1)}{W} = \left(\underset{(N-1)\times N}{T} \underset{N\times N}{P} \underset{N\times(N-1)}{T^T} \right)^{-1} = \frac{1}{c^2\sigma_\tau^2} (T T^T)^{-1}, \quad \underset{N\times N}{P} = \begin{bmatrix} c^2\sigma_\tau^2 & 0 & 0 \\ \dots & \dots & \dots \\ 0 & 0 & c^2\sigma_\tau^2 \end{bmatrix}, \text{ therefore:}$$

$$Cov(\Delta \hat{X}_u) = (G^T W G)^{-1} = (H^T T^T (P T^T)^{-1} T H)^{-1} = c^2\sigma_\tau^2 (H^T T^T (T T^T)^{-1} T H)^{-1} = c^2\sigma_\tau^2 (G^T (T T^T)^{-1} G)^{-1} \qquad \text{(C-12)}$$

and σ_τ = the timing error in seconds, c σ_τ = ranging error in meters.

Calculation of HDOP:

The dilution of precision matrix (U) can be calculated from (C-12) as follows [12]:

$$\underset{3\times 3}{U} = \frac{1}{c^2\sigma_\tau^2} Cov(\Delta \hat{X}_u) = (H^T T^T (T T^T)^{-1} T H)^{-1} = (G^T (T T^T)^{-1} G)^{-1}$$

The horizontal dilution of precision (HDOP) can be calculated using the following equation:

$$HDOP = \sqrt{U(1,1) + U(2,2)} \qquad \text{(C-13)}$$

Augmentation with Barometric Altimeter:

Equation (C-9) becomes:

$$\begin{bmatrix} \Delta\rho_1 - \Delta\rho_2 \\ \Delta\rho_1 - \Delta\rho_3 \\ \dots \\ \Delta\rho_1 - \Delta\rho_N \\ \Delta B \end{bmatrix} = \begin{bmatrix} a_{x1} - a_{x2} & a_{y1} - a_{y2} & a_{z1} - a_{z2} \\ a_{x1} - a_{x3} & a_{y1} - a_{y3} & a_{z1} - a_{z3} \\ \dots & \dots & \dots \\ a_{x1} - a_{xN} & a_{y1} - a_{yN} & a_{z1} - a_{zN} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \Delta x_u \\ \Delta y_u \\ \Delta z_u \end{bmatrix} + \begin{bmatrix} \varepsilon_1 - \varepsilon_2 \\ \varepsilon_1 - \varepsilon_3 \\ \dots \\ \varepsilon_1 - \varepsilon_N \\ \varepsilon_{baro} \end{bmatrix} \tag{C-14}$$

$$\underset{N\times1}{\Delta R_{aug}} = \underset{N\times3}{G_{aug}}\ \underset{3\times1}{\Delta X_u} + \underset{N\times1}{\Delta E_{aug}} \tag{C-15}$$

where ΔB is the difference between the measured barometric altitude converted to WGS-84 altitude and the predicted altitude, and ε_{baro} is the error in barometric altitude measurement, and

$$\underset{N\times1}{\Delta R_{aug}} = \begin{bmatrix} \Delta\rho_1 - \Delta\rho_2 \\ \Delta\rho_1 - \Delta\rho_3 \\ \dots \\ \Delta\rho_1 - \Delta\rho_N \\ \Delta B \end{bmatrix}, \underset{N\times3}{G_{aug}} = \begin{bmatrix} a_{x1} - a_{x2} & a_{y1} - a_{y2} & a_{z1} - a_{z2} \\ a_{x1} - a_{x3} & a_{y1} - a_{y3} & a_{z1} - a_{z3} \\ \dots & \dots & \dots \\ a_{x1} - a_{xN} & a_{y1} - a_{yN} & a_{z1} - a_{zN} \\ 0 & 0 & 1 \end{bmatrix}, \underset{N\times1}{\Delta E_{aug}} = \begin{bmatrix} \varepsilon_1 - \varepsilon_2 \\ \varepsilon_1 - \varepsilon_3 \\ \dots \\ \varepsilon_1 - \varepsilon_N \\ \varepsilon_{baro} \end{bmatrix} \tag{C-16}$$

The WLS solution becomes:

$$\underset{3\times1}{\Delta\hat{X}_u} = \left(\underset{3\times N}{G_{aug}^T}\ \underset{N\times N}{W_{aug}}\ \underset{N\times3}{G_{aug}} \right)^{-1} \underset{3\times N}{G_{aug}^T}\ \underset{N\times N}{W_{aug}}\ \underset{N\times1}{\Delta R_{aug}} \tag{C-17}$$

And the augmented covariance matrix is given by:

$$\underset{3\times3}{Cov\left(\Delta\hat{X}_u\right)} = \left(\underset{3\times N}{G_{aug}^T}\ \underset{N\times N}{W_{aug}}\ \underset{N\times3}{G_{aug}} \right)^{-1} \tag{C-18}$$

Where:

$$\underset{N\times N}{W_{aug}} = \begin{bmatrix} & & 0 \\ & \underset{(N-1)\times N}{T}\ \underset{N\times N}{P}\ \underset{N\times(N-1)}{T^T} & \dots \\ 0 & \dots & \sigma_{baro}^2 \end{bmatrix}^{-1} \tag{C-19}$$

Calculation of HPE:

The horizontal position error (HPE) with probability of 0.95 can be calculated as follows:

$$HPE = k \cdot d_{major} \tag{C-20}$$

Where the factor (k) is given by [12]:

$$k = \frac{2.4477 - 1.9625}{\left(\frac{d_{major}}{d_{\min or}}\right)^3} + 1.9625 = \frac{0.4852}{\left(\frac{d_{major}}{d_{\min or}}\right)^3} + 1.9625 \tag{C-21}$$

where d_{major} and d_{minor} are the semimajor and semiminor axes of the error ellipse. These are equal to the maximum and minimum eigenvalues, respectively, of the 1st 2x2 elements of the covariance matrix shown in Equation (C-18).

Figure C-1 shows the k-factor as a function of the ratio d_{major}/d_{minor}. It is seen from this figure that, the k-factor ranges between k =1.9625 when $d_{major} >> d_{minor}$ (e.g., when the error ellipse collapses to a straight line) and k = 2.4477 when $d_{major} = d_{minor}$ (when the error ellipse becomes a circle).

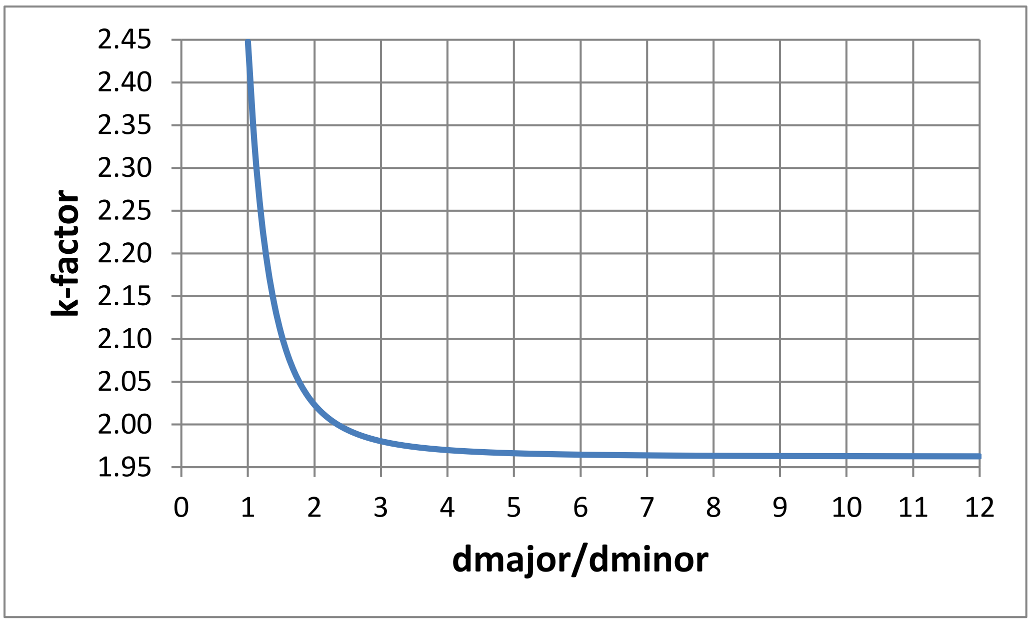

Figure C-1. The k-factor as a function of d_{major}/d_{minor}

Appendix D: List of Acronyms

ADS-B	Automatic Dependent Surveillance Broadcast
AGL	Above Ground Level
APNT	Alternative Positioning, Navigation and Timing
ARP	Airport Reference Point
ATCRBS	Air Traffic Control Radio Beacon System
CDF	Cumulative Distribution Function
DME	Distance Measuring Equipment
FAA	Federal Aviation Administration
GA	General Aviation
GBT	Ground Based Transceivers
GPS	Global Positioning System
HDOP	Horizontal Dilution of Precision (2D)
HPE	Horizontal Position Error
HPL	Horizontal Protection Level
IMC	Instrument Meteorological Conditions
LOS	Line of Sight
MLAT	Multilateration
MSL	Mean Sea Level
NACp	Navigation Accuracy Category for Position
NAS	National Airspace System
NextGen	Next Generation
NIC	Navigation Integrity Category
OI	Operational Improvement
RAIM	Receiver Autonomous Integrity Monitor
RNAV	Area Navigation
SIL	Source Integrity Level
SSR	Secondary Surveillance Radar
TIS-B	Positioning Sources for Traffic Information Service-Broadcast
VHF	Very High Frequency
VOR	VHF Omnidirectional Range
WAM	Wide-Area Multilateration