

# Interplanetary Travel

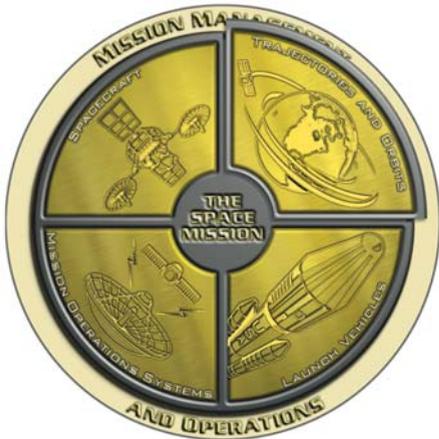
## 4.1.6

### In This Section You'll Learn to...

- ☛ Describe the basic steps involved in getting from one planet in the solar system to another
- ☛ Explain how we can use the gravitational pull of planets to get “free” velocity changes, making interplanetary transfer faster and cheaper

### Outline

- 4.1.6.1 **Planning for Interplanetary Travel**  
Interplanetary Coordinate Systems
- 4.1.6.2 **Gravity-assist Trajectories**

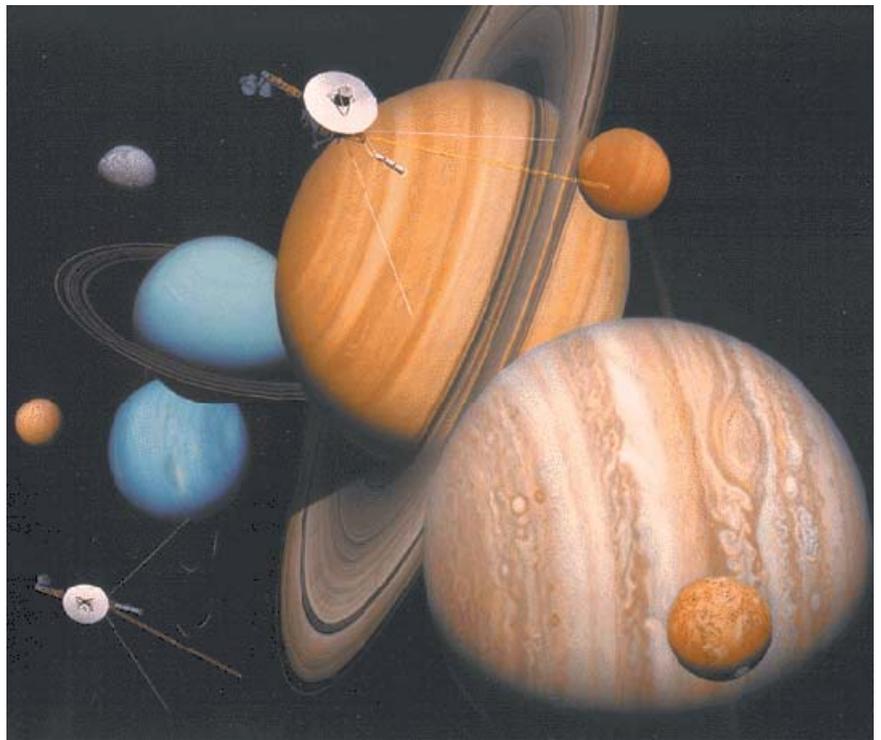


**Space Mission Architecture.** This chapter deals with the Trajectories and Orbits segment of the Space Mission Architecture.

The wealth of information from interplanetary missions such as Pioneer, Voyager, and Magellan has given us insight into the history of the solar system and a better understanding of the basic mechanisms at work in Earth’s atmosphere and geology. Our quest for knowledge throughout our solar system continues (Figure 4.1.6-1). Perhaps in the not-too-distant future, we’ll undertake human missions back to the Moon, to Mars, and beyond.

How do we get from Earth to these exciting new worlds? That’s the problem of interplanetary transfer. In Chapter 4 we laid the foundation for understanding orbits. In Chapter 6 we talked about the Hohmann Transfer. Using this as a tool, we saw how to transfer between two orbits around the same body, such as Earth. Interplanetary transfer just extends the Hohmann Transfer. Only now, the central body is the Sun. Also, as you’ll see, we must be concerned with orbits around our departure and destination planets.

We’ll begin by looking at the basic equation of motion for interplanetary transfer and then learn how we can greatly simplify the problem using a technique called the “patched-conic approximation.” We’ll see an example of how to use this simple method to plot a course from Earth to Mars. Finally, we’ll look at gravity-assist or “slingshot” trajectories to see how we can use them for “free”  $\Delta V$ , making interplanetary missions faster and cheaper.



**Figure 4.1.6-1. Voyager Trajectory.** Here we show an artist’s concept of the Voyager spacecraft and their trajectories during their grand tours of the outer planets. (Courtesy of NASA/Ames Research Center)

## 4.1.6.1 Planning for Interplanetary Travel

### In This Section You'll Learn to...

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- ☞ Describe the basic process for planning an interplanetary transfer from one planet to another
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To develop an understanding of interplanetary transfer, we need to start by gaining some perspective on the problem. If you were planning a trip across town, you'd pull out a city map. If you were planning a trip across the country, you'd pull out a national road atlas. If you were planning a space mission around Earth, you could use the Classic Orbital Elements discussed in Section 4.1.4 along with a Hohmann Transfer presented in Section 4.1.5.

To plan a trip from Earth to Mars, we have to expand our point of view even more to encompass the entire solar system! We'll start by looking at the coordinate systems involved in the defining the problem of interplanetary travel. Then we'll see how we can break this seemingly mind-blowing problem into a just a few bite-sized pieces.

### Interplanetary Coordinate Systems

Earlier, when we were first trying to understand orbital motion, we made several assumptions to simplify the problem. Two of our assumptions were

- There are only two bodies—the spacecraft and Earth
- Earth's gravitational pull is the only force acting on the spacecraft

We defined a convenient coordinate system, called the geocentric-equatorial frame, that we could use for orbits around Earth. This works fine as long as we want to hang out around the Earth. But eventually, we'd like to leave home and travel to other planets in the solar system such as Mars, Venus, or Jupiter.

Once our spacecraft crosses a boundary into interplanetary space, however, Earth's gravitational pull becomes less significant and the Sun's pull becomes the dominant force. Therefore, because the Sun is central to interplanetary transfer, we must develop a heliocentric coordinate system.

*Heliocentric* means the origin is the center of the Sun. In choosing a fundamental plane, we use the plane of Earth's orbit around the Sun, also known as the *ecliptic plane*. Next, we need a principal direction,  $\hat{I}$ , fixed with respect to the universe. Good thing we can bring back the vernal equinox direction ( $\odot$ ) for an encore performance. With the fundamental plane and principal direction chosen, we set the  $\hat{J}$  axis in the ecliptic plane,  $90^\circ$  from the  $\hat{I}$  axis in the direction of Earth's motion. Finally, the  $\hat{K}$  axis is

perpendicular to the ecliptic plane and it completes our right-handed system. Now we can relate any trajectory from Earth to another planet, or even to the edge of the solar system, to this *heliocentric-ecliptic coordinate system* (see Figure 4.1.6-2).

Now that we have a useful coordinate frame to describe the motion of a planet or spacecraft around the Sun, the next step is to understand what forces affect that motion. Any spacecraft (unless Martians build it) begins its mission under the influence of Earth's gravity, so that's one force we have to deal with. But when it gets far enough away from Earth, the Sun's gravitational pull begins to dominate. The Sun's gravity holds the spacecraft until it reaches the destination planet, so we have to deal with that force too. Finally, when the spacecraft reaches the planet it's going to, we have to account for gravity from the planet, as well. This gravitational force could range from Mercury's slight tug to Jupiter's huge pull.

Luckily, Sir Isaac Newton has told us all we need to know about gravity. Recall from Section 4.1.3 this

*Important Concept*

**Newton's Law of Universal Gravitation.** The force of gravity between two bodies is directly proportional to the product of their two masses and inversely proportional to the square of the distance between them.

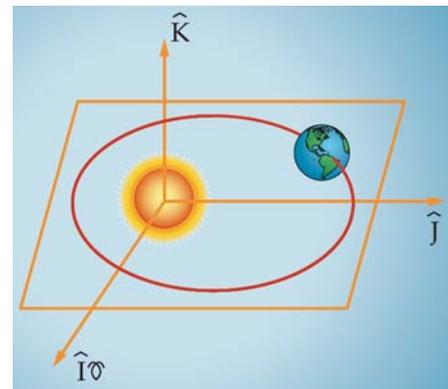
Figure 4.1.6-3 illustrates this concept, but in a trip from Earth to Mars, how many different bodies and their gravitational effects do we have to deal with? Let's count them.

- The Earth
- The Sun
- Mars
- Our spacecraft

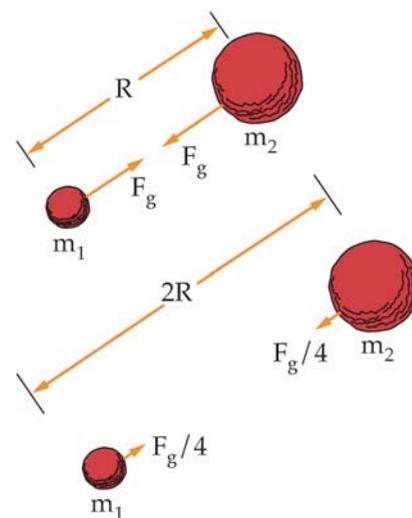
To work out the effects from all these gravitational forces acting at once on our tiny spacecraft, as shown in Figure 4.1.6-4, would almost make your calculator explode. In fact, rocket scientists don't know how to solve for the effects of even three bodies acting at once, let alone four.

So how can we tackle this problem? We use the old "divide and conquer" approach, taking one big problem and splitting it into three little ones. We learned that, as long as we're dealing with only two bodies at once, we are OK. (Typical bodies would be the spacecraft plus the Earth, or the spacecraft plus the Sun.) We know how to calculate those effects. For interplanetary transfers we call this approach the *patched-conic approximation*. The *patched-conic approximation* breaks the interplanetary trajectory into three separate regions and considers only the gravitational attraction on the spacecraft from one body in each region.

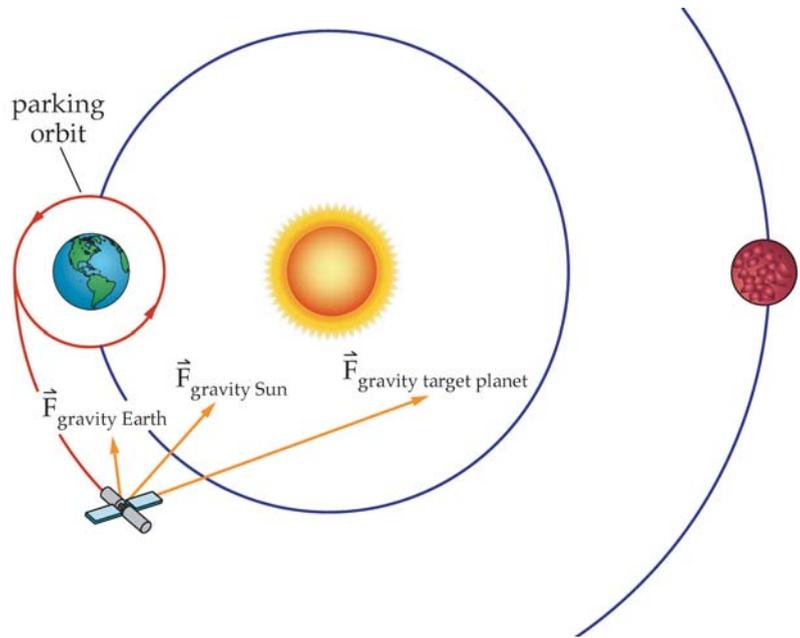
By looking at the problem with respect to only one attracting body at a time, we can use the results we came up with earlier. Remember, Newton spent a long time thinking about motion and concluded all bodies moving under the influence of gravity must follow a path that is one of the four



**Figure 4.1.6-2. Heliocentric-ecliptic Coordinate System for Interplanetary Transfer.** Origin—center of the Sun; fundamental plane—ecliptic plane (Earth's orbital plane around the Sun); principal direction—vernal equinox direction.



**Figure 4.1.6-3. Newton's Law of Universal Gravitation.** The force of attraction between any two masses is directly proportional to the product of their masses and inversely proportional to the square of the distance between them. Thus, if we double the distance between two objects, the gravitational force decreases to 1/4 the original amount.



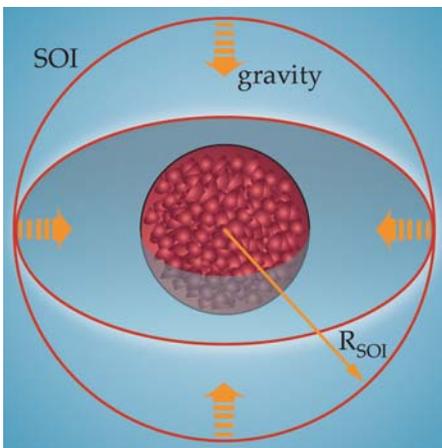
**Figure 4.1.6-4. Gravitational Forces on an Interplanetary Spacecraft.** Consider the forces on an interplanetary spacecraft as it makes its way from Earth to the target planet. We have the gravitational forces due to Earth, the Sun, and the target planet making it a four-body problem—Earth, Sun, target planet, and spacecraft.

conic sections (circle, ellipse, parabola, or hyperbola). Thus, the individual pieces of the spacecraft's trajectory are conic sections. By solving one two-body problem at a time, we "patch" one conic trajectory onto another, arriving at what's called the patched-conic approximation.

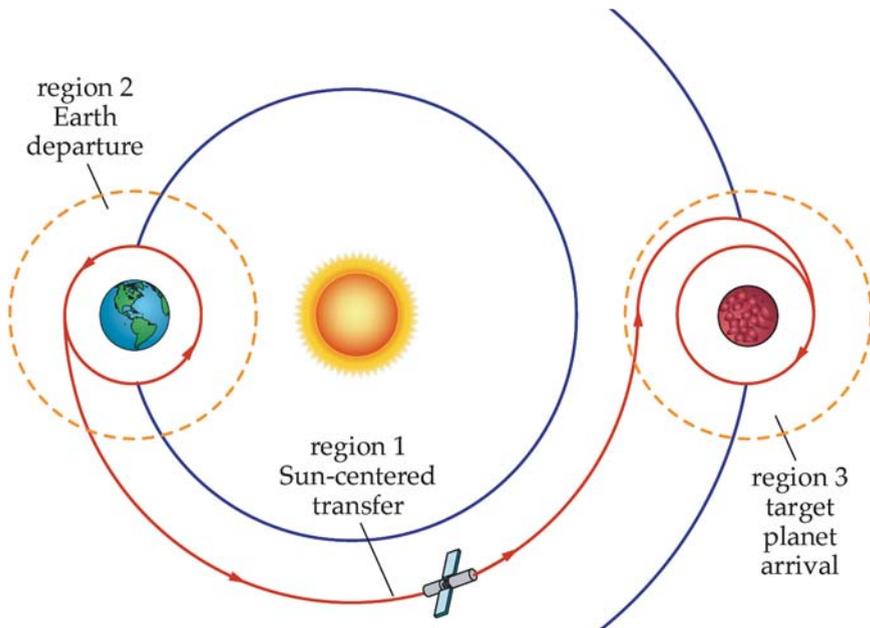
The patched-conic approximation is a way of breaking the interplanetary trajectory into pieces (regions) we can handle, using methods we already know. By working within only one region at a time, we have to deal with the gravity from only one body at a time. Figure 4.1.6-5 shows the three regions of the interplanetary transfer

- Region 1 (solved first)—*Sun-centered transfer from Earth to the target planet.* In this region, the Sun's gravitational pull dominates.
- Region 2 (solved second)—*Earth departure.* In this region, Earth's gravitational pull dominates.
- Region 3 (solved third)—*Arrival at the target planet.* In this region, the target planet's gravitational pull dominates.

To deal with gravity from only one body at a time, we need to know how gravity operates in space. Any mass in space exerts a gravitational pull on other bodies. Newton's Law of Universal Gravitation describes this force as varying inversely with the square of the distance from the central body. Theoretically, a body's gravitational attraction reaches out to infinity, but practically, it's effective only within a certain volume of space called the body's *sphere of influence (SOI)*, as shown in Figure 4.1.6-6. For



**Figure 4.1.6-6. Sphere of Influence (SOI).** A planet's SOI is the volume of space within which the planet's gravitational force dominates.



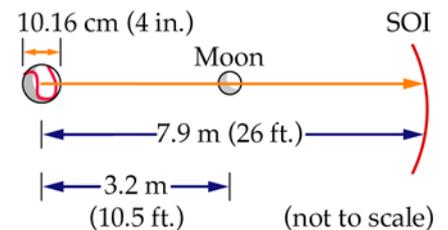
**Figure 4.1.6-5. Three Regions of the Patched-conic Approximation.** We break the trajectory for interplanetary transfer into three distinct regions in which the gravitational pull of only one body dominates the spacecraft's motion.

instance, within Earth's SOI, Earth's gravity dominates a spacecraft's motion. But at some point Earth's gravitational pull becomes insignificant and the pull of other bodies, such as the Moon and Sun, begins to dominate.

The size of the SOI depends on the planet's mass (a more massive planet has a longer "gravitational reach") and how close the planet is to the Sun (the Sun's gravity overpowers the gravity of closer planets). Earth's SOI is about 1,000,000 km in radius, well beyond the Moon's orbit but only a small fraction of the distance from Earth to the Sun (149.6 million km). To put this into perspective, imagine Earth being the size of a baseball, as in Figure 4.1.6-7. Its SOI would extend out 78 times its radius or 7.9 m (26 ft.). Appendix D.5 lists the sizes of the spheres of influence for other planets in the solar system.

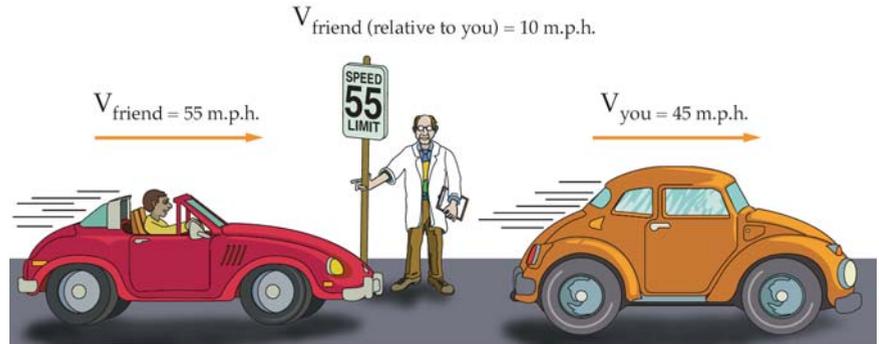
To simplify the complex interactions between a spacecraft and the spheres of influence for the Earth, Sun, and target planet, we use the patched-conic approximation. By separately considering the regions, we set up three distinct two-body problems, solve each one, and then "patch" them together to get a final solution. Our ultimate goal is to determine the total velocity change,  $\Delta V_{\text{total}}$ , a spacecraft needs to leave Earth orbit and get into orbit around another planet. (Later, we'll learn how to use this total  $\Delta V$  requirement to determine the amount of rocket propellant needed for the trip.)

Let's use the patched-conic approach to analyze a down-to-Earth problem. Imagine you're driving along a straight section of highway at 45 m.p.h. Your friend is chasing you in another car going 55 m.p.h. A



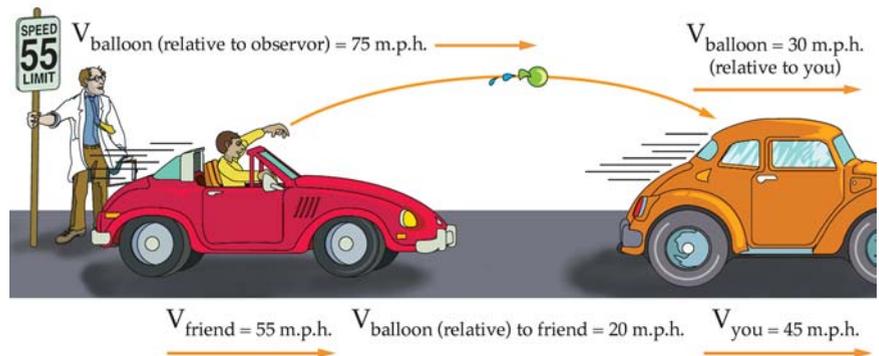
**Figure 4.1.6-7. Earth's Sphere of Influence (SOI) Extends Well beyond the Orbit of the Moon.** To put this in perspective, imagine if Earth were the size of a baseball; then the Moon would be 3.2 m (10.5 ft.) away, and the SOI would have a radius of 7.9 m (26 ft.).

stationary observer on the side of the road sees the two cars moving at 45 m.p.h. and 55 m.p.h., respectively. But your friend's velocity with respect to you is only 10 m.p.h. (she's gaining on you at 10 m.p.h.) as illustrated in Figure 4.1.6-8.



**Figure 4.1.6-8. Relative Velocity.** From your perspective at 45 m.p.h., you see your friend at a speed of 55 m.p.h. gaining on you at a relative speed of 10 m.p.h.

Now suppose your friend throws a water balloon toward your car at 20 m.p.h. How fast is the balloon going? Well, that depends on the perspective. From your friend's perspective, it appears to move ahead of her car at 20 m.p.h. (ignoring air drag). From the viewpoint of the stationary observer on the side of the highway, your friend's car is going 55 m.p.h., and the balloon leaves her car going 75 m.p.h. What do you see? The balloon is moving toward you with a closing speed of 30 m.p.h. (10 m.p.h. closing speed for your friend's car plus 20 m.p.h. closing speed for the balloon, as shown in Figure 4.1.6-9.)



**Figure 4.1.6-9. Transfer from Car to Car.** If your friend throws a water balloon at you at 20 m.p.h. (ignoring air drag) relative to your friend, it will be going 75 m.p.h. relative to a fixed observer and will appear to you to be gaining on you at 30 m.p.h.

By analyzing the balloon's motion, we see the three problems we use in a patched-conic approximation

- **Problem 1:** A stationary observer watches your friend throw a water balloon. The observer sees your friend's car going 55 m.p.h., your car going 45 m.p.h., and a balloon traveling from one car to the other at

75 m.p.h. The reference frame is a stationary frame at the side of the road. This problem is similar to Problem 1 of the patched-conic approximation (in region 1), where the Sun is similar to the observer and the balloon is similar to the spacecraft.

- **Problem 2:** The water balloon departs your friend's car with a relative speed of 20 m.p.h., as shown in Figure 4.1.6-9. The reference frame in this case is your friend's car. This problem relates to the patched-conic's Problem 2 (in region 2), where Earth is like your friend's car and the balloon is like the spacecraft.
- **Problem 3:** The water balloon lands in your car! It catches up to your car at a relative speed of 30 m.p.h. The reference frame is your car. This problem resembles the patched-conic's Problem 3 (in region 3), where the target planet is similar to your car and the balloon is still like the spacecraft.

Dividing interplanetary transfers into three problems requires us to keep track of velocities relative to a reference frame, which changes from one problem to the next. Thus, the spacecraft's velocity with respect to Earth isn't the same as its velocity with respect to the Sun. **This is a very important distinction to understand.** We find only one velocity common to Problems 1 and 2, and only one common to Problems 1 and 3. Still, these velocities allow us to "patch" together the trajectories from the three regions.

So let's see how all this works operationally. Imagine a new Mars Global Surveyor mission that is first launched into a circular orbit around Earth. We may spend a few days in orbit around Earth while ground controllers checkout onboard systems. When everything checks out and the timing is right, the order is given to fire the spacecraft's engines. This creates a large  $\Delta V$ , putting the spacecraft on a completely new trajectory with respect to Earth. SOI on a hyperbolic escape trajectory.

Now change perspective to that of an observer on the Sun (hope his feet aren't too hot). He sees the spacecraft leaving the vicinity of Earth on an elliptical transfer orbit (the same we used for the Hohmann Transfer). As the observer watches, he sees the spacecraft follow along this elliptical transfer orbit until it reaches the SOI of Mars.

Now change perspective a third time to that of any martians who might be watching. They see this alien zooming out of interplanetary space on a hyperbolic trajectory with respect to Mars. When it reaches the desired altitude above the planet, the martians see the spacecraft fire its rockets, providing the  $\Delta V$  to slow down into a circular orbit around Mars.

For mission planning, we need to know the sizes of the two  $\Delta V$ s needed to go from Earth orbit to Mars orbit. By using these three different perspectives of the patched-conic approximation, we can do that. Once we know the  $\Delta V$ , we can use rocket-science tools from Chapter 14 to determine how much propellant will need.

In the next section, we'll see a way to "steal" this  $\Delta V$  from other planets.

## Section Review

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### Key Concepts

- The coordinate system for Sun-centered or interplanetary transfers is the heliocentric-ecliptic system
    - The origin is the Sun's center
    - The fundamental plane is the ecliptic plane (Earth's orbital plane)
    - The principal direction ( $\hat{I}$ ) is the vernal equinox direction
  - Taken together, the interplanetary-transfer problem involves four separate bodies
    - The spacecraft
    - Earth (or departure planet)
    - The Sun
    - The target or destination planet
  - Because the four-body problem is difficult to solve, we split it into three, two-body problems using a method called the patched-conic approximation
    - Problem 1: A Hohmann Transfer between the departure and target planets
    - Problem 2: A hyperbolic departure from the departure planet
    - Problem 3: A hyperbolic arrival at the target planet
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## 4.1.6.2 Gravity-assist Trajectories

### In This Section You'll Learn to...

- ☛ Explain the concept of gravity-assist trajectories and how they can help spacecraft travel between the planets

In the previous sections we saw how to get to other planets using an interplanetary Hohmann Transfer. Even this fuel-efficient maneuver requires a huge amount of rocket propellant, significantly driving up a mission's cost. Often, we can't justify a mission that relies solely on rockets to get the required  $\Delta V$ . For example, if the Voyager missions, which took a "grand tour" of the solar system, had relied totally on rockets to steer between the planets, they would never have gotten off the ground.

Fortunately, spacecraft can sometimes get "free" velocity changes using gravity assist trajectories as they travel through the solar system. This *gravity assist* technique uses a planet's gravitational field and orbital velocity to "sling shot" a spacecraft, changing its velocity (in magnitude and direction) with respect to the Sun.

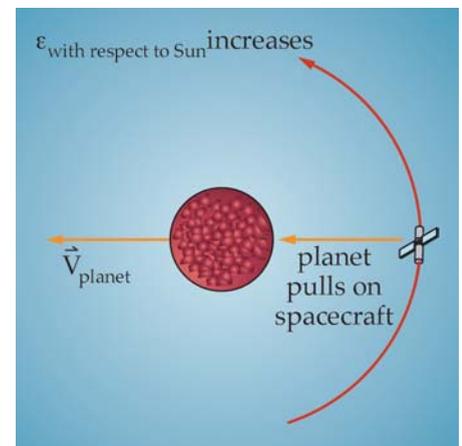
Gravity-assisted velocity changes aren't totally free. Actually, the spacecraft "steals" velocity from the planet, causing the planet to speed up or slow down ever so slightly in its orbit around the Sun. Gravity assist can also bend the spacecraft's trajectory to allow it to travel closer to some other point of interest. The Ulysses spacecraft used a gravity assist from Jupiter to change planes, sending it out of the ecliptic into a polar orbit around the Sun.

How does gravity assist work? As a spacecraft enters a planet's sphere of influence (SOI), it coasts on a hyperbolic trajectory around the planet. Then, the planet pulls it in the direction of the planet's motion, thus increasing (or decreasing) its velocity relative to the Sun. As it leaves the SOI at the far end of the hyperbolic trajectory, the spacecraft has a new velocity and direction to take it to another planet.

Of course, as Isaac Newton said, for every action there is an equal but opposite reaction. So, the spacecraft also pulls the planet a small amount. But, because the spacecraft is insignificantly small compared to the planet, the same force that can radically change the spacecraft's trajectory has no significant affect on the planet. (Imagine a mosquito landing on a dinosaur; the dinosaur would never notice the landing, but the mosquito would!)

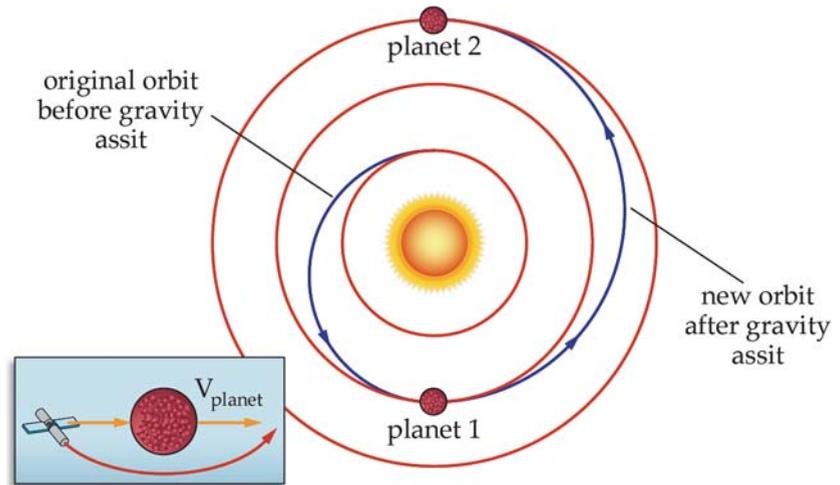
As we saw in the previous sections, a spacecraft's velocity depends on the perspective of the beholder. During a gravity assist, we want to change the spacecraft's velocity with respect to the Sun, putting it on a different heliocentric orbit, so it can go where we want it to.

Let's consider what's going on from the planet's perspective. As a spacecraft flies by on a hyperbolic trajectory, the planet pulls on it. If the spacecraft passes behind the planet, as shown in Figure 4.1.6-10, it's pulled in the direction of the planet's motion and thus gains velocity (and

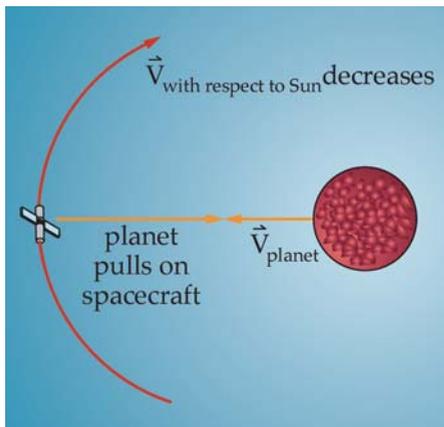


**Figure 4.1.6-10. Spacecraft Passing behind a Planet.** During a gravity-assist maneuver, a spacecraft's energy will increase with respect to the Sun if it passes behind the

hence energy) with respect to the Sun. This alters the spacecraft's original orbit around the Sun, as shown in Figure 4.1.6-11, sending it off to a different part of the solar system to rendezvous with another planet.



**Figure 4.1.6-11. Gravity Assist.** During a gravity assist, a planet pulls a spacecraft, changing its velocity with respect to the Sun and thus altering its orbit around the Sun. The planet's orbit also changes, but very little.



**Figure 4.1.6-12. Spacecraft Passing in Front of a Planet.** During a gravity-assist maneuver, a spacecraft's energy will decrease with respect to the Sun if it passes in front of the

When a spacecraft passes in front of a planet as in Figure 4.1.6-12, it's pulled in the opposite direction, slowing the spacecraft and lowering its orbit with respect to the Sun.

Gravity-assist trajectories often make the difference between possible and impossible missions. After the Challenger accident, the Galileo spacecraft's mission was in trouble. NASA officials banned liquid-fueled upperstages from the shuttle's payload bay, and solid-fuel upper stages simply weren't powerful enough to send it directly to Jupiter. Wisely, mission designers hit on the idea of going to Jupiter by way of Venus and Earth. They used one gravity assist from Venus and two from Earth to speed the spacecraft on its way; hence, the name VEEGA (Venus, Earth, Earth Gravity Assist) for its new trajectory.

A gravity assist that changes the magnitude of a spacecraft's velocity is called *orbit pumping*. Using a planet's gravity to change the direction of travel is called *orbit cranking*. The gravity of Jupiter "cranked" the Ulysses solar-polar satellite out of the ecliptic plane into an orbit around the Sun's poles.

Realistically, mission requirements will constrain which planets we can use for gravity assist or whether it's even possible. For example, NASA used gravity assist for Voyager II's flights past Jupiter, Saturn, and Uranus (saving 20 years of trip time to Neptune). But they couldn't use its flyby past Neptune to send it to Pluto. Doing so would have required a trip beneath Neptune's surface, which would have been a bit hard on the spacecraft! Instead, they used this last flyby to send Voyager II out of the solar system, where it travels today. In 1999, scientists received healthy

transmissions from both Voyager spacecraft at distances of more than 11.2 billion km (6.9 billion mi.) and 8.7 billion km (5.4 billion mi.) for Voyager I and II, respectively. The messages take over 20 hrs, 40 min for Voyager I and 15 hrs, 40 min for Voyager II to travel to the spacecraft and return, at light speed.

## Section Review

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### Key Concepts

- ▶ Gravity-assist trajectories allow a spacecraft to get “free” velocity changes by using a planet’s gravity to change a spacecraft’s trajectory. This changes the spacecraft’s velocity with respect to the Sun and slows the planet (but by a very small amount).
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