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SPANWISE AIR-LOAD DISTRIBUTION

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0.0 FOREWORD

0.00 The necessity for the use of a span load distribution approaching as closely as possible the true distribution occurring in flight has long been recognized. The increase in the use of cantilever monoplanes has made this matter of vital importance since the old arbitrary distribution is unsafe when applied to highly tapered wings of this type.

0.01 The National Advisory Committee for Aeronautics has made a study of span load distribution with the object of formulating a method for determining this distribution for any wing. This study has resulted in the conclusion that the theoretical distribution is in sufficiently good agreement with experimental results to justify its use as a basis for the desired method, except, in some cases, in the vicinity of the wing tip where empirical corrections to the theoretical curves have been found desirable.

0.02 The theoretical procedure described herein is one of several similiar methods that may be used, but this is believed to be the most satisfactory for the great majority of wings. Its application is demonstrated by several examples. Distribution curves of lift coefficient for a wide range of aspect ratios are included for simple uniformly tapered wings. For wings which meet the specifications for which these curves were prepared, the theoretical load distribution may be obtained directly, and the empirical tip correction applied in the same manner as when the theoretical distribution is obtained by the general method.

0.03 It is admitted that the load distribution found by this procedure may not be the same as that experienced in flight, because of fuselage, nacelle or other interference, but it is believed to be much closer to the correct distribution in most cases than that which has been used in the past, and probably correct within the accuracy of the structural calculations which follow.

0.04 Although the method is intended primarily for wings, it may be used also for obtaining the distribution along the span of large tail surfaces. In such cases, however, the influence of slipstream, downwash and fuselage interference is apt to cause a much greater discrepancy between the actual distribution and that indicated by theory.

0.05 The procedure given herein is confined to the determination of the section aerodynamic coefficients along the span for definite angles of attack or for given wing coefficients. Where the coefficients are affected by Reynolds Number, as in the case of the profile-drag coefficient, an explanation of this effect is given. The transformation of these coefficients into actual loads on the wing, including the determination of the design speeds and wing coefficients or angles of attack, and the consideration of the inertia loads will be covered in separate publications.

0.06 Special acknowledgment is due to the National Advisory Committee for Aeronautics and in particular to Mr. R.V. Rhode and to Mr. H.A. Pearson of the Staff of the Langley Memorial Laboratory, from whose reports much of that contained in the following pages has been taken.

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0.5 SYMBOLS

b	Wing span
S	Wing area
A	Aspect ratio, b^2/S
V	Wind velocity
ρ	Mass density of air
q	Dynamic pressure, $1/2\ \rho\ V^2$
w	Induced downflow at a section
L	Lift on wing
C_L	Wing lift coefficient, L/qS
c_s	Chord at plane of symmetry
c_t	Fictitious tip chord
c	Chord at any section
t	Maximum thickness at any section
c_{ave}	Arithmetical mean or average chord, S/b
α	Angle of attack of any section referred to its chord line
α_o	Effective angle of attack of any section
α_a	Angle of attack of any section referred to its zero-lift direction, absolute angle of attack.
α_s	Angle of attack of section at plane of symmetry referred to its zero-lift direction.
α_{so}	Angle of attack of section at plane of symmetry referred to its zero-lift direction when $C_L = 0$
δ	Flap deflection, positive downward.
$k\delta$	Angular change in zero-lift direction of any section produced by flap displacement δ

λ Ratio of fictitious tip chord, obtained by extending leading and trailing edges of wing to extreme tip, to the chord at the plane of symmetry, c_t/c_s

γ Ratio of rounded tip length to length of semispan

l Section lift (per unit length along span)

c_l Section lift coefficient, l/qc, perpendicular to wind at infinity.

Subscripts

- b Refers to basic lift ($C_L = 0$)
- b_1, b_2, b_3 Refers to specific basic lifts, several of which may exist simultaneously.
- a Refers to additional lift for any C_L
- a_1 Refers to additional lift for $C_L = 1.0$
- 1 Refers to relative lift ($C_L = 1.0$)

Δc_l Increment in section lift coefficient obtained empirically and required on some wings to make theoretical values near the tip agree with experiment.

Δc_{l_1} Δc_l when $C_L = 1.0$, $A = 6$, and $\lambda = 1.0$

c_{d_o} Section profile-drag coefficient

c_{d_i} Section induced-drag coefficient

Subscripts

- b Refers to portion caused by basic lift (constant)
- a Refers to portion caused by additional lift (varies as C_L^2)
- a_1 Refers to portion caused by additional lift for $C_L = 1.0$
- ab Refers to portion caused by interaction of basic and additional lifts (varies as C_L)
- a_1b Refers to portion caused by interaction of basic and additional lifts for $C_L = 1.0$

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bb	Refers to portion caused by interaction of two basic lifts (constant).
1	Refers to relative induced drag ($C_{D_i} = 1.0$)
C_{D_i}	Wing induced-drag coefficient
m	$dC_L/d\alpha$ of entire wing, per radian
m_o	$dc_l/d\alpha_o$ of any section, per radian
m_s	$dc_l/d\alpha_o$ of section at plane of symmetry.
mt	$dc_l/d\alpha_o$ of section at extreme tip.
u_o	$m_s c_s/4b$
ΔC_L	Increment in wing lift coefficient which must be added at wing tips to the theoretical wing C_L to make it agree with test. Also used to denote change in lift produced by displaced flap at given angle of attack.
ΔC_{L_1}	ΔC_L for any wing when $C_L = 1.0$
σ	A factor correcting the induced drag to allow for a departure from elliptical span loading.
τ	A factor correcting the induced angle of attack to allow for a departure from elliptical span loading.
y	Coordinate of any point along span, measured from plane of symmetry
y'	Coordinate of a fixed point along span.
$\bar{y}_L$	Location of centroid of lift on semispan
$\cos\theta$	$2y/b$ (when $y = b/2$, $\theta = 0$; when $y = -b/2$, $\theta = \pi$)
A_n, B_n, C_{2n}	Coefficients in Fourier series.
(n)$_n$	Circle inclosing a number refers to the line number in a table, while the sub number refers to the column number
F_1	Lift coefficient factor for tip correction
F_2	Aspect ratio factor for tip correction
F_3	Taper factor for tip correction

p'	Angular velocity in roll about the wind axis
C_l'	Rolling-moment coefficient about the wind axis
C_n'	Yawing-moment coefficient about axis normal to wind
R	Reynolds Number

CHAPTER 1

G E N E R A L

GENERAL

1.0 AERODYNAMIC COMPONENTS

1.00 The resultant aerodynamic load at a given location on the span of a wing is conveniently resolved into lift and drag forces, perpendicular and parallel, respectively, to the flight path and applied at the aerodynamic center with a couple about the spanwise axis. The drag force may be further broken down into profile and induced drag, the profile drag being the sum of a minimum and an additional profile drag. The curves of distribution of lift, moment, and the three component drags along the span are, in general, different and so must be separately obtained. Whether or not they are used separately in the stress analysis may be determined by practical considerations.

1.01 By the use of well known methods, the structural coefficients may be obtained in terms of the aerodynamic coefficients - for example, a beam coefficient will be made up of components due to each of the aerodynamic coefficients, viz: (a) Beam component of lift coefficient; (b) Beam component of induced-drag coefficient; (c) Beam component of minimum profile-drag coefficient; (d) Beam component of additional profile-drag coefficient; and (e) In some cases a beam component due to moment coefficient.

1.02 Structurally, the most important aerodynamic component is that of lift and the greater portion of this bulletin is devoted to the distribution of the lift coefficient. Since the induced drag is actually the flight path component of the section lift, its distribution is readily obtained during the process of calculating the lift distribution. The distribution of the profile drag (minimum and additional) and of the moment coefficient and the locations of the aerodynamic center are discussed in Chapter 6.

1.1 BASIC CONSIDERATIONS

1.10 In a wing of infinite aspect ratio and constant airfoil section, the forces on all sections are the same, and the lift and drag coefficients which such a wing would have are known as the section characteristics for the particular section used. In such a wing, there is no downwash and the true absolute angle of attack is constant along the span and equal to the angle between the zero-lift chord and the flight path. After these characteristics are once determined for a given section, they may be used with that section regardless of where the section occurs in any wing, provided the true or local angle of attack can be determined at that particular location.

1.11 In a wing with finite span, the angle between the zero-lift chord and the flight path is no longer the true angle of attack of the sections (except when zero), because the downwash reduces the local angles of attack. When the zero-lift chords of a wing are in one plane, and the plan form is such that the chord lengths are proportional to the ordinates of an ellipse, and the same section is used throughout, the downwash angle is constant along the span. The true angles of attack of the sections, although less than the angle between the zero-lift chord and the flight path, are constant, and the section lift coefficient is the same from tip to tip and equal to the wing lift coefficient.

1.12 In a wing of any other shape or in which there is twist or a change in section along the span, the downwash angle is no longer constant along the span, and the section lift coefficient will, in general, vary. The problem then is to determine this variation in lift coefficient along the span for any given wing.

1.13 The lift coefficient at any section may be expressed as

$$c_l = m_o \alpha_o \qquad (1.1)$$

where m_o is $dc_l/d\alpha_o$ for the section, and α_o is the effective angle of attack. Since, as explained above, the effective angle of attack at any section is the difference between the absolute angle of attack with respect to the flight path and the downwash or induced angle, this may be written

$$c_l = m_0 \, (\alpha_a - w/V) \qquad (1.2)$$

where α_a is the absolute angle of attack of the section with respect to the flight path, and w is the downwash or induced velocity at the section. V is the velocity of the airplane or the free-stream velocity. The downwash angle at any section at distance y' from the plane of symmetry may be expressed as follows (reference 1):

$$\frac{w}{V} = \frac{1}{8\pi} \int_{-\frac{b}{2}}^{+\frac{b}{2}} \frac{\frac{d}{dy} (c \; c_l) \; dy}{y'-y} \qquad (1.3)$$

where c and c_l are respectively the chord and lift coefficient of the various sections along the span. The integration of this expression, most of which must be performed graphically, is explained in reference 1. The solution of (1.3) for a given location when substituted in (1.2) gives the value of the lift coefficient for that location. The induced-drag coefficient for the same location is given by

$$c_{d_i} = c_l \frac{w}{V} \qquad (1.4)$$

1.14 This method would be relatively simple if it were not for the fact that c_l must be known all along the span before (1.3) can be solved. The distribution can, therefore, be determined only by successive approximations. A span loading curve must first be assumed, after which several points along the span must be checked by (1.3) and (1.2). If the computed values of c_l or cc_l do not fall on the assumed curve, the curve must be modified and the process repeated until reasonable agreement is obtained. It is needless to say that, unless the shape of the loading curve is fairly well known at the start, this method is very tedious.

1.15 Several methods have been proposed for obtaining the distribution of lift directly, in which the circulation ($c\ c_l\ V/2$) is generally expressed, either as a power or a Fourier series. The Fourier series method was first used by Glauert (reference 2) in which the value of the lift is computed at four points along the semispan. In the case of a wing of irregular plan form or in which the section angle of attack changes abruptly, as when ailerons or part-span flaps are deflected, a larger number of points must be used to obtain an accurate distribution. The points may be chosen anywhere along the span and there is no limit to the number of points that may be used with this method. It is necessary, however, to solve as many simultaneous equations as the number of points taken on the semispan. Other methods are described in references 4 and 5.

1.2 THE LOTZ METHOD

1.20 Miss Lotz has developed a method based on the Fourier solution which is preferred by many (reference 11). She introduced two additional series the coefficients of which are independent of the load distribution and may be separately computed. Shenstone (reference 3) gave a brief outline of the Lotz method, described a mechanical method for determining the coefficients and included a numerical example. The method of obtaining the coefficients has been simplified by Pearson (reference 9). He developed forms for the use of ten and twenty points along the semispan and gave a numerical example for a tapered wing with a part-span flap, using twenty points. This method has been used in the development of the procedure herein presented. It has the advantage of giving points closer together near the wing tip where the value of the lift coefficient usually varies most rapidly. This proves a disadvantage, however, in the case of a wing with a cut-out in the center section, because here the points are relatively far apart and it is impossible to vary the point spacing. Also, in the case of a displaced flap or aileron, the load distribution at the end of the flap or aileron is not sharply defined by this method. In such cases it may be necessary to use a greater number of points, or it may be preferable to use one of the other available methods. No attempt is made here to explain the theory of the Lotz method, as that can be obtained from the above references with the help of a text book on Fourier series. The purpose of this bulletin is to present a procedure that can be readily followed by anyone with a knowledge of arithmetic. With this in mind, tables for the use of ten points along the semispan are given with an explanation of their use and, later, several numerical examples are carried through to further demonstrate the method. If judgment is used in modifying the chord or angle of attack in the vicinity of discontinuities, such as at the end of a cut-out or at the end of a flap, as explained later in Paragraph 4.253 and illustrated in Examples 3 and 4 of Chapter 7, ten points should be sufficient for all practical purposes in almost all cases. A procedure including the necessary tables for the use of twenty points along the semispan may be found in reference 9.

1.3 SCOPE AND LIMITATIONS OF THE THEORY

1.31 Fuselage and Nacelle Interference - The method may be applied to any monoplane wing or to the individual wings of a biplane, in which case each wing is treated as a monoplane by itself, although the agreement with experimental data is not apt to be quite so good in the case of the biplane. In most cases the wing can be assumed to extend through the fuselage unaffected, although where sufficient test data are available to show definitely the effect of a fuselage or nacelle on the lift curve of a wing, the chord or angle of attack in the portion blanketed may be modified if necessary. Where a wing tapers from the edge of the fuselage, the leading and trailing edges should be extended to the plane of symmetry to complete the wing. The distance between intersections of the leading and trailing edges on the plane of symmetry is called the root chord. Likewise, the tip chord is the distance between the intersections of the extended leading and trailing edges with a plane tangent to the tip and parallel to the plane of symmetry. The ratio of the tip chord to the root chord is called the taper ratio and is herein denoted by λ.

1.32 Empirical Tip Corrections - It has been found by experiment that near the tip the actual distribution differs slightly from that indicated by theory and it is necessary, in some cases, to apply empirical corrections to cause agreement. These corrections are a maximum for wings of constant chord, and become negligible for small values of the taper ratio λ . The first step is, therefore, to compute λ . If λ is less than about 0.5, no correction is needed and the calculations may proceed, using the actual chords of the wing tip. If λ is greater than approximately 0.5, the magnitude of the correction is sufficient to require consideration, but the necessity of applying it will depend somewhat upon the type of wing structure, as discussed later in Paragraph 2.34. Attention is called to the fact that if the correction is found to be necessary, the calculations for the theoretical distribution should be applied to the wing having square tips; that is, the leading and trailing edges of the wing should be extended to the extreme tip, and these lines used to define the chords of the tip. This will require a slight modification of the area of the wing to allow for the amount thus added. This empirical correction consists in adding Δc_l to the theoretical curve near the tip in accordance with curves furnished herein. Since these curves are intended for use with the theoretical curves for square-tip wings, it must be definitely determined, before starting the theoretical calculations by the general method, whether or not the tip correction is to be applied.

1.33 Sweepback and Dihedral - Equation (1.3) is based on the assumption that the section aerodynamic centers all fall on a straight line reaching from tip to tip and is not theoretically correct when the wing has sweepback or dihedral. For small values of either, the effect is small.

The theoretical effect of dihedral is to cause the sections near the center to take a greater proportion of the load, while sweepback tends to shift the load outward toward the tips. It does this by increasing the downwash at the center and by decreasing it near the tips, which is an effect similar to that produced by increasing the taper. Sufficient pressure distribution tests on sweptback wings are not yet available to check the theory, and nothing more than a warning will be included in the bulletin at this time. It is hoped that eventually the effect of sweepback can be included by using an equivalent taper that will produce approximately the same distribution. An approximate calculation indicates that a wing with uniform chord and 30-degrees sweepback may have a distribution of lift coefficient along the span somewhat similar to a wing without sweepback but tapered 2:1.

1.34 <u>Lift Coefficient Near the Stall</u> - The wing theory on which this procedure, except for the tip correction, is based assumes that the lift coefficient of any section is proportional to its effective angle of attack. This is approximately true for most wing sections except at angles of attack near the stall. When stalling begins at any section, the distribution, as determined by this method, is no longer valid. It is fortunate, from a structural standpoint, that the critical wing loads usually occur at angles of attack appreciably below the stall, and there is also reason to believe that the dynamic lift curve is more apt to be a straight line and extend to values well beyond those determined in the wind tunnel under static conditions. Since most loads critical for the structure occur during a rapid change in angle of attack, such as when encountering a gust or during a maneuver, the assumption that all section lift coefficients are proportional to the angle of attack is justified. For dynamic loading, therefore, it is permissible to use the procedure of this bulletin for a value of lift coefficient at least 25 percent in excess of that for stalling under steady conditions.

1.35 <u>Effect of Reynolds Number</u> - At angles of attack below the stall the effect of a change in Reynolds Number on the lift and induced drag is negligible. The magnitude of the minimum profile drag, however, is a function of the Reynolds Number, as described in Paragraph 6.112, which means that the distribution along the span is modified by this effect, although the relative distribution may be assumed to remain constant. (See Paragraph 7.192). The additional profile-drag coefficient is also affected, since its value for any value of lift coefficient is a function of the maximum lift coefficient which in turn is dependent on Reynolds Number. This is discussed in Paragraph 6.12 and the determination of the profile-drag coefficient for

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a given condition is illustrated in Paragraph 7.193 of the first example in Chapter 7. As stated, the angle of attack at which a section stalls, and the value of the maximum lift coefficient, are dependent on the value of the Reynolds Number. Except insofar as it affects the additional profile drag, this phenomenon is relatively unimportant for dynamic loading and has no bearing on the relative span distribution. If an investigation of the stalling characteristics of a wing is desired which involves a computation of the lift distribution under static conditions, the effect of Reynolds Number on the maximum lift coefficients of the sections is very important, as described in Paragraph 1.4 below. The location of the aerodynamic center and the magnitude of the moment coefficient about the aerodynamic center are not appreciably affected by a variation in Reynolds Number.

1.4 DETERMINATION OF THE STALLING CHARACTERISTICS OF A WING

1.40 While the purpose of determining the span distribution is primarily structural, it is also of interest from an aerodynamic standpoint. Since it is essential to have good lateral control when approaching a landing while flying near the stall, it is desirable to know where stalling begins along the span of the wing. In reference 8 Anderson explains how this can be determined. The maximum lift coefficients for the sections are determined from the section characteristics and their individual Reynolds Numbers, and a curve plotted along the semispan. Several curves of lift distribution as calculated are plotted on the same graph, and one found which is tangent to the curve of maximum lift coefficients, thus showing approximately where stalling begins. This method assumes that the maximum lift coefficient for a section depends only on its effective Reynolds Number and is independent of its location in the whole wing. This has not been definitely proved. There is evidence to indicate that the lift coefficient at which a section stalls depends somewhat on the plan-form shape of the wing, and perhaps on other factors. Furthermore, if there is appreciable curvature in the section lift curves below their maximum lift points, the section lift coefficients computed by the usual method, which assumes linear variation of c_l with α_o, must be recomputed using modified values of m_o determined for the values of α_o found in the first calculation for the particular angle of attack which resulted in a curve tangent to the curve of maximum lift coefficients. Since this will result in slightly different values of α_o, it may be necessary to recompute once more.

CHAPTER 2

METHOD OF OBTAINING THE LIFT AND INDUCED-DRAG DISTRIBUTION DIRECTLY FOR CERTAIN TYPES OF UNTWISTED UNIFORMLY TAPERED WINGS BY MEANS OF CHARTS

2. METHOD OF OBTAINING THE LIFT AND INDUCED-DRAG DISTRIBUTION DIRECTLY FOR CERTAIN TYPES OF UNTWISTED UNIFORMLY TAPERED WINGS BY MEANS OF CHARTS

2.0 Scope and Limitations of Special Method - Before describing the general method of computing the span distribution, a more simple method will be presented in which the relative distribution for the lift coefficient is read directly from charts contained herein. This method is applicable only to wings with certain definite characteristics, however. The charts give the additional-lift distribution which is equal to the total distribution only when the basic distribution is zero, as will be explained later in Chapter 4 during the description of the general procedure. In general, this method is applicable only to wings having the following characteristics:

(a) The zero-lift chords must all be parallel, that is, the wing must have no aerodynamic twist.

(b) The plan form of the semiwing must be trapezoidal except for the tip, which means that the leading and trailing edges must be straight from the plane of symmetry, or from the side of the fuselage or hull, to the beginning of a normally shaped wing tip.

(c) The charts were calculated for wings having a constant section lift-curve slope m_o throughout the span, but an approximate method of using them for wings which taper in thickness ratio is given in Section 2.2. The approximation consists in assuming that $c\ m_o$ varies linearly along the span.

(d) The individual wings of a biplane which fall within the above classification may be treated as monoplanes with reasonable accuracy, although interference effects may cause a slightly greater divergence between the theoretical and the actual distributions.

(e) The use of a large cut-out, the use of a straight center section with tapered outer panels, or the use of wings with curved leading or trailing edges (except at the tip) makes these curves inapplicable and the general method must be used.

(f) For wings which have aerodynamic twist, symmetrical or antisymmetrical (See Paragraph 4.20), but which meet the plan-form requirements given above, these charts may still be used for determining the distribution of the additional portion of the lift coefficient and the distribution of that portion of the induced-drag coefficient which varies as C_L^2, but since the C_{2n} coefficients are necessary in order to calculate the basic distributions, there is not much to be gained by using the curves for this type of wing.

2.1 Procedure for Wing with Constant m_o - Given a wing that meets the specifications (a) and (b) above and has a constant value of m_o throughout the span, the procedure is as follows:

(a) Determine the taper ratio, $\lambda = c_t/c_s$, where c_t is the chord of the extended wing at the extreme tip and c_s is the chord at the plane of symmetry or the intercept produced by extending the leading and trailing edges to the center.

(b) Calculate the true aspect ratio of the actual wing, $A = b^2/S$.

(c) Calculate the aspect ratio for the extended wing, $A' = 2b/c_s\ (1 + \lambda)$

(d) Obtain the section slope of the lift curve per radian, m_o, for the airfoil used in the wing. All recent N.A.C.A. publications on airfoils, such as reference 15, include a table of "Fundamental Section Characteristics" from which m_o may be found by multiplying the value given for a_o by 57.3. If this information is not available for the airfoil section used, the slope may be approximated by the following formula given by Diehl in reference 10:

$$m_o = 2\pi - 4.0\ (t/c) \qquad (2.1)$$

where t/c is the thickness ratio of the section.

(e) If m_o is constant throughout the span, calculate A'/m_o, where A' is the aspect ratio of the extended wing found in (c) above.

(f) Calculate γ, the ratio of the mean tip length to length of semispan (See Paragraph 2.1(h)).

(g) The relative distribution of the additional lift coefficient $c_{l a_1}$, which for this type of wing is the distribution of the total lift coefficient when the wing lift coefficient $C_L = 1.0$, may now be found from Figures 2-1, 2-2 and 2-3 for the designated locations along the span. These figures consist of curves of $c_{l a_1}$, plotted versus taper ratio λ for four values of A'/m_o at eight locations along the semispan. The coefficient $c_{l a_1}$ is found, therefore, for the given value of λ by interpolating between the curves for the value of A'/m_o found in (e) above which can best be done by cross plotting.

(h) These theoretical values of $c_{l a_1}$ determined for the wing with square or trapezoidal tips should, when multiplied by the chords and the products integrated over the span, check the wing coefficient of 1.0. In order to obtain the values of $c_{l a}$ at any given section for any given value of C_L, it is then only necessary to multiply the value of $c_{l a_1}$ for this section by C_L. If, however, the wing actually has rounded tips, the values of $c_{l a_1}$

when applied to the actual chords will not check the wing coefficient of 1.0 and it is necessary to correct the distribution by multiplying by a constant determined by integrating over the span. This is more fully explained in Paragraph 2.2(h) for the wing with varying m_o.

(i) After the correct distribution of $c_{l\,a_1}$ has been determined, the corresponding distribution of induced-drag coefficient $c_{di_{a_1}}$ can be calculated. It is first necessary, however, to find the angle of attack required to produce a wing lift coefficient of 1.0. This is given by

$$\alpha_a = C_L/m = 1/m \qquad (2.2)$$

in which α_a is the absolute angle of attack in radians, or the angle of attack of the wing referred to its zero-lift direction, which must be constant along the span for this procedure to apply. The slope of the lift curve of the actual wing per radian is represented by m and is determined by

$$m = \frac{m_o}{1 + \frac{m_o}{\pi A}(1 + \tau)} \qquad (2.3)$$

in which m_o is the same as determined in (d) above, A is the aspect ratio of the actual wing, while τ is a factor for the induced angle of attack that may be found from Figure 2-4. Figure 2-4 is an alignment chart, the scale on the right being A/m_o, while on the left are two sets of curves; one set representing constant values of λ with varying amounts of tip rounding γ , while the other set represents constant values of γ with varying λ . The tip-rounding ratio γ is the ratio of the tip length, or mean distance along the span from the extreme tip to the sections where the tip is tangent to the straight leading and trailing edges, to the semispan. In the preparation of this chart the tip shape was assumed to be elliptical, but the chart may be used with sufficient accuracy regardless of the shape of the tip rounding. To use the chart a point should be located by interpolating between both sets of curves for the values of λ and γ for the wing under consideration. A straight line should then be drawn from this point to the correct value of A/m_o on the right hand scale and where this line crosses the central scale is read the correct value of τ . It will be noted that rounding the tip causes a marked reduction in the value of τ and that, for a given tip length, τ is a minimum when λ is approximately 0.3, depending somewhat on the aspect ratio. τ is, of course, zero for an untwisted wing completely elliptical in plan form.

(j) The distribution of induced drag can now be calculated. At any given section the effective angle of attack is

$$\alpha_o = \frac{c_{l\,a_1}}{m_o} \tag{2.4}$$

Then the downwash angle is given by

$$\frac{w}{V} = \alpha_a - \alpha_o \tag{2.5}$$

and

$$c_{d_{i\,a_1}} = c_{l\,a_1} \frac{w}{V} \tag{2.6}$$

In this way the section values of the induced-drag coefficient are found for $C_L = 1.0$. In order to obtain the values c_{d_i} for any given value of C_L, it is then only necessary to multiply these values of $c_{d_{i\,a_1}}$ by $C_L{}^2$.

(k) When it is desired to determine the distributions at a given angle of attack, it is first necessary to determine the wing lift coefficient C_L corresponding to this angle. This is given by

$$C_L = m\,\alpha_a \tag{2.7}$$

(l) For a given value of C_L, the corresponding value of C_{D_i} is given for this type of wing by the usual formula

$$C_{D_i} = \frac{C_L{}^2}{\pi A} \quad (1 + \sigma) \tag{2.8}$$

in which A is the aspect ratio of the actual wing as determined in (b) above and σ is the induced drag factor found from Figure 2-5 in a manner similar to that used in determining τ from Figure 2-4.

2.2 Procedure for Wing with Variable m_o - In the procedure just described for a wing meeting the requirements of Paragraphs 2.0(a) and 2.0(b), it was assumed that m_o was constant along the span. This is not usually the case for tapered wings. A procedure will now be given for a wing in which m_o varies. The method consists of substituting for the actual wing with varying m_o an equivalent wing having a constant value of m_o, but with a different taper so as to produce the same span loading. The results must necessarily be approximate unless $c\ m_o$ happens to vary linearly along the span. A very common type of tapered wing is one in which both the chord c and the thickness t vary linearly along the span, in which case, if m_o varies linearly with thickness ratio t/c, $c\ m_o$ also varies linearly along the span and an exact solution can be obtained. The method is illustrated by example 1 in Chapter 7.

(a) The procedure is similar to that of Section 2.1 except that the taper ratio of the equivalent wing must first be determined. If a table of "Fundamental Section Characteristics" is available for several thickness ratios of the airfoil section used, the respective values of a_o should be multiplied by 57.3 and plotted versus t/c. A curve to represent these points may then be used from which to read the values of m_o for the various sections of the wing. If a table of characteristics is not available, Equation 2.1 may be used to determine m_o. If the curve of m_o versus t/c is a straight line and the thickness tapers linearly, the taper ratio of the equivalent wing is

$$\lambda' = \frac{m_t \; c_t}{m_s \; c_s} = \frac{m_t}{m_s} \lambda \qquad (2.9)$$

in which m_t and c_t are the values of m_o and the chord at the extreme tip of the trapezoidal wing, and m_s and c_s are the corresponding values at the plane of symmetry. If the thickness does not taper linearly or if m_o does not vary linearly with t/c, it is necessary to plot a curve of c m_o along the semispan. On this plot draw a straight line to represent approximately a mean for this curve. On this straight line read the value of $m_t' \; c_t$ at the tip and $m_s' \; c_s$ at the centerline. Then the taper ratio of the equivalent wing is

$$\lambda' = \frac{m_t' \; c_t}{m_s' \; c_s} = \frac{m_t'}{m_s'} \lambda \qquad (2.10)$$

(b) Calculate the aspect ratio of the actual wing, $A = b^2/S$.

(c) Calculate the aspect ratio for the equivalent square-tip wing,

$$A' = 2b/c_s \quad (1 + \lambda')$$

(d) Calculate the aspect ratio for the equivalent round-tip wing

$$A'' = A \frac{(1 + \lambda)}{(1 + \lambda')}$$

(e) Calculate A''/m_s' and A'/m_s'

(f) Calculate γ, the ratio of the mean tip length to length of semispan.

(g) From Figures 2-1, 2-2 and 2-3 obtain the values of $c_{l\,a_1}$ for the equivalent wing, using λ' and A'/m_s' for λ and A'/m_o.

(h) Calculate the values of c_{l_a} for the true chords by multiplying the values of $c_l'{}_{a_1}$ obtained above for the equivalent wing by the respective ratios of m_o/m_s'.

$$c_{l_a} = c_l'{}_{a_1} \frac{m_o}{m_s'}; \text{ or } c_l'{}_{a_1} \frac{m_o}{m_s} \text{ if } m_s' = m_s \quad (2.11)$$

(i) If these values of c_{l_a} are multiplied by the true chords of the wing, including those of the actual rounded tip, and integrated over the span and divided by the true wing area, the result is the wing lift coefficient which, if the tip were not rounded and m_o were constant, should equal 1.0. When the tips are rounded, or m_o varies along the span, however, the value of C_L will differ slightly from 1.0 and it is necessary to apply a correction to the c_{l_a} values to obtain the correct $c_{l_{a_1}}$ values. The correction factor is found as follows:

$$C_L' = \frac{2}{S}\int_0^{b/2} c\, c_{l_a}\, dy = \frac{2}{S}\int_0^{b/2} c\, c_{l_{a_1}}' \frac{m_o}{m_s'}\, dy \quad (2.12)$$

The corrected relative values of $c_{l_{a_1}}$ are found by dividing the values of c_{l_a} by C_L'.

(j) The procedure for calculating the induced-drag distribution and the wing coefficients is the same as that of Section 2.1. Care must be taken to distinguish between the equivalent and actual wings, however. The slope of the lift curve for the equivalent wing is given by

$$m' = \frac{m_s'}{1 + \frac{m_s'}{\pi A''}(1 + \tau')} \quad (2.13)$$

in which τ' is the value of τ obtained from Figure 2-4 for λ' and A''/m_s' of the equivalent wing with proper rounding. Then for the actual wing

$$m = m' \frac{S''}{S} = m' \frac{A}{A''} = m' \frac{(1 + \lambda')}{(1 + \lambda)} \quad (2.14)$$

and, as before,

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$$C_L = m\,\alpha_a \qquad (2.7)$$

(k) The induced-drag coefficient is given as usual by

$$C_{D_i} = \frac{C_L{}^2}{\pi A}\,(1 + \sigma) \qquad (2.8)$$

where A is the aspect ratio of the actual wing while σ is determined for the equivalent wing using λ', A''/m_s' and the ratio of tip length to semispan.

(l) Figure 2-6 gives the theoretical location of the lateral center of pressure as a fraction of the semispan for trapezoidal wings. These curves may be found useful for checking purposes. For wings with a varying m_o, the characteristics λ' and A'/m_s' for the equivalent wing should be used with the curves.

2.3 Application of Empirical Tip Correction.

2.30 As previously mentioned, the theoretical lift distribution, whether obtained from these charts or by means of the general method, is not in good agreement with experiment in the region of the tips for rectangular wings or for wings with only a small amount of taper; although the disagreement is very much reduced if the tips are well rounded. In reference 14 Pearson has developed an empirical correction based on a limited amount of test data which, when added to the theoretical lift distribution for the wing with square tip, serves to bring the calculated values into approximate agreement with experiment. Reference 14 has been used to prepare Figures 2-7 to 2-11 from which the lift increment to be added over the affected tip distance to the theoretical curve may be determined. The affected tip distance was found by Pearson to be 40 per cent of the average wing chord determined by dividing the area by the span. The distribution of this ΔC_L is given in Figure 2-7 where Δc_{l_1} is plotted against per cent of affected tip distance and represents the actual values of the increment of section lift coefficient for a rectangular wing of aspect ratio 6 when the wing $C_L = 1.0$. The shape of this curve is independent of angle of attack or wing lift, aspect ratio, and taper ratio; although its magnitude depends on all three in accordance with Figures 2-8, 2-9 and 2-10, respectively. For a given wing, therefore, the lift coefficient increment at any section near the tip is Δc_{l_1}, from Figure 2-7, times F_1 F_2 F_3 from Figures 2-8, 2-9 and 2-10.

2.31 It should be particularly noted that this tip increment is centered near the trailing edge of the wing and its center of pressure--or aerodynamic center since it does not change with angle of attack--is given in per cent of the wing chord by Figure 2-11. In case it is desired to add the increment section lift directly to the theoretical section lift and use only one aerodynamic center for the section, it can be done if an increment moment coefficient is also added to the basic moment coefficient for the section. This moment correction is given in reference 14, Figure 3b, or it may be computed.

2.32 For a wing with no twist, the theoretical lift distribution curve does not change in shape with angle of attack; which means that the section lift coefficient at any section for a given wing C_L can be found by multiplying the relative lift coefficient $c_{l\,a_1}$ for the section by C_L. If the ΔC_L also varied directly with C_L, the delta distribution could be added directly to the theoretical and the two treated as one, thereafter. Unfortunately, ΔC_L does not vary directly but more nearly as C_L^2. In fact, the factor F_1 of Figure 2-8 is so nearly equal to C_L^2, that the expression for the increment lift, instead of

$$\Delta c_l = F_1 \, F_2 \, F_3 \quad \Delta c_{l\,1} \qquad (2.15)$$

may be written for all practical purposes

$$\Delta c_l = C_L^2 \, F_2 \, F_3 \quad \Delta c_{l\,1} \qquad (2.16)$$

Because of this difference in the rate at which these two distributions vary with angle of attack, it is believed best to treat the tip increment as a separate and additional load to be handled separately for each design condition. In this way the theory for determining C_L and C_{D_i} for any angle of attack may be used without regard for the tip correction. For any given angle of attack, therefore, the total lift on the wing is determined by

$$L = q \quad S \quad (C_L + \Delta C_L) \qquad (2.17)$$

in which S is the total wing area, C_L is the theoretical value for the given angle of attack and ΔC_L is twice the load added to each wing tip expressed as a coefficient in terms of the total wing area, or

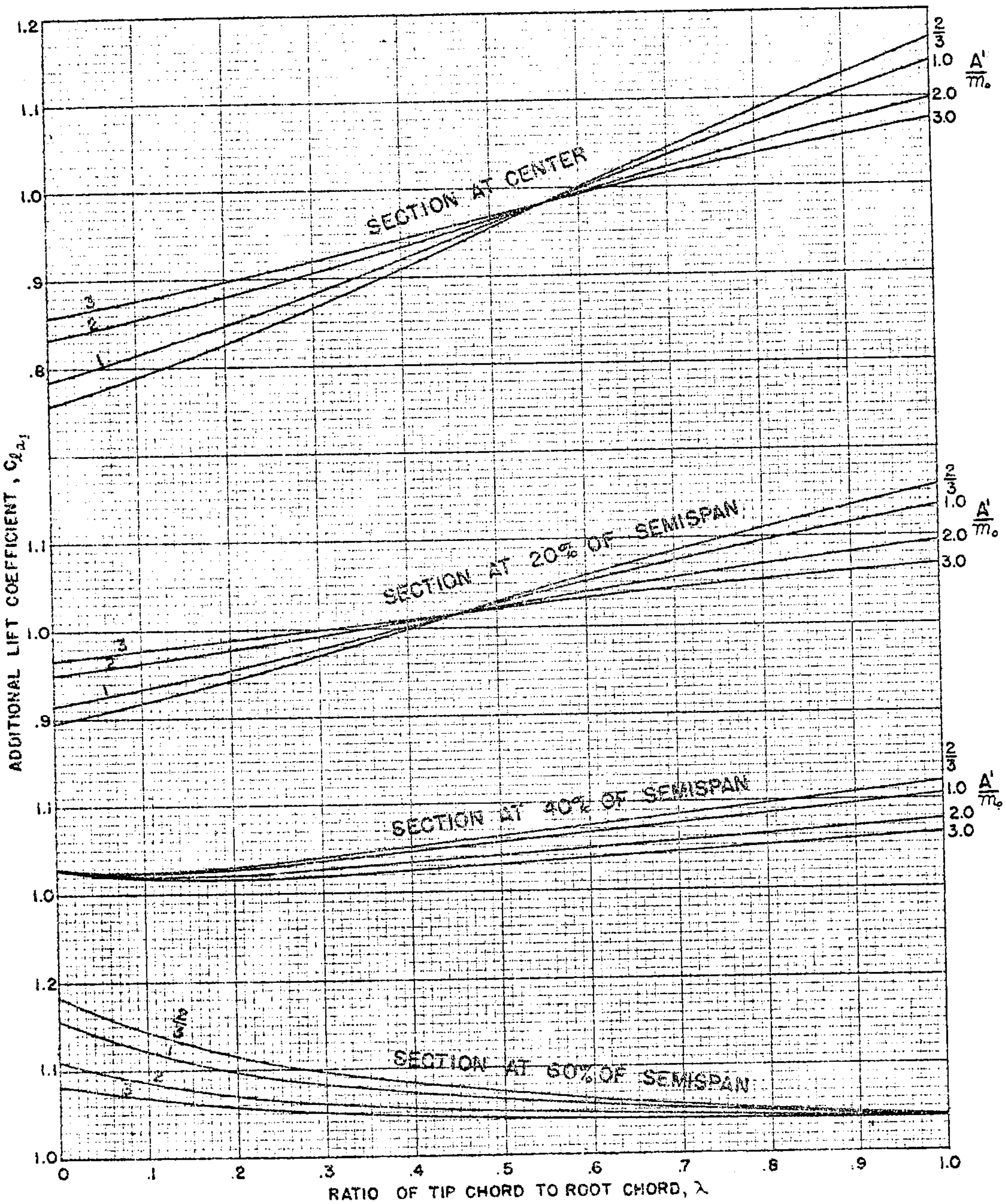
$$\Delta C_L = \frac{2}{S} \times F_1 \times F_2 \times F_3 \int_{y = \left(\frac{b}{2} - 0.4 \frac{S}{b}\right)}^{y = \frac{b}{2}} \Delta c_{l_1} \; c \; dy$$

$$= \Delta C_{L_1} \times F_1 = \Delta C_{L_1} \times C_L^2 \text{ approx.} \qquad (2.18)$$

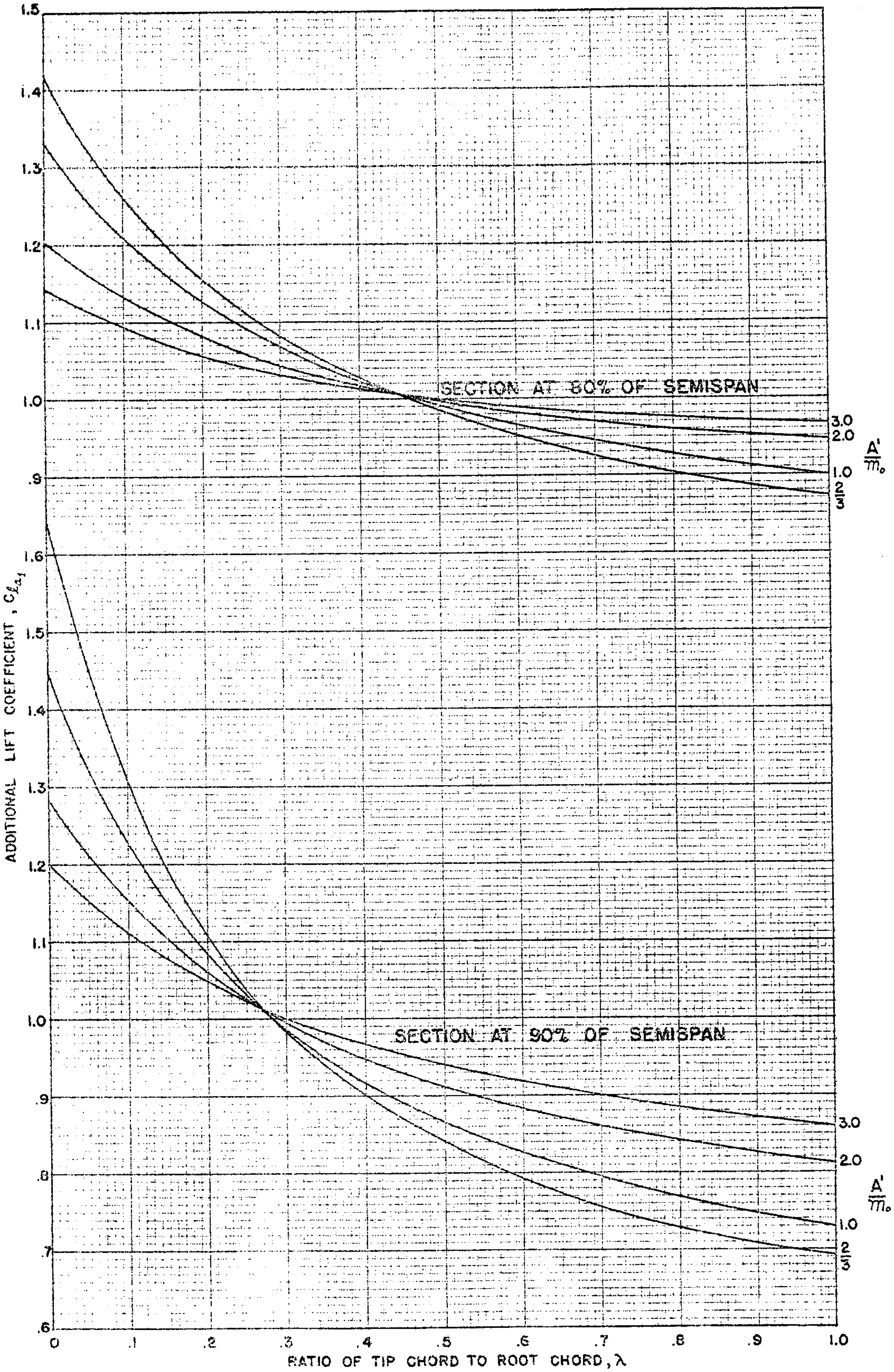
2.33 This empirical tip correction, when used, must be added to the theoretical distribution determined for a wing with square or trapezoidal tips, regardless of the actual shape of the tip. Since the curves, from which the distribution may be read directly, were calculated using square tips, the tip correction may be used in connection with them. When the taper ratio λ does not exceed about 0.5, the correction is so small as to be negligible in most cases—as can be seen by looking at Figure 2-10—and the theoretical values read from the curves may be used without correction. When the wing is such that the curves do not apply and the general method must be used instead, it is necessary to know—if the wing has rounded tips—before starting the calculations whether or not the empirical tip corrections are to be applied. If the wing has sufficient taper or is otherwise of such a nature that it is decided not to apply the empirical tip correction, the true chords of the wing should be used. On the other hand, if it is intended to apply the correction, the chords of an extended square tip should be used in the general calculations instead, because the use of the true chords would tend to give too large a tip load after adding the correction.

2.34 For the usual cantilever monoplane with well rounded tip, the effect of the empirical tip correction is never large, because the percentage increase in bending moment produced at the root of such a wing by the additional tip load is, in most cases, small, even at high angles of attack. Under these conditions it is permissible to make conservative approximations that may save a considerable amount of work, such as calculating the tip load for a high lift coefficient and then allowing it to vary directly with C_L, instead of as C_L^2, by adding it directly to the theoretical lift for the corresponding angle of attack. If the wing is sensitive to torsional loads, however, it may be necessary to investigate, because whatever tip correction there is has a center of pressure well toward the trailing edge, thus producing a diving moment on the wing in the positive lift range. For semi-cantilever or braced monoplanes and for biplanes, the tip correction is much more important. This is especially true in the case of a straight biplane wing panel with two beam construction. In such a wing the extra load on the tip not only has a far greater percentage effect on the bending moment at the strut than on the root moment of a monoplane, but the center of pressure is such that all of the extra load, or even more, may fall on the rear beam. The addition of the tip load has been known to double the bending moment in the rear beam at the strut and also double the load in the rear interplane strut over the load from the theoretical distribution alone. Because of the small amount of experimental data available for the derivation of these empirical factors, it is essential that extreme caution be used in designs where the tip loading is critical. In the case of biplanes, a conservative method would be to design the general wing structure for the theoretical loading for square tips and then design the beams in the overhang, the interplane struts, and other parts affected, taking into account the empirical tip loading and being particularly conservative because of the uncertainties involved.

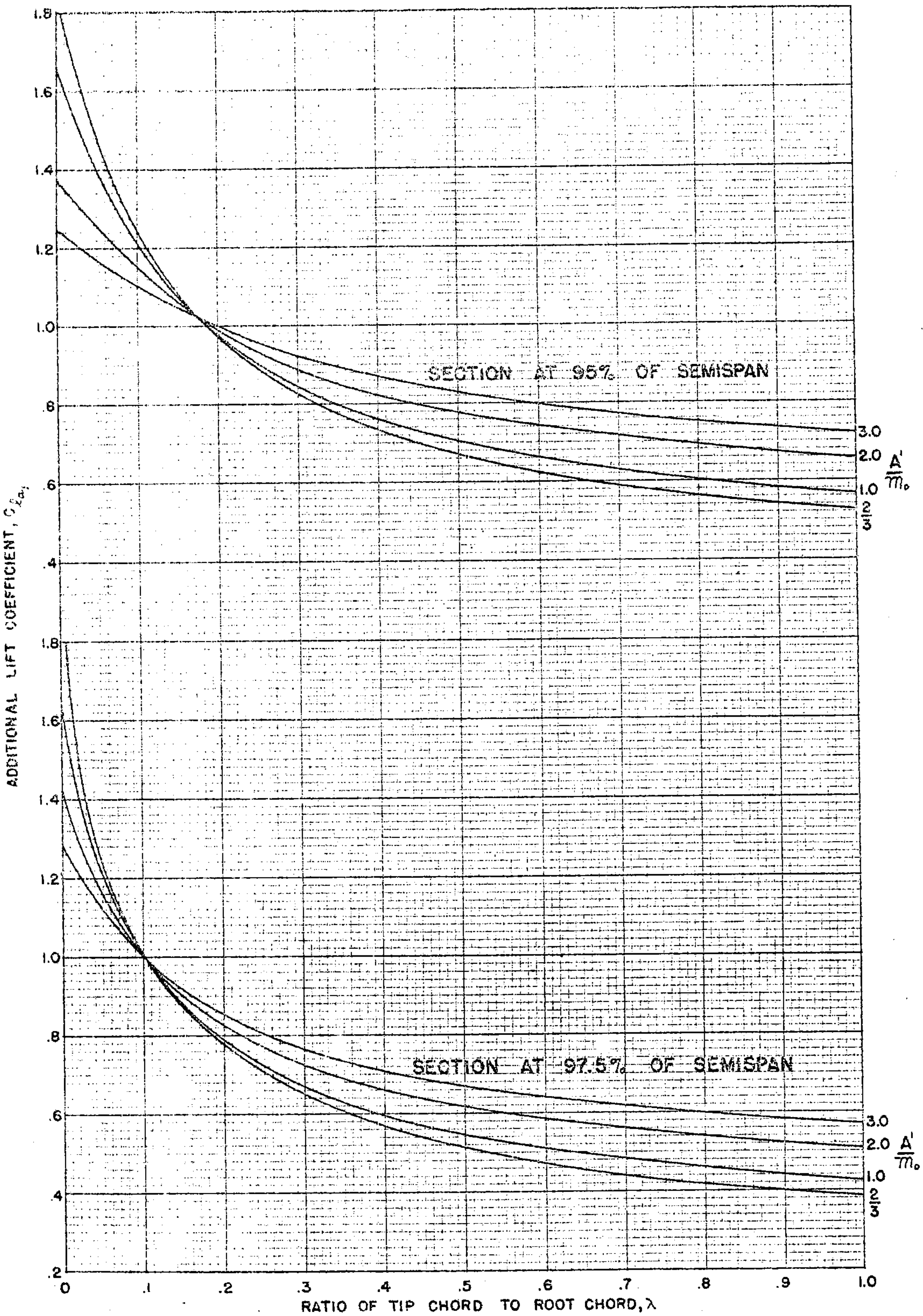
2.35 No mention has been made of the increase in drag caused by this empirical tip increment of lift. In most cases its effect would be small and it is believed conservative for the wing structure to neglect it. If desired, however, the tip increment may be assumed to act normal to the chord instead of to the line of flight, in which case there would be a component in the drag direction.



THEORETICAL SPAN DISTRIBUTION
OF
ADDITIONAL LIFT COEFFICIENT
FIG. 2-1



THEORETICAL SPAN DISTRIBUTION
OF
ADDITIONAL LIFT COEFFICIENT
FIG. 2-2



THEORETICAL SPAN DISTRIBUTION
OF
ADDITIONAL LIFT COEFFICIENT
FIG. 2-3

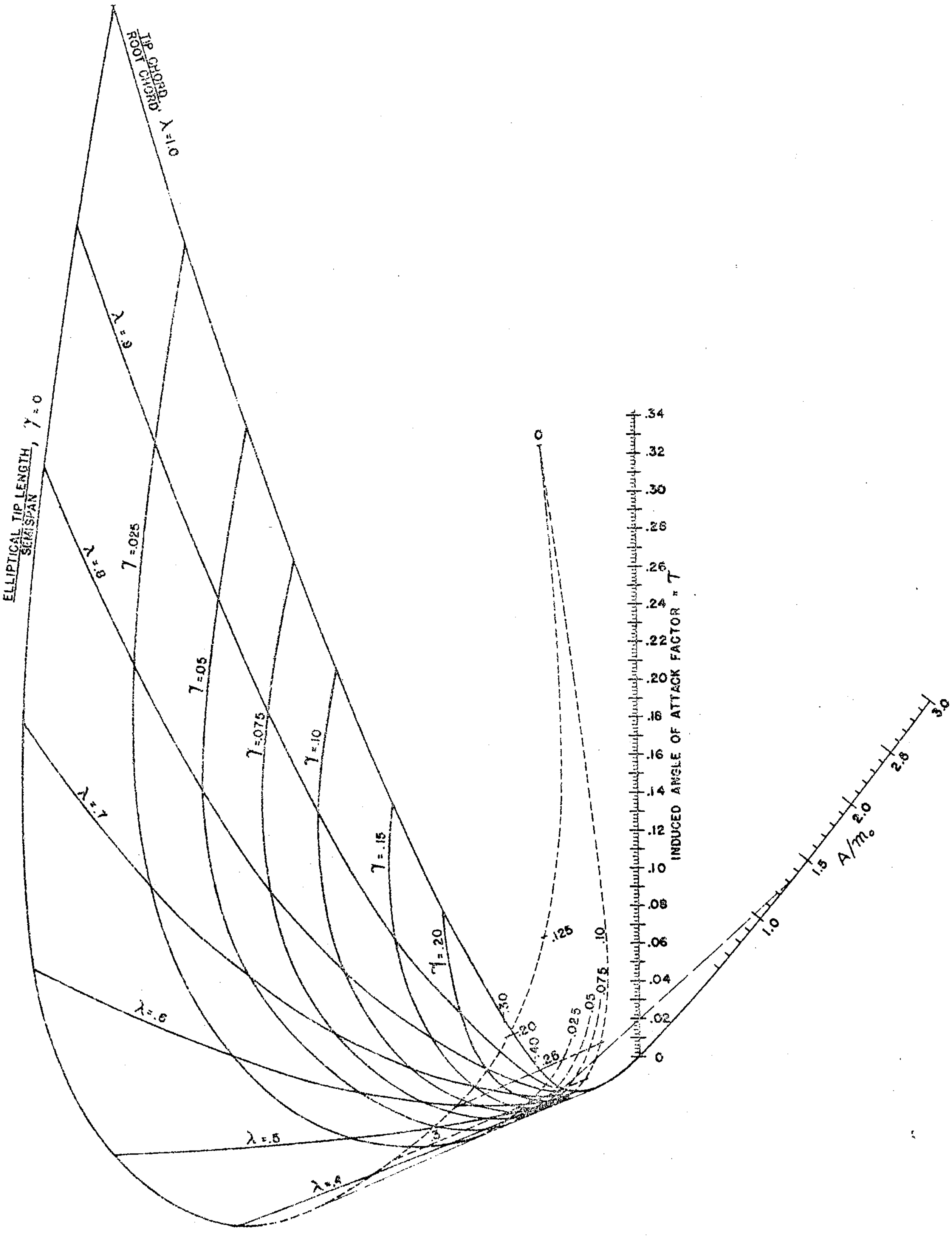


FIG. 2-4 FACTOR FOR INDUCED ANGLE OF ATTACK, T

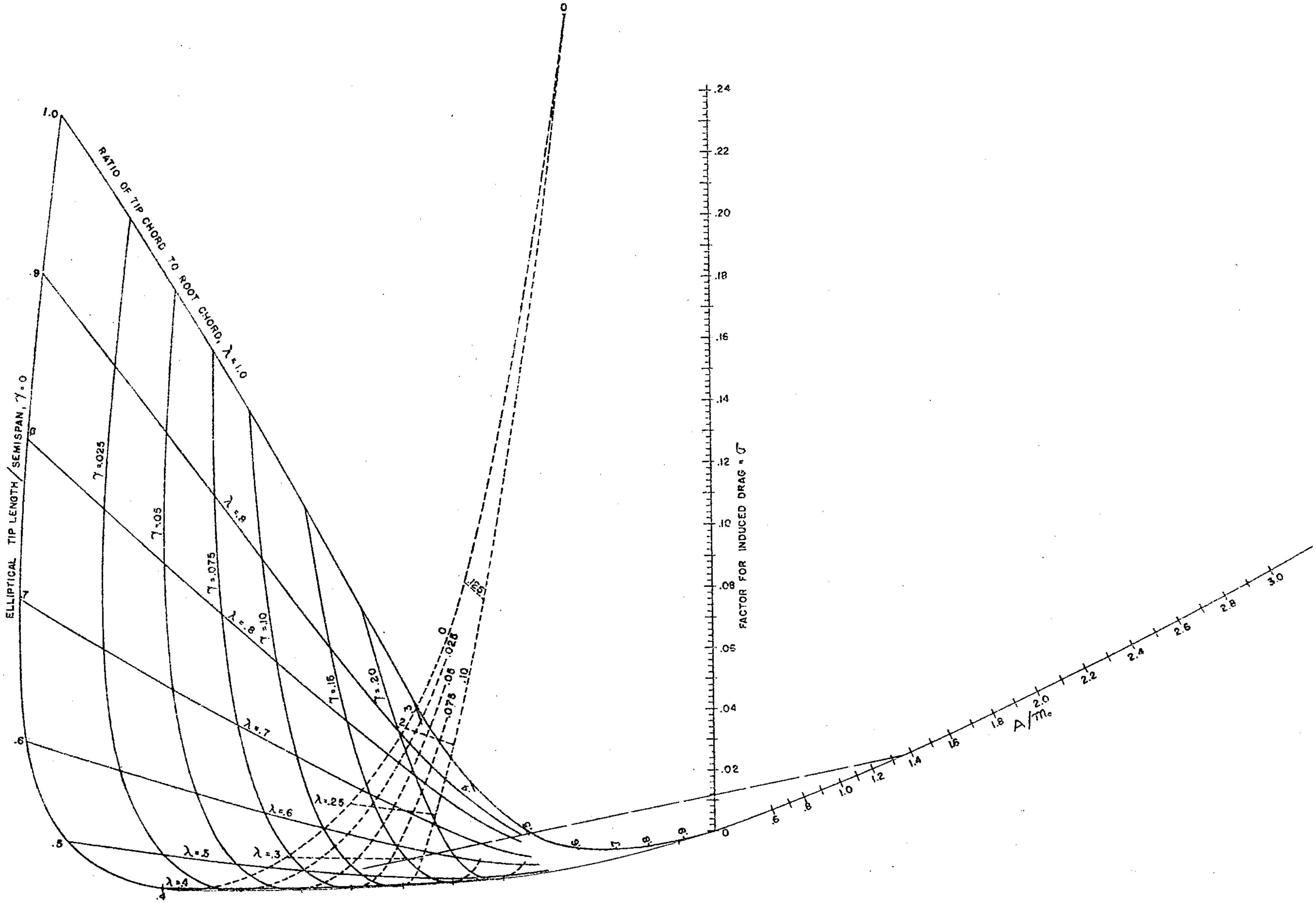


FIG. 2-5 FACTOR FOR INDUCED DRAG, σ

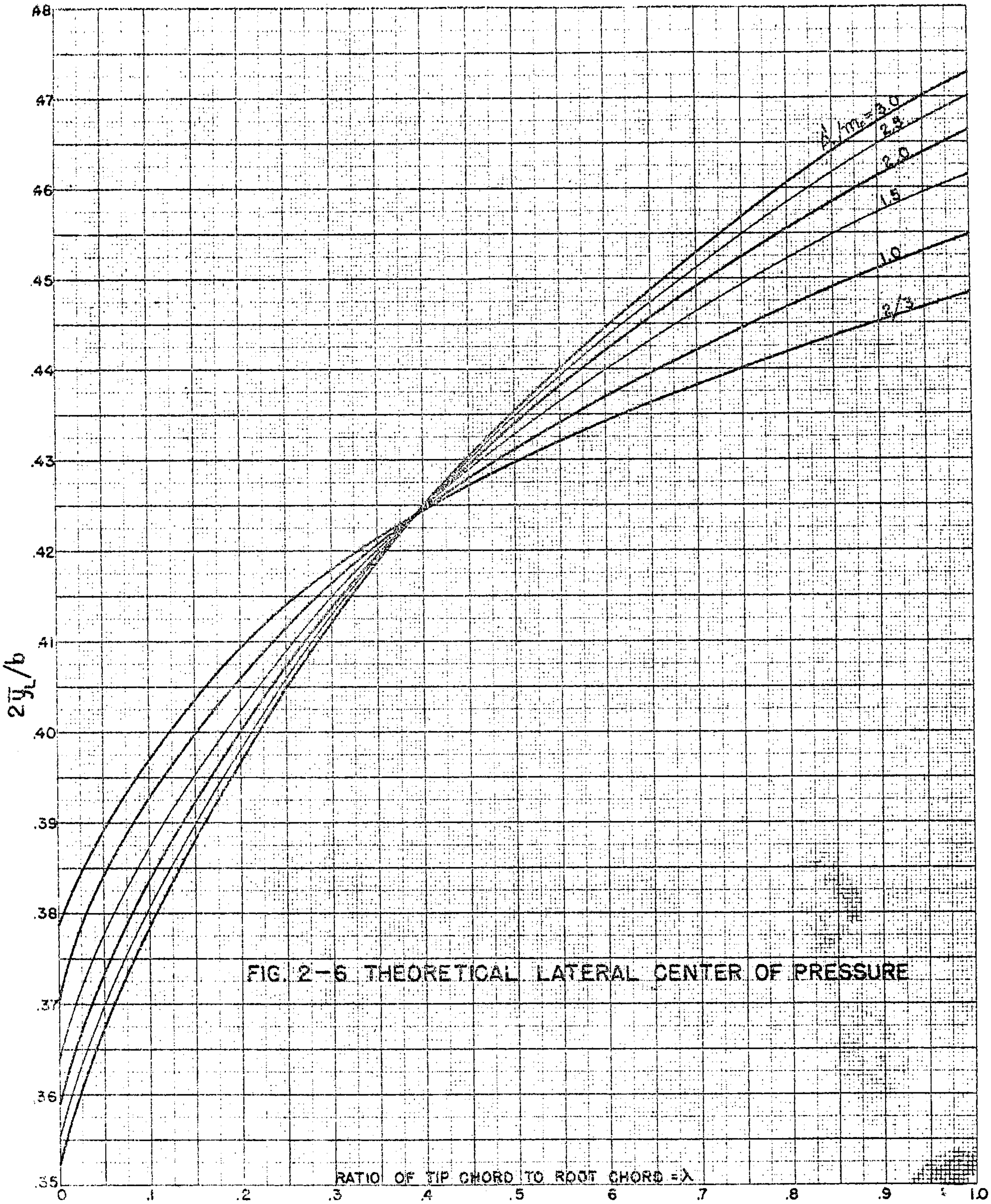


FIG. 2-6 THEORETICAL LATERAL CENTER OF PRESSURE

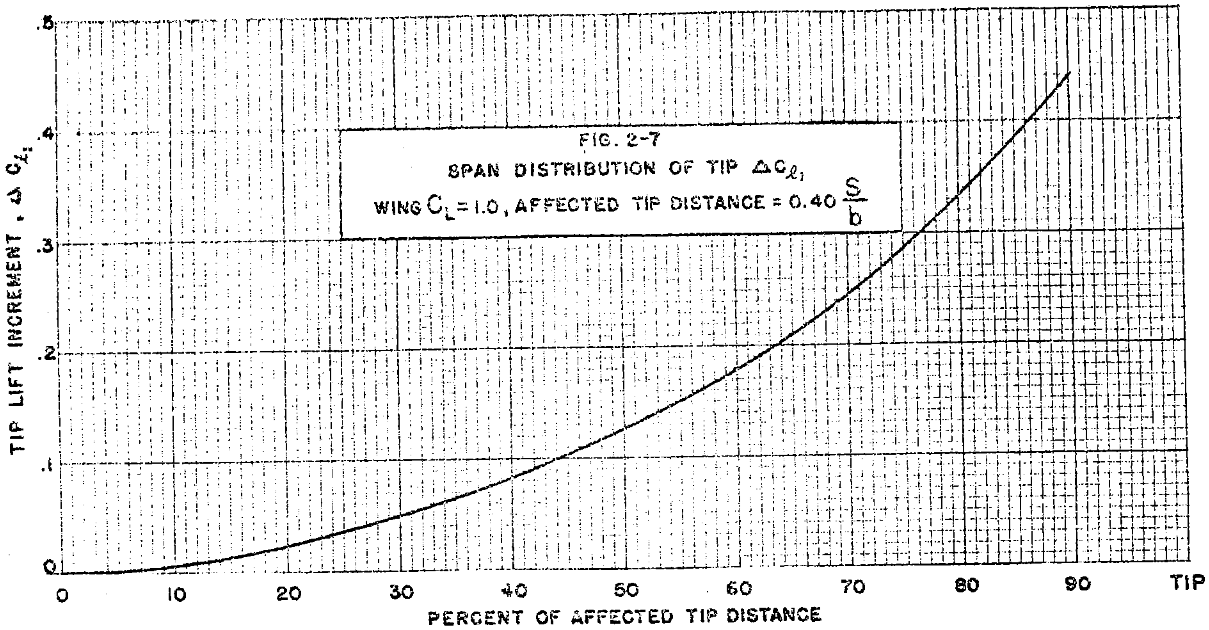


FIG. 2-7
SPAN DISTRIBUTION OF TIP Δc_{ℓ_1}
WING $C_L = 1.0$, AFFECTED TIP DISTANCE = $0.40 \frac{S}{b}$
TIP LIFT INCREMENT, Δc_{ℓ_1}
0
.1
.2
.3
.4
.5
0
10
20
30
40
50
60
70
80
90
TIP
PERCENT OF AFFECTED TIP DISTANCE

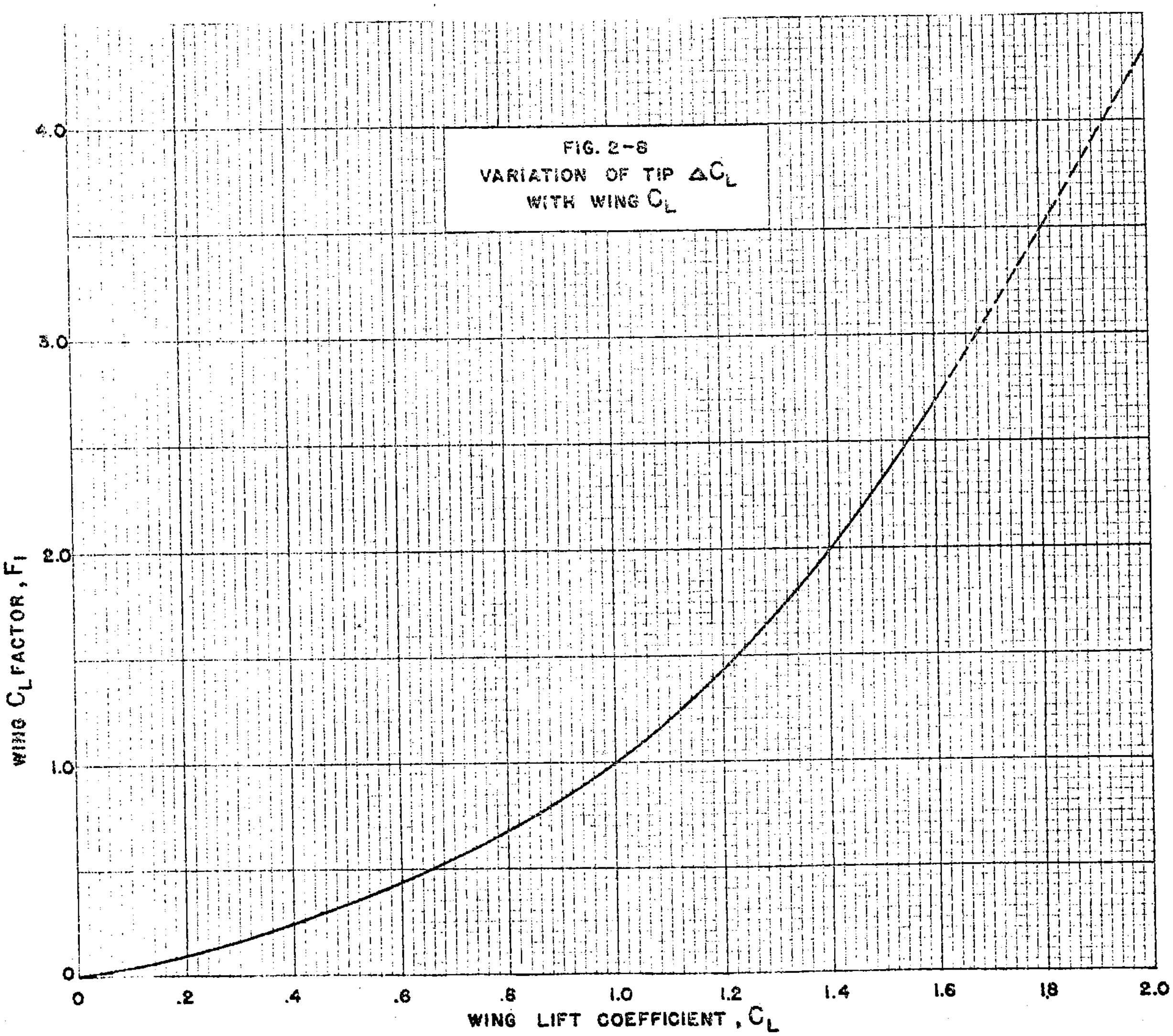


FIG. 2-8
VARIATION OF TIP ΔC_L
WITH WING C_L
WING C_L FACTOR, F_1
0
1.0
2.0
3.0
4.0
0
.2
.4
.6
.8
1.0
1.2
1.4
1.6
1.8
2.0
WING LIFT COEFFICIENT, C_L

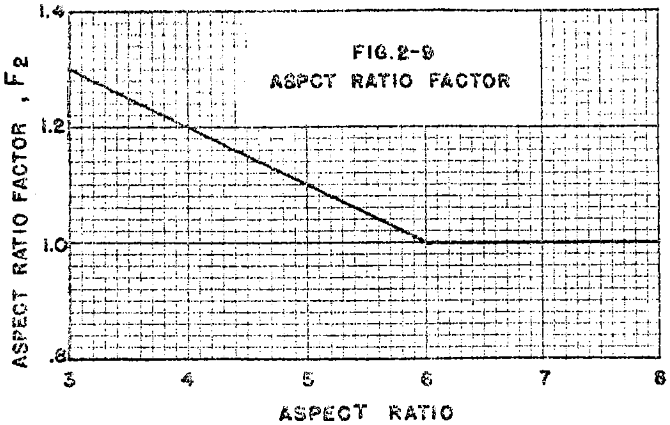


FIG. 2-9
ASPCT RATIO FACTOR
ASPECT RATIO FACTOR , F_2
1.4
1.2
1.0
.8
3
4
5
6
7
8
ASPECT RATIO

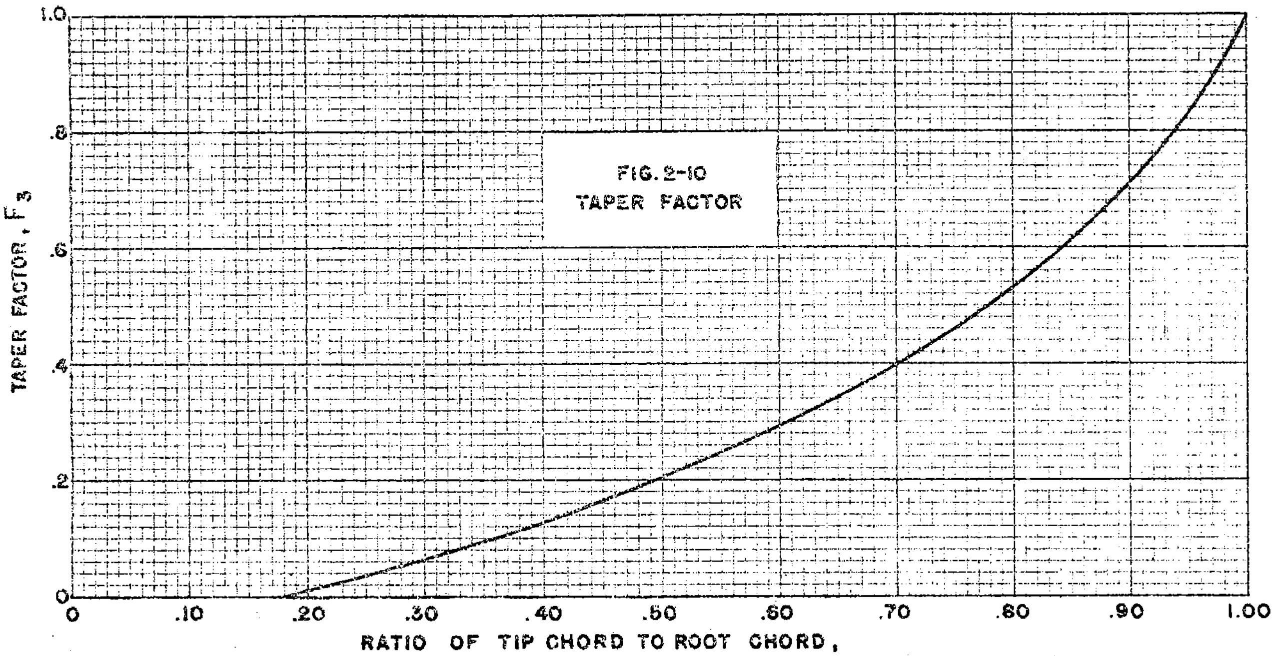


FIG. 2-10
TAPER FACTOR
TAPER FACTOR , F_3
1.0
.8
.6
.4
.2
0
0
.10
.20
.30
.40
.50
.60
.70
.80
.90
1.00
RATIO OF TIP CHORD TO ROOT CHORD ,

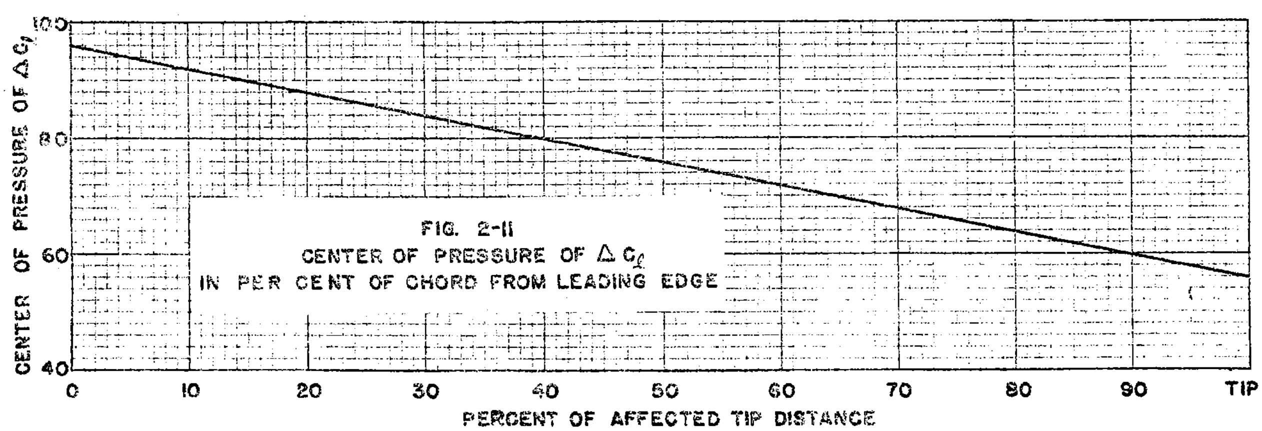


FIG. 2-11
CENTER OF PRESSURE OF Δc_l
IN PER CENT OF CHORD FROM LEADING EDGE
CENTER OF PRESSURE OF Δc_l
100
80
60
40
0
10
20
30
40
50
60
70
80
90
TIP
PERCENT OF AFFECTED TIP DISTANCE

CHAPTER 3

GENERAL PROCEDURE FOR CALCULATING LIFT AND INDUCED-DRAG DISTRIBUTIONS FOR ANY SYMMETRICAL WING AT A GIVEN ANGLE OF ATTACK

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GENERAL PROCEDURE FOR CALCULATING LIFT AND INDUCED-DRAG DISTRIBUTIONS FOR ANY SYMMETRICAL WING AT A GIVEN ANGLE OF ATTACK

3.0 Scope - When the wing is such that the preceding charts (Chapter 2) are not applicable, it is necessary to use the general method. The procedure which follows is for the purpose of calculating the theoretical span distribution of lift and induced-drag coefficients for any wing symmetrical about the center line for a given angle of attack. Chapter 4 will explain how to use this information to obtain the distributions for any angle of attack, and Chapter 5 will give the procedure for obtaining the basic distribution caused by an antisymmetrical distribution of angle of attack, such as that produced by displaced ailerons. The following method is illustrated by examples 2 and 3 in Chapter 7.

3.1 Table I - Fill in Table I in accordance with the instructions of columns a and b. Extreme care should be used with this table, since there is no check on the work done here. In column a the numbers in circles represent line numbers which are made to designate the value in a particular column of a designated line by the use of a sub-number.

(1) Line 1 of this table gives the designa-

tions of the points along the span for which the coefficients are to be calculated.

(2) Line 2 gives the cosines of the equally spaced angles listed in line 3 which determine the location of the points.

(3) Line 3 gives the angles, $\theta = \cos^{-1} 2y/b$

(4) Line 4 gives sin θ.

(5) Line 5 gives the location of the points in inches from the centerline or plane of symmetry.

(6) Line 6, the absolute angle of attack in radians will be the same for all points if the wing has no aerodynamic twist. For such wings it is convenient to use an angle of 1.0. If the wing has a flap of constant chord ratio extending over part of the span; so that the angle of attack has one constant value in this portion, which differs from the angle of attack of the remainder of the span, it is usually convenient to use an angle of 1.0 in the portion covered by the flap and zero for the remainder. In any case, this line should not be filled in without first reading Chapter 4 on the procedure for determining the distribution at any angle of attack, especially Section 4.25, if the wing has a part-span flap.

(7) Line 7 is obtained by multiplying these values of angle of attack in line 6 by the values

of sin θ in the respective columns of line 4.

(8) Line 8 gives the maximum thickness of each section in inches.

(9) Line 9 is the chord in inches at each point along the span. Note that when the tip correction is to be applied, the chords of the squared tip are to be used instead of those of the actual tip, if the tip is rounded.

(10) Line 10 is the thickness ratio for use in determining the fundamental airfoil characteristics of the sections along the span.

(11) Line 11 gives the slope of the lift curve per radian for infinite aspect ratio. This value is given per degree as a_o under "Fundamental Section Characteristics" in all recent N.A.C.A. publications of airfoil characteristics. These values should be faired, when taken from a table, before using for a tapered wing. Diehl, in reference 10, gives a formula by means of which the approximate section slope may be determined for any normal section, which may be used when the information is not otherwise available. Expressed in radians, it is

$$m_o = 2\pi - 4.0\,(t/c) \qquad (2.1)$$

where t/c is the thickness ratio of the section.

(12) Line 12 is the ratio of the slope of

the lift curve for the section at the plane of symmetry to the slope of the section at each point along the span.

(13) Line 13 is the ratio of the chord at the plane of symmetry to the chord at each point along the span.

(14) Line 14 is the product of line 12 times line 13 times line 4.

(15) Below the table are listed several parameters including total span, gross area, aspect ratio, average chord and taper ratio. It will be found convenient to list those items, which have dimensions, in both feet and inches. In cases where the tip correction is to be applied and the squared tip is being used in the calculations, all of these items involving S should be given, both for the actual wing and for the wing with squared tip.

3.2 Table II - Fill in Table II in accordance with the following, except when the wing has no twist, in which case this table is not necessary:

(1) At the top of the page in the space indicated, copy the values from Table I, line 7, except that one-half the value given in Table I, column 0, is to be written under $y_o/2$. The full values in the other columns are to be copied

consecutively so that the subscript numbers of the y's immediately above the spaces are the same as the respective column numbers in Table I.

(2) Compute r_1 and r_2 as indicated.

(3) Fill in all the spaces in the table which contain y's or r's as follows:

(a) For line 1, multiply .1564 by the values given at the top of the page for y_9, $-y_3$, $-y_7$ and y_1 and write the products in the respective spaces as indicated, being careful to use the proper signs.

(b) For line 2, multiply .3090 by the values given at the top of the page for y_8 and y_4 and fill in the four spaces.

(c) Fill in the next eight lines in the same way, using the correct multiplier given at the left of the table for each line, and using the values for r_1 and r_2 where indicated in the same way as the values for the y's are used.

(4) Total the columns and write the results in line 11.

(5) Obtain the sum of the values in line 11

for each of the five pairs of columns (a) and (b), and write the sums in line 12 in the respective columns (a).

(6) Subtract each column (b) value from its preceding column (a) value in line 11, and write the results in line 13 under the respective columns (b).

(7) Multiply the values in lines 12 and 13 by 0.4, or divide them by 2.5, and enter the results in line 14. These are double the values for the B_* coefficients as indicated in line 15.

(8) Make the check indicated at the bottom of the page. An error is indicated if this fails to check exactly, but a perfect check does not necessarily mean that no error has been made, because an incorrect value may occur twice with different sign so as to cancel out when the sum is obtained. When the wing has no twist, all B_* coefficients except B_1 are zero and it is not necessary to fill in Table II.

3.3 Table III - Fill in Table III as follows:

(1) Fill in the spaces containing y's in lines 1 and 2, by copying the values from the respective columns of Table I, line 14; that is, y_1 is taken from column 1 of Table I, y_2 from column 2, and so on, while $y_0/2$ is one-

half the value given in column 0.

(2) Add lines 1 and 2, and write the sums in line 3 in the spaces containing v's.

(3) Subtract line 2 from line 1, and write the results in line 4 in the spaces containing w's.

(4) Copy the v's obtained in line 3 into the last three columns of lines 1 and 2 as designated.

(5) Add the last three columns of lines 1 and 2, and write the sums in line 3 in the spaces containing p's.

(6) Subtract the last three columns of line 2 from line 1, and write the results in line 4 in the spaces containing q's.

(7) Line 5 contains column headings corresponding to the C_{2r} coefficients to be obtained in the respective columns below, the first column containing multipliers by which all values are multiplied before entering in the columns to the right.

(8) Next fill in all the spaces to the right of the column of multipliers which contain w's, p's and q's as follows:

(a) For line 6, multiply .3090 by the values given in lines 3 and 4 for w_4, q_2, p_2, $-w_2$, p_1 and $-q_1$, and write the products in the respective spaces as indicated, being careful to use the proper signs.

(b) Fill in lines 7, 8, 9 and 10 in the same way, using the correct multiplier given at the left for each line.

(c) It will be noted while filling in line 10 that the value for column 0 is the sum of all three p's, the value for column 20 is $(q_0 - q_1 + q_2)$, while the value for column 10 is $w_0 - w_2 + w_4$.

(9) Total the columns and write the results in line 11.

(10) Obtain the sum of the quantities in line 11 for each of the two pairs of columns (a) and (b), and write the sums in line 12 in the respective columns (a).

(11) Subtract each column (b) value from its preceding column (a) value in line 11, and write the results in line 13 under the respective columns (b).

(12) Divide these values in lines 12 and 13, and all values in line 11 in those columns not headed by an a or b, by 5 (or multiply them by 0.2) and write the results in line 14. These are the values for the C_{2n} coefficients as indicated in line 15, except C_0 and C_{20} for which double their values are obtained.

(13) The sum of all C_{2n} coefficients from C_o to C_{20}, both inclusive, should be zero. Failure to exactly equal zero indicates an error but, as in the case of the B_n coefficients, a perfect check does not necessarily mean that no error has been made. It should be noted that these C_{2n} coefficients are dependent only on the shape of the wing and the distribution of m_o, and are independent of angle of attack. In a twisted wing where at least two solutions are required, it is necessary to determine the C_{2n} coefficients only once.

3.4 Table IV - In Table IV are listed and computed the constants necessary in solving for the A_n coefficients.

(1) The constant u_o equals Table I, line 11, column 0, times Table I, line 9, column 0, divided by 4 times the full span of the wing in inches.

(2) The $2P_n$ values are next computed, using this value of u_o and obtaining the values of $2C_o$, C_2, C_6, C_{10}, C_{14} and C_{18} from Table III, line 14.

(3) From the same source are taken the C_{2n} values for computing the items designated D to M inclusive.

3.5 Table V - The general equations are given below and may be solved simultaneously for the A_n values, after substituting into the equations the values of constants given in Table IV and the values of $2B_n$ from Table II.

$$2A_1 P_1 + A_3 D + A_5 H + A_7 K + A_9 M = 2B_1$$
$$A_1 D + 2A_3 P_3 + A_5 E + A_7 I + A_9 L = 2B_3$$
$$A_1 H + A_3 E + 2A_5 P_5 + A_7 F + A_9 J = 2B_5$$
$$A_1 K + A_3 I + A_5 F + 2A_7 P_7 + A_9 G = 2B_7$$
$$A_1 M + A_3 L + A_5 J + A_7 G + 2A_9 P_9 = 2B_9$$

The best way to solve these equations is by successive approximations. Since, for any normal wing, all the An coefficients are small with the exception of A_1, an approximation can be quickly made by assuming them zero. By substituting the approximate value of A_1 thus obtained in the other equations, approximate values of the other coefficients are obtained which, when substituted in the first equation, results in a closer approximation for A_1. A repetition results in closer approximations for all the coefficients. Table V is provided for a solution in this manner. This table is divided into 5 divisions of 7 lines, each division of which represents one of the five equations. Column 1 gives the operations required to obtain each coefficient when all the others are known. In column 2 should be listed the items D to M inclusive from Table IV, as called for in column 1. The values of $2B_n$ and $2P_n$ should be taken from Tables II and IV respectively, and entered in the spaces indicated. In column 3a are listed the values of the A_n coefficients as first approximated. Since none are known at the start, A_1 is determined as though the others were all zero and listed in column 4a, line 7, and column 3a, lines 8, 15, 22 and 29. A_3 is next approximated in the same way, except that the value first found for A_1 is used and the items listed in column 1 are evaluated in column 4a, where A_3 is listed in line 14 and in column 3a, lines 16, 23 and 30. A_5, A_7 and A_9 are next approximated in order in the same manner, accomplishing the necessary operations in column 4a. After all the A_n coefficients have been approximated in this way, they are written in column 3b, lines 1 to 4, and the whole process repeated, using the latest approximated value for each coefficient as it appears, and using columns 3b and 4b in the same manner that columns 3a and 4a were used in the first approximation. The values of A_n obtained in column 4b may be considered final, but the process may be repeated once more in columns 3c and 4c as a check. The most common error in solving these equations by this method is to misplace a decimal point and to carry the same error through each approximation.

3.6 <u>Table VI</u> - List in Table VI, column 1, the values of the A_n coefficients found from the solution of the equations. Calculate columns 2 and 3 as shown, and total the values in columns 2 and 3. Calculate the values of C_L and C_{Di} as shown. These represent the wing lift coefficient and wing induced-drag coefficient for the angle of attack for which the calculations were made. For checking purposes, the lateral center of pressure of lift $\bar{y}_L$ may be found as indicated.

3.7 Table VII - Fill in Table VII as follows:

(1) Copy the values of the A_n coefficients into column a as indicated.

(2) Fill in all columns of lines (8 - 12) by multiplying the A's by the proper sines, the values of which are listed above.

(3) Enter the totals of these products in line 13.

(4) Multiply each of the values in Table I, line 13, by the value in Table I, line 11, column 0, and enter the results in the respective columns of line 14.

(5) Line 15, found by multiplying the values in line 13 by the respective values in line 14, gives the theoretical values of the section lift coefficient at the ten points along the semispan for this particular angle of attack when C_L equals the value computed in Table VI.

(6) In line 16 is copied the values of the section slopes from Table I.

(7) The angles of attack are likewise entered in line 17 from Table I.

(8) The effective angle of attack of each section is computed in line 18 by dividing the c_l values in line 15 by the respective values of m_o in line 16.

(9) The induced angle is found in line 19 by subtracting the effective angle from the absolute angle of attack.

(10) Line 20 gives the theoretical values of the section induced-drag coefficient, the product of the induced angle times the lift coefficient, for this particular angle of attack.

(11) The chords of the sections are copied in line 21 from Table I.

(12) In line 22 the relative lift coefficients are found by dividing the c_l values of line 15 by the value of C_L found in Table VI. If the wing has no twist, so that the basic lift is zero at all sections, c_{l_1} equals $c_{l_{a1}}$ and represents the lift coefficients when $C_L = 1.0$.

(13) Line 23 represents the products of chord and section lift coefficient and, when plotted versus span, produces what is known as the span-loading curve. In cases where the wing has discontinuity in plan form as when cut-outs or a straight center section with tapered outer panels are present, it is better to fair the span-loading curve

and then compute the c_{l_1} values from this in the vicinity of the break. The c_l curve is not necessarily a continuous curve in such cases.

(14) Line 24 is computed for the purpose of a check as indicated below the table. It is simply a method of integrating the span-loading curve to obtain the value of C_L and comparing it with the value found in Table VI. The sum of all ten values in line 24 minus one-half the value in column 0 when multiplied by π and divided by 20 times the average chord should equal 1.0. This check may be accomplished graphically, if preferred.

(15) In line 25 the relative induced-drag coefficients are found by dividing the c_{d_i} values in line 20 by the value of C_{D_i} found in Table VI. If the wing has no twist, these represent the section induced-drag coefficients when $C_{D_i} = 1.0$.

(16) The values in line 26 may be used for fairing, as described above for the lift.

(17) Line 27 is used for the purpose of checking, in the same way that line 24 was used.

(18) In cases where squared tips are used in the above calculations in anticipation of applying the tip correction, it is necessary to modify slightly the relative values of c_{l_1} and

$c_{d_{i_1}}$ before applying them to the wing with actual chords as was explained in Chapter 2. This can be most readily accomplished by using the actual chords in line 21 and the actual wing area in the checks at the bottom of the table. Instead of these integrations being checks in this case, they give the respective values of the factors by which c_{l_1} and $c_{d_{i_1}}$ must be divided to correct for the rounded tip.

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TABLE I.

GEOMETRIC CHARACTERISTICS OF WING

1	a	b	0	1	2	3	4	5	6	7	8	9	10
2	FRACTION SEMI-SPAN	$\frac{2y}{b} = \cos\theta$	0	.1564	.3090	.4540	.5878	.7071	.8090	.8910	.9511	.9877	1.0000
3		θ	90	81	72	63	54	45	36	27	18	9	0
4		$\sin\theta$	1.0000	.9877	.9511	.8910	.8090	.7071	.5878	.4540	.3090	.1564	0
5	(2) × $\frac{b}{2}$	y											
6	ABSOLUTE IN RADIANS	α_a											
7	(6) × (4)	$\alpha_a \sin\theta$											
8	MAXIMUM THICKNESS	t											
9	CHORD	c											
10	(8)/(9)	$\frac{t}{c}$											
11	$dc_l/d\alpha_o$	m_o											
12	$(11)_o/(11)_n$	m_s/m_o											
13	$(9)_o/(9)_n$	c_s/c											
14	(12)×(13)×(4)	$\frac{m_s c_s}{m_o c}\sin\theta$											

b =

S =

$A = \frac{b^2}{S} =$

$C_{AVE.} = S/b =$

$\lambda =$

TABLE II.

COMPUTATION OF ANGLE COEFFICIENTS, B_n

TABLE I. LINE 7	$y_0/2$	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9

$y_1 - y_3 - y_5 + y_7 + y_9 = r_1$	$y_0/2 - y_4 + y_8 = r_2$

	MULTI-PLIER	1 – 19 a	1 – 19 b	3 – 17 a	3 – 17 b	5 – 15 a	5 – 15 b	7 – 13 a	7 – 13 b	9 – 11 a	9 – 11 b
1	.1564	y_9		$-y_3$				$-y_7$		y_1	
2	.3090		y_8		y_4				y_4		y_8
3	.4540	y_7		y_9				$-y_1$		$-y_3$	
4	.5878		y_6		$-y_2$				y_2		$-y_6$
5	.7071	y_5		y_5		r_1		$-y_5$		y_5	
6	.8090		y_4		y_8				y_8		y_4
7	.8910	y_3		$-y_1$				y_9		$-y_7$	
8	.9511		y_2		y_6				$-y_6$		$-y_2$
9	.9877	y_1		y_7				y_3		y_9	
10	1.0000		$y_0/2$		$-y_0/2$		r_2		$-y_0/2$		$y_0/2$
11	TOTAL										
12	$a + b$										
13	$a - b$										
14	(12) & (13) / 2.5										
15	$2B_n$	$2B_1$	$2B_{19}$	$2B_3$	$2B_{17}$	$2B_5$	$2B_{15}$	$2B_7$	$2B_{13}$	$2B_9$	$2B_{11}$

CHECK: $2B_1 - 2B_3 + 2B_5 - 2B_7 + 2B_9 - 2B_{11} + 2B_{13} - 2B_{15} + 2B_{17} - 2B_{19} = 2y_0 = 2\alpha_s$

NOTE: IF α_a IS CONSTANT ALONG THE SPAN, $B_1 = \alpha_s$ & B_3 TO B_{19} ARE ZERO.

TABLE III

COMPUTATION OF PLAN FORM COEFFICIENTS, C_{2n}

1	TABLE I			y_9		y_8		y_7		y_6		y_5		V_0		V_1		V_2	
2	LINE 14	$\frac{y_0}{2}$		y_1		y_2		y_3		y_4				V_5		V_4		V_3	
3	①+②	v_0		v_1		v_2		v_3		v_4		v_5		p_0		p_1		p_2	
4	①−②	w_0		w_1		w_2		w_3		w_4				q_0		q_1		q_2	

5	MULTI-PLIER	0	20	2 — a	18 — b	4	16	6 — a	14 — b	8	12	10
6	.3090			w_4		q_2	p_2	$-w_2$		p_1	$-q_1$	
7	.5878				w_3				w_1			
8	.8090			w_2		q_1	$-p_1$	$-w_4$		$-p_2$	$-q_2$	
9	.9511				w_1				$-w_3$			
10	1.0000	$p_0+p_1+p_2$	$q_0-q_1+q_2$	w_0		q_0	p_0	w_0		p_0	q_0	$w_0-w_2+w_4$
11	TOTAL											
12	a + b											
13	a − b											
14	DIVIDE BY 5											
15	C_{2n}	$2C_0$	$2C_{20}$	C_2	C_{18}	C_4	C_{16}	C_6	C_{14}	C_8	C_{12}	C_{10}

CHECK: $\frac{2C_0 + 2C_{20}}{2} + C_2 + C_4 + C_6 + C_8 + C_{10} + C_{12} + C_{14} + C_{16} + C_{18} = 0$

TABLE IV

CONSTANTS REQUIRED IN SOLUTION OF A_n COEFFICIENTS

ITEM		
U_0	$\frac{m_s C_s}{4b}$	
$2P_1$	$2C_0 - C_2 + 2U_0$	
$2P_3$	$2C_0 - C_6 + 6U_0$	
$2P_5$	$2C_0 - C_{10} + 10U_0$	
$2P_7$	$2C_0 - C_{14} + 14U_0$	
$2P_9$	$2C_0 - C_{18} + 18U_0$	
D	$C_2 - C_4$	
E	$C_2 - C_8$	
F	$C_2 - C_{12}$	
G	$C_2 - C_{16}$	
H	$C_4 - C_6$	
I	$C_4 - C_{10}$	
J	$C_4 - C_{14}$	
K	$C_6 - C_8$	
L	$C_6 - C_{12}$	
M	$C_8 - C_{10}$	

TABLE V.
SOLUTION OF A_n COEFFICIENTS

	1	2	3a	3b	3c	4a	4b	4c
1	$(C_2 - C_4)A_3$							
2	$(C_4 - C_6)A_5$							
3	$(C_6 - C_8)A_7$							
4	$(C_8 - C_{10})A_9$							
5	1 + 2 + 3 + 4							
6	$2B_1$ - 5	$2B_1$ = ________						
7	A_1 = 6 / $2P_1$	$2P_1$ = ________						
8	$(C_2 - C_4)A_1$							
9	$(C_2 - C_8)A_5$							
10	$(C_4 - C_{10})A_7$							
11	$(C_6 - C_{12})A_9$							
12	8 + 9 + 10 + 11							
13	$2B_3$ - 12	$2B_3$ = ________						
14	A_3 = 13 / $2P_3$	$2P_3$ = ________						
15	$(C_4 - C_6)A_1$							
16	$(C_2 - C_6)A_3$							
17	$(C_2 - C_{12})A_7$							
18	$(C_4 - C_{14})A_9$							
19	15 + 16 + 17 + 18							
20	$2B_5$ - 19	$2B_5$ = ________						
21	A_5 = 20 / $2P_5$	$2P_5$ = ________						
22	$(C_6 - C_8)A_1$							
23	$(C_4 - C_{10})A_3$							
24	$(C_2 - C_{12})A_5$							
25	$(C_2 - C_{16})A_9$							
26	22 + 23 + 24 + 25							
27	$2B_7$ - 26	$2B_7$ = ________						
28	A_7 = 27 / $2P_7$	$2P_7$ = ________						
29	$(C_8 - C_{10})A_1$							
30	$(C_6 - C_{12})A_3$							
31	$(C_4 - C_{14})A_5$							
32	$(C_2 - C_{16})A_7$							
33	29 + 30 + 31 + 32							
34	$2B_9$ - 33	$2B_9$ = ________						
35	A_9 = 34 / $2P_9$	$2P_9$ = ________						

TABLE VI

COMPUTATION OF WING LIFT AND DRAG COEFFICIENTS

1	2	3
A_1 =	$(A_1)^2$ =	$A_1/3$ =
A_3 =	$3(A_3)^2$ =	$A_3/5$ =
A_5 =	$5(A_5)^2$ =	$-A_5/21$ =
A_7 =	$7(A_7)^2$ =	$A_7/45$ =
A_9 =	$9(A_9)^2$ =	$-A_9/77$ =
	$\sum nA_n^2$ =	$\sum$ =

$$1 + \sigma = \frac{\sum n A_n^2}{A_1^2} =$$

$$C_L = \pi A u_0 A_1 =$$

OR

$$\frac{\pi}{4} \frac{m_s c_s}{C_{AVE.}} A_1 =$$

$$C_{D_i} = \left(\frac{1+\sigma}{\pi A}\right) C_L^2 = C_{D_{i_1}} \times C_L^2 = \underline{\qquad\qquad}$$

FOR WING WITH NO AERODYNAMIC TWIST, $C_{D_{i_1}} = C_{D_{i_{a_1}}}$

$$\bar{y}_L = \left(\frac{4}{\pi} \times \frac{\Sigma \text{COL. } 3}{A_1}\right) \frac{b}{2} =$$

TABLE VII

COMPUTATION OF LOAD DISTRIBUTIONS

C_{D_i} =

C_L =

1	a	b	0	1	2	3	4	5	6	7	8	9
2	TABLE I LINE 3	θ	90	81	72	63	54	45	36	27	18	9
3	TABLE I LINE 4	SIN θ	1.0000	.9877	.9511	.8910	.8090	.7071	.5878	.4540	.3090	.1564
4		SIN 3θ	-1.0000	-.8910	-.5878	-.1564	.3090	.7071	.9511	.9877	.8090	.4540
5		SIN 5θ	1.0000	.7071	0	-.7071	-1.0000	-.7071	0	.7071	1.0000	.7071
6		SIN 7θ	-1.0000	-.4540	.5878	.9877	.3090	-.7071	-.9511	-.1564	.8090	.8910
7		SIN 9θ	1.0000	.1564	-.9511	-.4540	.8090	.7071	-.5878	-.8910	.3090	.9877
8	A_1=	A_1 SIN θ										
9	A_3=	A_3 SIN 3θ										
10	A_5=	A_5 SIN 5θ										
11	A_7=	A_7 SIN 7θ										
12	A_9=	A_9 SIN 9θ										
13	(8)+(9)+(10)+(11)+(12)	$\sum A_n \sin n\theta$										
14	TABLE I [(11)$_0$ x (13)]	$\frac{m_s c_s}{c}$										
15	(13) x (14)	c_ℓ										
16	TABLE I LINE 11	m_0										
17	TABLE I LINE 6	α_a										
18	(15) / (16)	α_o										
19	(17) - (18)	$\frac{w}{V}$										
20	(19) x (15)	c_{d_i}										
21	TABLE I LINE 9	c										
22	(15) / C_L	c_{ℓ_1}										
23	(22) x (21)	$c\,c_{\ell_1}$										
24	(23) x (3)	cc_{ℓ_1} SIN θ										
25	(20) / C_{D_i}	$c_{d_{i_1}}$										
26	(25) x (21)	$cc_{d_{i_1}}$										
27	(26) x (3)	$cc_{d_{i_1}}$ SIN θ										

CHECKS:

LINE 24: $\frac{\pi}{20}\frac{b}{S}\sum cc_{\ell_1}\sin\theta = \frac{\pi}{20C_{AVE}}\left(\sum (24) - \frac{1}{2}(24)_0\right) = 1.0$

LINE 27: $\frac{\pi}{20}\frac{b}{S}\sum cc_{d_{i_1}}\sin\theta = \frac{\pi}{20C_{AVE}}\left(\sum (27) - \frac{1}{2}(27)_0\right) = 1.0$

CHAPTER 4

PROCEDURE FOR DETERMINING

THE LIFT AND INDUCED-DRAG DISTRIBUTION

FOR ANY ANGLE OF ATTACK.

PROCEDURE FOR DETERMINING THE LIFT AND INDUCED-DRAG DISTRIBUTIONS FOR ANY ANGLE OF ATTACK

4.0 General

4.00 Introduction - A method has been outlined for calculating the lift and induced-drag distributions at a given angle of attack for any wing symmetrical, both in plan form and in angle of attack, about its centerline or X axis. The same method can be used, of course, to determine the distributions at any other angle of attack, but this is not usually necessary as will now be explained.

4.01 Lift - The theoretical lift distribution on any wing can be divided into two parts called the basic-lift distribution and the additional-lift distribution. The basic is sometimes referred to as the zero-lift distribution because it represents the section lift coefficients along the span when the wing lift coefficient is zero. It depends only on the characteristics of the wing and is independent of the wing angle of attack, but is mostly affected by the distribution of angle of attack along the span. When the absolute angle of attack is constant along the span, which means that all zero-lift chords lie in the same plane or that the wing has no effective twist, the section basic-lift coefficients are all zero. The additional-lift coefficient distribution also depends on the characteristics of the wing including the chords and section lift-curve slopes, but is independent of the twist or angle-of-attack distribution. Its shape is also independent of the wing angle of attack, but the section lift coefficient at any section is directly proportional to the wing angle of attack or to the wing lift coefficient. This can best be expressed in the form of an equation where the lift coefficient at any section for a wing lift coefficient of C_L is

$$c_l = c_{l_b} + C_L \; c_{l_{a_1}} \qquad (4.1)$$

where c_{l_b} is the lift coefficient which that particular section has when the wing lift C_L is zero and $c_{l_{a_1}}$ is the additional lift coefficient which the section has when the wing lift coefficient C_L is 1.0.

4.02 Induced Drag - In a similar manner the distribution of induced-drag coefficient may be divided into partial distributions except that it is necessary to use three instead of two parts, as in the case of the lift. One part is constant or independent of angle of attack and represents the induced drag when the wing lift is zero; another part is directly proportional to the wing lift coefficient, while the third part is proportional to the square of the wing lift coefficient. When the wing has no effective twist, the first two distributions disappear and only the third part which varies as C_L^2 is present. This can also be expressed in equation form, where for a wing lift coefficient of C_L the induced-drag coefficient at any section is

$$c_{d_i} = c_{d_{i_b}} + C_L \ c_{d_{i_{a_1 b}}} + C_L^2 \ c_{d_{i_{a_1}}} \qquad (4.2)$$

where $c_{d_{i_b}}$ represents the induced-drag coefficient for this section when $C_L = 0$, and $c_{d_{i_{a_1}}}$ represents the induced-drag coefficient at this section if the wing had no twist when $C_L = 1.0$, while $c_{d_{i_{a_1 b}}}$ represents the part of the induced-drag coefficient that varies directly with C_L when $C_L = 1.0$, and is caused by the interaction of the basic and additional lifts. The latter coefficient may be calculated by means of the formula

$$c_{d_{i_{a_1 b}}} = \frac{c_{d_{i_{a_1}}} \times c_{l_b}}{c_{l_{a_1}}} + \frac{c_{d_{i_b}} \times c_{l_{a_1}}}{c_{l_b}} \qquad (4.3)$$

4.03 Tip Correction - As explained in Chapter 2, certain wings require an empirical tip correction to be applied to the theoretical lift distribution to make it agree with experiment. This correction is calculated and applied in the same way whether the theoretical distribution is obtained by the general method or by the use of the curves and is fully described in Section 2.3. It must be remembered, however, that when using the general procedure, the correction must be applied to the theoretical distribution based on a wing with square tips.

4.1 Wings with no Effective Twist.

4.11 Lift - When the wing has no effective twist, all basic distributions are zero and drop out of the above expressions. The c_{l_1} in line 22 of Table VII then becomes $c_{l_{a_1}}$ and the value of c_l for any of the ten points along the semispan can be found for any wing C_L by multiplying the value in line 22 by the desired C_L.

4.12 <u>Induced Drag</u> - The $c_{d_{i_1}}$ in line 25 of Table VII represents the section values of the induced-drag coefficient when C_{D_i} for the wing is 1.0. To obtain the distribution of c_{d_i} or $c_{d_{i_a}}$ for any given value of C_L, it is first necessary to calculate the value of C_{D_i} by means of the formula from Table VI

$$C_{D_i} = \frac{C_L^2}{\pi A} (1 + \sigma) = C_{D_{ia_1}} \; C_L^2 \qquad (4.4)$$

and then to multiply the values in line 25 by this C_{D_i}. An alternative and perhaps preferable method is to first obtain the distribution of $c_{d_{ia_1}}$ by dividing the values of c_{d_i} in line 20 of Table VII by the square of C_L in Table VI. It is then necessary, in order to obtain the c_{d_i} distribution for any C_L, to simply multiply these values of $c_{d_{ia_1}}$ by the desired value of C_L^2.

4.13 <u>Angle of Attack</u> - Before resolutions can be made into the planes of the wing structure, it is necessary to know the angle of attack at which the chosen design value of wing lift coefficient occurs. For the wing without twist, the slope of the lift curve for the wing is simply

$$m = \frac{C_L}{\alpha_a} \text{ per radian with } \alpha_a \text{ in radians} \qquad (2.7)$$

or

$$a = \frac{m}{57.3} \quad \text{per degree}$$

where C_L is the value of wing lift coefficient calculated in Table VI and α_a is the angle of attack used in line 6 of Table I (usually 1.0 radian). Since the usual expression for the slope of the lift curve for a finite wing in terms of the section slope is

$$m = \frac{m_o}{1 + \frac{m_o}{\pi A}(1 + \tau)} \qquad (2.3)$$

it is well at this time to calculate the value of the induced-angle factor τ. Since the value of m_o may vary along the span, it is only possible to do this in terms of the value at one particular section, and if the section at the plane of symmetry is chosen, the above formula becomes

$$m = \frac{m_s}{1 + \frac{m_s}{\pi A}(1 + \tau_s)} \qquad (4.5)$$

and

$$(1 + \tau_s) = (m_s - m) \frac{\pi A}{m\, m_s} \qquad (4.6)$$

4.14 <u>Example</u> - The general procedure for this type of wing is illustrated by example 2 in Chapter 7.

4.2 Wings with Effective Twist.

4.20 Types of Twist - If the zero-lift chords of a wing are not all parallel, the wing is said to have effective twist. This twist may consist of any one or of any combination of the following:

(1) Symmetrical

(a) A built-in twist, warp or washout in the wing which results in the zero-lift chords not being parallel.

(b) A flap or other device extending symmetrically along only part of the span, the displacement or operation of which causes the zero-lift chords of that portion of the wing to be angularly displaced from their original position and from those of the remainder of the wing.

(c) Ailerons or other lateral control devices which are drooped symmetrically to obtain increased lift and which in so doing cause the zero-lift chords of the wing to be angularly displaced.

(d) Twist caused by the elastic deformation of the structure under load.

(e) Ailerons which are differentially displaced may be considered to be given a symmetrical displacement plus an antisymmetrical displacement equal and opposite on the two sides. The symmetrical part may be treated similarly to item (c) above except that its action is intermittent.

(2) Antisymmetrical:

(a) Twist caused by equal and opposite aileron displacement or the antisymmetrical portion of the displacement of differential ailerons.

(b) Twist caused by roll or by an angular velocity of the airplane about an axis parallel to the direction of flight which causes the angle of attack to vary linearly between the center and the tip.

(c) Gusts which impart a rolling acceleration are unsymmetrical and produce effective twist. Regardless of the spanwise distribution of such a gust, it can be divided into two separate distributions, one symmetrical and one antisymmetrical. The symmetrical portion may be further divided into a

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twisted and an untwisted portion, either or both of which may be zero. If the gust were assumed to have a uniform gradient from tip to tip of the wing, the antisymmetrical distribution of lift and induced drag would be similar to that caused by a rolling velocity and there would be no effective symmetrical twist.

4.21 Procedure for Wing with a Single Symmetrical Twist

4.210 Introduction - When a wing has effective twist, the basic distributions are no longer zero. In order to obtain the lift and induced-drag distributions at any wing lift coefficient for a wing having a single twist—or a combination of twists that may be treated as one—symmetrical about the centerline, the necessary unit distributions may be determined by following the procedure outlined below. This applies to a wing with built-in twist, as type 1(a), or to a wing with part-span flap or with drooped ailerons, as long as the distribution for only one flap or aileron angle is desired. It will be described in Section 4.23 how to use the results from these computations for one flap angle to determine the distribution for any other flap angle. The method is illustrated by example 3 of Chapter 7.

4.211 Preliminary Computations for Wing Without Twist - The theoretical lift and induced-drag coefficient distributions are first found in the normal manner, using Tables (I-VII) for the subject wing, except that the wing is assumed to have no twist, and $\alpha_a = 1.0$ is used in line 6 of Table I for the entire span. As usual, when there is no twist, it is not necessary to compute Table II, because $B_1 = \alpha_a$ and all other B_n coefficients are zero.

4.212 Preliminary Computations for Wing with Twist - Next repeat the above calculations except that this time the true angles of the wing are used. Any arbitrary angle of attack of the section at the plane of symmetry may be used, but where the effective twist is the result of a part-span flap, some work can be saved by using zero angle of attack for the portion of the span not covered by the flap. In these computations it is necessary to fill in only lines 6 and 7 of Table I, since everything else in this table is the same as in the first computation. If the wing has a part-span flap, Section 4.25 should be read before filling in line 6. Table II must be computed this time, but Tables III and IV do not need to be touched, because the C_{2n} coefficients are independent of twist. The computations in Table V must be repeated using the new B_n coefficients and the same C's as before. Table VI is computed in the usual manner, but lines (16-20) and (25-27) of Table VII may be omitted for the twisted wing.

4.213 Table VIII - The distributions found in Table VII and the wing lift coefficients in Table VI for the two cases, one the untwisted wing and one the twisted wing, furnish sufficient information for obtaining the various unit distributions which is accomplished in Table VIII as follows:

(1) The column numbers in line 1 correspond to the same numbers in Tables I and VII and serve to identify the span locations at which the section coefficients are determined.

(2) Line 2 is copied from Table VII, line 22, for the untwisted wing and represents the additional-lift coefficient, $c_{l_{a_1}}$, distribution when $C_L = 1.0$.

(3) Line 3 is copied from Table VII, line 15, for the twisted wing and represents the distribution of total lift coefficient for the actual wing for the particular angle of attack and flap angle used in the computations. In this table the presence of (T) after C_L or after a table number indicates the twisted wing, while its absence indicates the normal or untwisted wing.

(4) Line 4 is obtained by multiplying the values of $c_{l_{a_1}}$ in line 2 by the value of C_L found in Table VI for the twisted wing and represents

the additional part of the total distribution given in line 3.

(5) Line 5 is obtained by subtracting line 4 from line 3 and represents the distribution of the basic-lift coefficient for the subject wing with the twist used in the computations.

(6) Line 6 is copied from Table I, line 11, and represents the section lift-curve slopes.

(7) Line 7 is copied from Table I, line 6, for the twisted wing and represents the absolute angles of attack for the sections as used in the twisted wing calculations.

(8) Line 8 contains the absolute angles of attack when the wing $C_L = 0$, and is obtained by subtracting from the values in line 7 the constant angle obtained by dividing the value of C_L from Table VI for the twisted wing by the slope of the wing lift curve which is equal to C_L for the untwisted wing at an angle of attack of one radian.

(9) Line 9 is obtained by dividing each value in line 5 by the corresponding value in line 6 and represents the effective angles of attack when $C_L = 0$.

(10) Line 10 is obtained by subtracting line 9 from line 8 and represents the downwash or in-

duced angles when $C_L = 0$.

(11) Line 11 is obtained by multiplying the values in line 5 by the corresponding values in line 10 and represents the distribution of the basic induced-drag coefficient for the subject wing with the twist used in the computations.

(12) Line 12 is copied from Table VII, line 20, for the untwisted wing and represents the induced-drag coefficient distribution for the untwisted wing at the particular angle of attack used in the computations (1 radian).

(13) Line 13 is obtained by dividing line 12 by the square of C_L found in Table VI for the untwisted wing and represents the distribution of that portion of the induced drag which varies with C_L^2 when $C_L = 1.0$.

(14) Line 14 is obtained by dividing the values in line 5 by the corresponding values in line 2.

(15) Line 15 is obtained by multiplying the values in line 13 by the corresponding values in line 14.

(16) Line 16 is obtained by dividing the values in line 11 by the corresponding values in line 14.

(17) Line 17 is the sum of lines 15 and 16

and represents that portion of the induced-drag distribution which varies directly with C_L, when $C_L = 1.0$.

(18) Line 18 is already filled in.

(19) Line 19 is copied from Table I, line 9.

(20) Line 20 is obtained by multiplying the values in line 11 by the corresponding values in line 19.

(21) Line 21 is obtained by multiplying the values in line 18 by the corresponding values in line 20.

(22) Line 22 is obtained by multiplying the values in line 17 by the corresponding values in line 19.

(23) Line 23 is obtained by multiplying the values in line 18 by the corresponding values in line 22.

(24) At the bottom of the table, the values of the wing coefficients $C_{D_{i_b}}$ and $C_{D_{i_{a1b}}}$ are found by integrating lines 21 and 23.

(25) A check on these values is then obtained by using Equation 4.10 to compute the total wing induced-drag coefficient for this wing at the value of wing lift coefficient found in Table VI for the twisted wing and comparing it with the value of C_{D_i} previously computed in this Table VI.

4.214 Total Distributions - Equations 4.1 and 4.2 may now be used in connection with Table VIII to find the total distribution of lift and induced-drag coefficients for the subject wing at any C_L. The total section lift coefficient at any of the ten points along the semispan is

$$c_l = \text{Line (5)} + C_L \times \text{Line (2)} \qquad (4.7)$$

and the total section induced-drag coefficient is likewise

$$c_{d_i} = \text{Line (11)} + C_L \times \text{Line (17)} + C_L{}^2 \times \text{Line (13)} \qquad (4.8)$$

where the values are taken from the lines of Table VIII indicated, for the column corresponding to the span location for which the section coefficients are desired.

4.215 Wing Coefficients - The slope of the lift curve for the wing is independent of twist as long as the respective section slopes along the span are the same in all cases. (Paragraph 4.252). The computations for the untwisted wing at an angle of attack of one radian, therefore, serve to furnish the lift-curve slope, since the wing C_L found in Table VI is equal to m as long as α_a is one radian. The value of τ is also independent of twist and is given in terms of m_o for the section at the plane of symmetry by

$$(1 + \tau_s) = (m_s - m) \frac{\pi A}{m\, m_s} \qquad (4.6)$$

which is the same as given previously for an untwisted wing. In order to determine the wing lift coefficient for a given angle of attack, it is not only necessary to know the lift-curve slope, but in the case of the twisted wing, the angle of attack for zero lift also must be known. This is found in Table VIII, line 8. If the absolute angle of attack of the section at the plane of symmetry be represented by α_s for a condition for which it is desired to know C_L, and α_{s_o} be used to represent the angle of attack of this section when $C_L = 0$, as listed in Table VIII, line 8, column 0, then

$$C_L = m\ (\alpha_s - \alpha_{s_o}) \qquad (4.9)$$

The induced-drag coefficient for the wing for this value of C_L can be found by

$$C_{D_i} = C_{D_{i_b}} + C_L\ C_{D_{i_{a_1 b}}} + C_L^{\ 2}\ \ C_{D_{i_{a_1}}} \qquad (4.10)$$

where $C_{D_{i_b}}$ represents the basic induced-drag coefficient and $C_{D_{i_{a_1 b}}}$ represents the portion of the induced drag caused by the interaction of the basic and additional lifts when $C_L = 1.0$, both found at the bottom of Table VIII by integrating the respective distributions over the semispan, and $C_{D_{i_{a_1}}} = \dfrac{(1 + \sigma)}{\pi A}$ for the untwisted wing found in Table VI.

TABLE VIII

COMPUTATION OF UNIT LOAD DISTRIBUTIONS

SINGLE SYMMETRICAL TWIST

$C_{L(T)} =$ $\qquad\qquad\qquad\qquad$ $C_L = m =$

1	a	b	0	1	2	3	4	5	6	7	8	9
2	TABLE VII LINE 22	$C_{\ell a_1}$										
3	TABLE VII(T) LINE 15	C_ℓ										
4	(2) × $C_{L(T)}$	$C_{\ell a}$										
5	(3) - (4)	$C_{\ell b}$										
6	TABLE I LINE 11	m_o										
7	TABLE I(T) LINE 6	$\alpha_a(C_{L(T)} = \quad)$										
8	(7) - $C_{L(T)}/m$	$\alpha_a(C_L = 0)$										
9	(5) / (6)	α_o										
10	(8) - (9)	$\frac{w}{V}$										
11	(5) x (10)	$C_{d_{i_b}}$										
12	TABLE VII LINE 20	C_{d_i}										
13	(12) / C_L^2	$C_{d_{i a_1}}$										
14	(5) / (2)	$C_{\ell b} / C_{\ell a_1}$										
15	(13) × (14)	$\frac{C_{d_{i a_1}} \times C_{\ell b}}{C_{\ell a_1}}$										
16	(11) / (14)	$\frac{C_{d_{i_b}} \times C_{\ell a_1}}{C_{\ell b}}$										
17	(15) + (16)	$C_{d_{i a_1 b}}$										
18		SIN θ	1.0000	.9877	.9511	.8910	.8090	.7071	.5878	.4540	.3090	.1564
19	TABLE I LINE 9	c										
20	(11) × (19)	$cC_{d_{i_b}}$										
21	(18) × (20)	$cC_{d_{i_b}}$ SIN θ										
22	(17) × (19)	$cC_{d_{i a_1 b}}$										
23	(18) × (22)	$cC_{d_{i a_1 b}}$ SIN θ										

LINE 21: $\frac{\pi}{20}\frac{b}{S}\sum cC_{d_{i_b}} \sin\theta = \frac{\pi}{20 C_{AVE}}\left(\sum (21) - \frac{1}{2}(21)_0\right) = C_{D_{i_b}} =$

LINE 23: $\frac{\pi}{20}\frac{b}{S}\sum cC_{d_{i a_1 b}} \sin\theta = \frac{\pi}{20 C_{AVE}}\left(\sum (23) - \frac{1}{2}(23)_0\right) = C_{D_{i a_1 b}} =$

CHECK: $C_{D_i} = C_{D_{i_b}} + C_{D_{i a_1 b}} \times C_{L(T)} + C_{D_{i a_1}} \times C_{L(T)}^2 =$

4.22 Procedure for Wing with a Single Antisymmetrical Twist.

4.220 Introduction - It was explained in Chapter 3 how to compute the section lift and induced-drag coefficients for a given angle of attack for any wing, twisted or untwisted, as long as it is symmetrical about the centerline. It has been explained in this chapter how to use the results of Chapter 3 to determine the distributions for this type of wing for any angle of attack or wing lift coefficient. The consideration of a wing, which has a twist unsymmetrical about the centerline, has so far been omitted in order to avoid confusion. It was shown in Chapter 3 that for a symmetrical wing only the odd B_n and A_n coefficients appear in the computations. In an unsymmetrical wing, both odd and even coefficients would ordinarily occur and considerably lengthen the computations. In a wing with unsymmetrical twist, however, the distribution of angle of attack can always be divided into one that is symmetrical and one that is antisymmetrical; by which is meant one having equal and opposite angles on the two sides, as explained under type (2) in Paragraph 4.20. Separate computations can then be made for each, the former involving only the odd coefficients and the latter only the even. The general procedure for computing the antisymmetrical distributions is given in Chapter

5 and, although the method is similar to that of Chapter 3 for the symmetrical case, most of the tables are modified and the computations must always be made for zero angle of attack at the center, in order to completely eliminate symmetrical loading and resultant wing lift. Examples 4 and 5 of Chapter 7 include computations for antisymmetrical twist. The procedure described in the following paragraph is for a wing having no other twist except the antisymmetrical twist under consideration. The procedure when one or more other twists, symmetrical or antisymmetrical, are present is described in Section 4.24.

4.221 Computations - The procedure for a wing with type 2 or antisymmetrical twist is not far different from the regular procedure for a twisted wing as described in Section 4.21. The computations are first made for the wing without twist for an angle of attack of one radian, as before. The second set of computations, which in the other procedure could be made for the twisted wing for any convenient angle of attack, must be made in this case for zero angle of attack at the center for which the wing C_L is zero. These computations are made in accordance with Chapter 5, using Tables II-A, IV-A, V-A, VI-A and VII-A in place of Tables II, IV, V, VI and VII. While in the case of symmetrical twist Table VIII was required to find all the basic distributions, in this case c_{l_b} and $c_{d_{i_b}}$ are found directly in Table VII-A. Table VIII-A is, therefore, needed only to determine $c_{d_{i_{a_1b}}}$ which is accomplished in a manner similar to that of Table VIII. In this case, however, $c_{d_{i_{a_1b}}}$ is equal and opposite on the two semiwings and it is this which produces the induced yawing moment. The yawing moment coefficient for $C_L = 1$ is found at the bottom of the table by integration. By comparing it with its value found in Table VI-A a check is obtained for the work in Table VIII-A.

TABLE VIII A

UNIT LOAD DISTRIBUTIONS

ANTI-SYMMETRICAL

C_L =

1	a	b	1	2	3	4	5	6	7	8	9
2	TABLE VIIA LINE 15	C_{l_b}									
3	TABLE VII LINE 22	$C_{l_{a_1}}$									
4	TABLE VIIA LINE 20	C_{di_b}									
5	TABLE VII LINE 20	C_{di}									
6	(5) / C_L^2	$C_{di_{a_1}}$									
7	(2) / (3)	$C_{l_b} / C_{l_{a_1}}$									
8	(6) × (7)	$C_{di_{a_1}} \times C_{l_b} / C_{l_{a_1}}$									
9	(4) / (7)	$C_{di_b} \times C_{l_{a_1}} / C_{l_b}$									
10	(8) + (9)	$C_{di_{a_1 b}}$									
11		SIN 2θ	.3090	.5878	.8090	.9511	1.0000	.9511	.8090	.5878	.3090
12	TABLE I LINE 9	C									
13	(10) × (12)	$C\,C_{di_{a_1 b}}$									
14	(11) × (13)	$CC_{di_{a_1 b}} \times$ SIN 2θ									

$$\frac{C'_n}{C_L} = \frac{\pi}{80 C_{AVE}} \Sigma (14) =$$

4.23 Procedure for Wing with Variable Twist - It has now been explained how to obtain for any angle of attack the distributions for any wing with symmetrical (Section 4.21) or antisymmetrical (Section 4.22) twist. In the case of a flap or aileron, however, it may be desirable to know the distributions for several different flap angles. These can, of course, be found by repeating the above calculations for each flap angle, but this is not necessary. After the basic distributions have been found for any one flap angle, they may be found for any other flap angle in the manner described below. The same method applies to any type of twist as long as the relative distribution of absolute angle of attack along the span remains the same.

(a) Let $(k\delta)_1$ represent the change in zero-lift angle from neutral flap condition at any one section as used in the computations for the original flap displacement. (See Section 4.25).

(b) Let $(k\delta)_2$ represent the change in zero-lift angle from neutral flap condition at the same section caused by the second flap displacement for which the distributions are desired.

(c) Let subscript 1 refer to all coefficients as computed for the original flap displacement and subscript 2 refer to the coefficients which it is desired to find for the second flap displacement.

(d) Then

$$c_{l_{b_2}} = c_{l_{b_1}} \frac{(k\delta)_2}{(k\delta)_1} \tag{4.11}$$

$$c_{d_{i_{b_2}}} = c_{d_{i_{b_1}}} \left[\frac{(k\delta)_2}{(k\delta)_1}\right]^2 \tag{4.12}$$

$$c_{d_{i_{a_1 b_2}}} = c_{d_{i_{a_1 b_1}}} \frac{(k\delta)_2}{(k\delta)_1} \tag{4.13}$$

or

$$\text{new} \quad c_l = c_{l_{b_1}} \frac{(k\delta)_2}{(k\delta)_1} + C_L \, c_{l_{a_1}} \tag{4.14}$$

$$\text{new} \quad c_{d_i} = c_{d_{i_{b_1}}} \left[\frac{(k\delta)_2}{(k\delta)_1}\right]^2 + C_L \, c_{d_{i_{a_1 b_1}}} \frac{(k\delta)_2}{(k\delta)_1} + C_L^2 \, c_{d_{i_{a_1}}} \tag{4.15}$$

$$\text{new} \quad C_{D_i} = C_{D_{i_b}} \left[\frac{(k\delta)_2}{(k\delta)_1}\right]^2 + C_L \, C_{D_{i_{a_1 b_1}}} \frac{(k\delta)_2}{(k\delta)_1} + C_L^2 \, C_{D_{i_{a_1}}} \tag{4.16}$$

(e) It should be noted that the amount of flap displacement has no effect on the additional distributions. The angle of zero lift is, however, changed so that for a given angle of attack of the reference section, α_s

$$C_L = m \left[\alpha_s - \alpha_{s_{o_1}} \quad \frac{(k\delta)_2}{(k\delta)_1} \right] \qquad (4.17)$$

where $\alpha_{s_{o_1}}$ is the angle of attack of this section for zero wing lift with the original flap displacement. This requires that the values of m_o remain constant for varying amounts of flap displacement.

(f) The above procedure is also based on the assumption that the relative distribution of $k\delta$ along the span remains the same with changing flap angle.

(g) Where $k\delta$ is constant along the span of the flap, it is best to let $k\delta$ = 1 radian for the initial computations which causes $(k\delta)_1$ to drop out of the above equations.

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4.24 Procedure for Wing with Multiple Twist.

4.240 Introduction - One procedure has now been given for a wing with one symmetrical twist (Section 4.21) and another procedure for a wing with one antisymmetrical twist (Section 4.22). Any number of these twists may exist simultaneously and, as long as they are added together to form one composite fixed twist, the above procedure may be used— provided they are all symmetrical or all antisymmetrical. Where several types exist, however, some of which may vary or some vanish entirely under certain conditions, it will be found convenient to obtain basic distributions for each type of twist separately as though the others did not exist, and it is a necessity where some of the twist is symmetrical and some antisymmetrical. The procedure is the same as before. The first step is to make complete computations for the wing without twist in order to obtain the additional distributions. Then one additional computation must be made for each of the twists in accordance with the procedures previously outlined. A basic distribution of lift and induced drag is thus found for each type of twist, each one of which may be modified to take care of other degrees of the same type of twist, if desired. The section lift at any point of the span can then be found by simple addition, but the induced-drag coefficient is not so simple, because there is not only interaction between the additional and each basic distribution, but there is also interaction between the various basic distributions.

4.241 Section Coefficients - Expressions are given below for a wing lift coefficient C_L for the section lift and induced-drag coefficients for a wing with three different twists, the basic coefficients for each being designated by sub-numbers 1, 2 and 3, respectively. From the general form of these equations it can be readily seen how to expand them to include any number of individual twists. The twists may be either symmetrical or antisymmetrical, or both.

$$c_l = (c_{l b_1} + c_{l b_2} + c_{l b_3}) + C_L\, c_{l a_1} \qquad (4.18)$$

$$\begin{aligned} c_{d_i} = &\left[c_{d_{i b_1}} + c_{d_{i b_2}} + c_{d_{i b_3}}\right] + \left[\left(\frac{c_{d_{i b_2}}}{c_{l b_2}} + \frac{c_{d_{i b_3}}}{c_{l b_3}}\right) c_{l b_1} \; + \right. \\ &\left. + \left(\frac{c_{d_{i b_1}}}{c_{l b_1}} + \frac{c_{d_{i b_3}}}{c_{l b_3}}\right) c_{l b_2} + \left(\frac{c_{d_{i b_1}}}{c_{l b_1}} + \frac{c_{d_{i b_2}}}{c_{l b_2}}\right) c_{l b_3}\right] + \\ &+ C_L \left[\left(\frac{c_{d_{i b_1}}}{c_{l b_1}} + \frac{c_{d_{i b_2}}}{c_{l b_2}} + \frac{c_{d_{i b_3}}}{c_{l b_3}}\right) c_{l a_1} \; + \right. \\ &\left. + \frac{c_{d_{i a_1}}}{c_{l a_1}} (c_{l b_1} + c_{l b_2} + c_{l b_3})\right] + C_L^2\, c_{d_{i a_1}} \qquad (4.19) \end{aligned}$$

This may be written -

$$c_{d_i} = \left(\sum \frac{c_{d_{i_b}}}{c_{l\,b}}\right)\left(\sum c_{l\,b}\right) + C_L\left[\left(\sum \frac{c_{d_{i_b}}}{c_{l\,b}}\right) c_{l\,a_1} + \left(\sum c_{l_b}\right) \frac{c_{d_{i_{a_1}}}}{c_{l\,a_1}}\right] + C_L^2 \; c_{d_{i_{a_1}}} \qquad (4.20)$$

where

$$\sum \frac{c_{d_{i_b}}}{c_{l\,b}} = \frac{c_{d_{i_{b_1}}}}{c_{l\,b_1}} + \frac{c_{d_{i_{b_2}}}}{c_{l\,b_2}} + \frac{c_{d_{i_{b_3}}}}{c_{l\,b_3}}$$

and

$$\sum c_{l\,b} = c_{l\,b_1} + c_{l\,b_2} + c_{l\,b_3}$$

The similarity between this formula and Equation 4.2 for a simple twist should be noted. When Equation 4.20 is used it is necessary to know the values of $c_{l\,b}$ and $c_{d_{i_b}}$ for each of the individual twists, but it is not necessary to know the individual values of $c_{d_{i_{a_1 b}}}$. When these have been calculated, however, in Tables VIII and VIII-A, they may be used, if Equation 4.20 is written as follows:

$$c_{d_i} = \left(\sum \frac{c_{d_{i_b}}}{c_{l\,b}}\right)\left(\sum c_{l\,b}\right) + C_L\left(\sum c_{d_{i_{a_1 b}}}\right) + C_L^2 \; c_{d_{i_{a_1}}} \qquad (4.21)$$

in which

$$\sum c_{d_{i_{a_1 b}}} = c_{d_{i_{a_1 b_1}}} + c_{d_{i_{a_1 b_2}}} + c_{d_{i_{a_1 b_3}}}$$

It is seen that, when the individual values of $c_{d_{i_b}}$ are known, it is a rather simple matter to compute the c_{d_i}

values for any composite twist. It may be found more convenient in some cases, however, to omit all induced-drag computations until the lift distribution is found for each design condition. The distribution of total induced-drag coefficient can then be found for each design condition by using a form similar to lines 15 to 20 inclusive of Table VII, and filling line 15 with the distribution of total section lift coefficient as determined for each design condition. This latter method is almost a necessity when any part of the effective twist is caused by unsymmetrical air flow, such as occurs during a roll.

4.242 Wing Coefficient - The relation between wing C_L and angle of attack is found, as before, by using the sum of the zero-lift angles of attack in place of one, as

$$C_L = m\left[\alpha_s - \left(\alpha_{s_{o1}} + \alpha_{s_{o2}} + \alpha_{s_{o3}}\right)\right] \qquad (4.22)$$

where, as before, α_s is the angle of attack of the reference section for which it is desired to know C_L, and $\alpha_{s_{o1}}$, $\alpha_{s_{o2}}$, and $\alpha_{s_{o3}}$ are the respective angles of the same section when $C_L = 0$ for each twist separately.

4.243 Examples - The application of this method to a wing having two twists, one symmetrical and one antisymmetrical, is illustrated in example 4 of Chapter 7. The antisymmetrical twist caused by roll requires special treatment that is best described by means of an example. Example 5 has been added, therefore, to illustrate the combination of roll with two other twists.

4.25 Value of $k\delta$ for Flaps and Ailerons.

4.250 Several references have been made to this Section as giving an explanation of the angle of attack to use in Table I, line 6, when it is desired to compute the distribution for a wing with a part-span flap in operation, or when the antisymmetrical distribution from displaced ailerons is wanted. It has been explained that when E, the ratio of flap chord to wing chord, is constant along the span of the flap, it is most convenient to assume an angle of attack of zero for the portion of the span without flap, and one radian for the portion covered by the flap. Before the distribution can be found for any given flap angle δ, it is necessary to know the absolute angle of attack for the section with this particular flap displaced to this angle. The change in this angle produced by the flap displacement is usually expressed as $k\delta$, where k is theoretically a function of the chord ratio E only, but actually varies with δ as well. Furthermore, when E varies with span, it is necessary to know how $k\delta$ varies with E before Table I, line 6, can be filled in. In this case when the angle of attack of the section without flap is taken as zero, the angles of attack for the points covered by the flap may be taken as $k\delta$ directly or some reference section may be chosen as having one radian and the other sections taken in proportion.

4.251 The angle $k\delta$ may be taken from N.A.C.A. reports which include tables of "Fundamental Section Characteristics", as the difference between α_{l_0} for the basic wing section and α_{l_0} for the same section with flap displaced, and the result divided by 57.3 in order to change to radians. Where wind tunnel tests are available of a rectangular wing with the flap extending over the full span, the value $k\delta$ is readily determined as the number of degrees between the intercepts at zero lift of the C_L vs. α curves for the wing with flap and the wing without flap, respectively, divided by 57.3. If the two curves are not quite straight, they should be approximated by straight lines before reading the intercepts on the zero-lift line.

4.252 Where no information of this type is available, the value of $k\delta$ for ordinary or plain flaps and for simple split flaps may be estimated by means of the curves in Figure 4-1 which were prepared from the data in reference 13. Most of the tests on which reference 13 is based were made with wings having a thickness ratio of about 12 percent. As pointed out in reference 16, there is a tendency for the effectiveness of flaps to increase slightly with the thickness of the section. No account of this has been taken in preparing Figure 4-1, as the approximate nature of the curves scarcely warrants such refinement. These curves give reasonable results when there is no leakage at the hinge. The effectiveness, and consequently the value of $k\delta$, is considerably reduced when there is a leakage slot at the hinge, as shown by Figure 11 of reference 17. These curves should not be used for other types of flaps, such as the slotted, external airfoil and Fowler. Most of these have the property of producing an equivalent $k\delta$ with a relatively smaller displacement. When the value of $k\delta$ is obtained from Figure 4-1, the value of m_o for the section with flap should be assumed to be the same as for the basic section. When obtaining $k\delta$ from other sources, however, or when using other types of flaps, it is necessary to consider also the value of m_o and if it differs appreciably from that for the basic section, it should be used in the computations. In general, the displacement of an ordinary flap does not change m_o, while moderate displacements of a split flap usually cause m_o to increase. Flaps which have the effect of increasing the chord, such as the Fowler, may be handled as though the chord remained constant and m_o increased. In cases where it is necessary to use different values of m_o for flaps open and closed, it is necessary also to make a separate additional-lift computation for use when the flaps are open. Otherwise, the normal distribution for an untwisted wing with flaps closed can be used as the additional distribution for flaps open.

4.253 When filling in Table I, line 6, if the end of the flap or aileron happens to fall midway between two of the ten points on the semispan, the angle of attack for one point is zero while that of the next point is $k\delta$ for that section with displaced flap. When the end of the flap coincides with one of the ten points, the value of the angle of attack should be taken as one-half the value of $k\delta$ at this point. When the end of the flap falls at any other location between two points, the value of the angle of attack at the nearer point should be modified by interpolation, using the angles rather than the linear distances in the interpolation. This is an approximation that seems sufficiently accurate in most cases. It really amounts to replacing a portion of the flap near the end by an equivalent portion of less effectiveness, but with greater length so that the total lift is kept the same. The distribution of coefficients near the flap end is made less abrupt and a slight error in the rolling moment also results. Where greater accuracy is desired, a system using more points should be used, such as that given in reference 9.

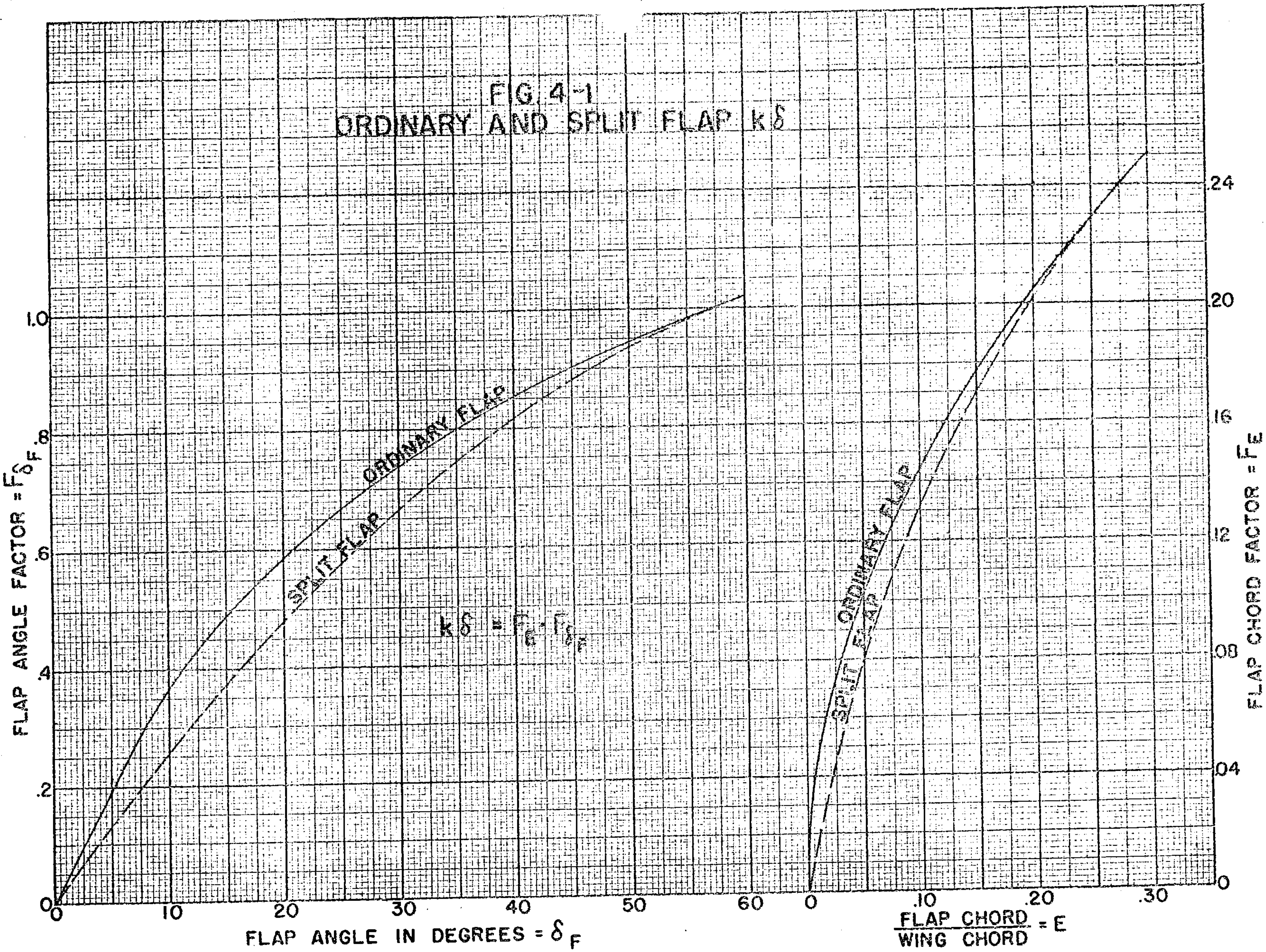

FIG. 4-1
ORDINARY AND SPLIT FLAP $k\delta$

CHAPTER 5

PROCEDURE FOR CALCULATING LIFT AND INDUCED-DRAG DISTRIBUTION FOR ANTISYMMETRICAL LOADING ONLY

PROCEDURE FOR CALCULATING LIFT AND INDUCED-DRAG DISTRIBUTIONS FOR ANTISYMMETRICAL LOADING ONLY

5.0 General

5.00 Introduction - The computations described herein must be made in connection with Section 4.22 of the preceding chapter. Since the distributions caused by ailerons or by roll may be considered as auxiliary loadings, it has been thought best to postpone, in so far as possible, all material concerning antisymmetrical loading until the procedure for symmetrical loading has been completed. Section 4.22 was included in the preceding chapter because Sections 4.23, 4.24 and 4.25 apply equally well to both types.

5.01 Scope - The purpose of this chapter is to outline the method for obtaining the basic-lift and basic-induced-drag distributions for antisymmetrical loading, or for type 2 twist of Section 4.20. The procedure is almost identical with that given in Chapter 3 for symmetrical loading. In the case of symmetrical loading, only the odd B_n and A_n coefficients appear in the computations, while only the even coefficients occur in the antisymmetrical calculations. The main difference in the two cases is that the computations for symmetrical loading may be made for any angle of attack, because both basic and additional distributions are symmetrical. The computations for antisymmetrical loading, on the other hand, must be made for zero angle of attack at the plane of symmetry in order to eliminate all additional distribution. Furthermore, the wing must have no symmetrical twist—or, if it does, it must be assumed zero for the purpose of these calculations—in order to eliminate all symmetrical basic distributions. Only in this way can the odd coefficients be kept zero. There is the advantage that when the antisymmetrical lift and induced-drag distributions are obtained, they are basic and can be combined directly with other distributions in accordance with the procedure of Section 4.22. In the case of symmetrical loading this is not possible, because the angle of attack for zero lift is not known until after the computations of Chapter 3 have been completed. Table VIII of Section 4.21 is, therefore, required in order to determine the basic distributions. Table VIII-A of Section 4.22 on the other hand, is required only for the purpose of determining $c_{d_{i_{a_1 b}}}$. The procedure of this chapter is illustrated in examples 4 and 5 of Chapter 7.

5.1 Table I - Table I will probably have been used for computing the symmetrical case previously in Chapter 3. It will, therefore, be necessary to fill in only lines 6 and 7 for this condition as all other lines remain the same. This assumes the same value for m_o for the respective sections in the two cases.

(a) If the effect of displaced ailerons is desired, the angle of attack of the inboard portion of the wing should be taken as zero and that for the aileron portion as $k\delta$ where $k\delta$ represents the change in zero-lift angle at any section produced by displacing the aileron. If the ratio of the chord of the aileron to the wing chord is constant over the span of the aileron, $k\delta$ may be assumed constant and equal to one radian for this initial calculation, the distributions then being modified later to take care of any desired aileron angle. In any case, line 6 should not be filled in without first reading Section 4.25 which explains how to determine the value of $k\delta$ for a wing section with flap. (See example 4 of Chapter 7).

(b) If the damping characteristics in roll are desired, the wing may be assumed to have a linear twist from an angle of attack of zero at the center to positive and negative maximums at the respective wing tips. The magnitude of the angle of attack depends on the ratio of the angular velocity in roll to the forward velocity of the airplane. Any convenient angle of attack, such as one radian at the tip, may be used in these calculations. How to modify the results to make them applicable for any value of this angle—which may be expressed as $p'b/2V$ in which p' is the angular velocity in roll about the wind axis—was explained in Section 4.23. Tables I and II-A are not needed for this calculation because $2B_2 = p'b/2V$ and all other B's are zero. The c_l and c_{d_i} distributions determined for the wing with an assumed twist are not in the correct direction for computing the damping and yawing moments, but must be resolved before they can be combined with other distributions. This is more fully explained in example 5 of Chapter 7. For small values of $p'b/2V$ the error in rolling moment in assuming c_l and c_{d_i} obtained in these calculations to be normal and parallel, respectively, to the line of flight is small but the error in yawing moment is too large to neglect.

5.2 Table II-A - Table II-A should next be filled in for the purpose of obtaining the B_n coefficients, which in the antisymmetrical case are all even. The y's are obtained from Table I, line 7, as in the symmetrical case, except that y_0 is always zero. It will be noted that this table is quite similar to Table III for obtaining the C_{2n} coefficients. It is impossible to obtain one check that includes all the B's, but the use of two checks is a help in locating the error in case one equation checks while the other does not.

5.3 Table III - The C_{2n} coefficients are obtained from Table III and are the same as for the symmetrical case, since they depend only on the plan form and the values of m_o for the sections.

5.4 Table IV-A - The constants required for the solution of the A_n coefficients are tabulated in Table IV-A. This is very similar to Table IV and is self-explanatory.

5.5 Table V-A - The general equations for solving for the A_n coefficients may be obtained from those given in Section 3.5 for the symmetrical case by adding one to each sub-number, the various known constants being taken from Table IV-A instead of from IV and the $2B_n$'s from Table II-A instead of from II. In solving these equations, Table V-A is used in place of Table V which results in the values for the even A_n coefficients. The procedure is the same as for the symmetrical case.

5.6 Table VI-A - Table VI-A is somewhat similar to Table VI for the symmetrical case. Since the total lift on the wing is zero, the expression is given for the lift coefficient on half the wing. The induced-drag coefficient is correct for the total wing when there is no symmetrical lift present. Note that the rolling moment about the wind axis is C_l'qbS and depends on A_2 in the same way that the

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lift depended on A_1 in the symmetrical case. Columns 1 and 5 have been added to this table in order to provide a check for $c_{d_{i_{a,b}}}$ to be calculated later in Table VIII-A. The odd A_n coefficients in column 1 are found by dividing the A's found for the wing without twist by the value of C_L corresponding to these A's; or, in other words, the A's in column 1 are the correct values for the wing without twist when $C_L = 1.0$. Then the yawing moment coefficient C_n', found by using the sum of column 5, when multiplied by qbS gives the induced yawing moment for this particular antisymmetrical twist when the wing lift coefficient C_L = 1.0. The value is directly proportional to the amount of twist and to C_L. It cannot be used for a wing that has also a symmetrical twist, except to check the values of $c_{d_{i_{a,b}}}$, as explained in Paragraph 4.221.

5.7 <u>Table VII-A</u> - Table VII-A is used for obtaining the distributions in the same way that Table VII was used for symmetrical loading. In this case line 15 gives the distribution of the basic-lift coefficient for the particular distribution of angle of attack used in Table I, and line 20 gives the distribution of basic-induced-drag coefficient for the same condition. It should be noted that while c_{l_b} is positive on one wing and negative on the other, resulting in zero wing lift, the induced drag coefficient $c_{d_{i_b}}$, on the other hand, is positive and equal on each semiwing, because c_{l_b} and w/V usually have the same sign. This does not mean, however, that, when an antisymmetrical distribution is combined with a symmetrical loading, the induced drag for the combination is symmetrical and there is no yawing moment; because the part of the induced drag produced by the interaction of the two distributions is antisymmetrical. (See Paragraph 4.221). Line 23 is used as a check similar to the way line 24 was used as a check for the symmetrical case, except that sin 2θ is used as a multiplier instead of sin θ and the check is against the rolling-moment coefficient instead of the lift coefficient. This is done below the table where the rolling-moment coefficient is taken from Table VI-A. Line 26 is used to check the value of the wing induced-drag coefficient found in Table VI-A, and if a check of the lift on one semiwing is desired, it may be found in a similar manner using c_{l_b} in place of $c_{d_{i_b}}$. If the rolling moment checks, however, it is proof that the values of c_{l_b} are correct in so far as the computations in this table are concerned.

TABLE II A

COMPUTATION OF ANGLE COEFFICIENTS, B_n

ANTI-SYMMETRICAL

1	TABLE I	y_9		y_8		y_7		y_6		y_5	
2	LINE 7	y_1		y_2		y_3		y_4			
3	(1) + (2)	V_1		V_2		V_3		V_4		V_5	
4	(1) − (2)	W_1		W_2		W_3		W_4			

5	MULTI-PLIER	2 − 18		4 − 16		6 − 14		8 − 12		10
		a	b	a	b	a	b	a	b	
6	.3090	V_1				V_3				
7	.5878		V_2	W_1	W_4		$-V_4$	$-W_3$	W_2	
8	.8090	V_3				V_1				
9	.9511		V_4	W_3	W_2		V_2	W_1	$-W_4$	
10	1.0000	V_5				$-V_5$				$V_1 - V_3 + V_5$
11	TOTAL									
12	a + b									
13	a − b									
14	$\frac{(12) \text{ or } (13)}{2.5}$									
15	$2B_n$	$2B_2$	$2B_{18}$	$2B_4$	$2B_{16}$	$2B_6$	$2B_{14}$	$2B_8$	$2B_{12}$	$2B_{10}$

CHECKS:

$2B_2 - 2B_6 + 2B_{10} - 2B_{14} + 2B_{18} = 2y_5 =$

$.5878(2B_2 + 2B_8 - 2B_{12} - 2B_{18}) + .9511(2B_4 + 2B_6 - 2B_{14} - 2B_{16}) = 2y_8 =$

TABLE IV A

CONSTANTS REQUIRED IN SOLUTION OF A_n COEFFICIENTS ANTI-SYMMETRICAL

ITEM		
U_0	$\frac{m_s C_s}{4\ b}$	
$2P_2$	$2C_0 - C_4 + 4U_0$	
$2P_4$	$2C_0 - C_8 + 8U_0$	
$2P_6$	$2C_0 - C_{12} + 12\ U_0$	
$2P_8$	$2C_0 - C_{16} + 16\ U_0$	
$2P_{10}$	$2C_0 - C_{20} + 20U_0$	
D	$C_2 - C_6$	
E	$C_2 - C_{10}$	
F	$C_2 - C_{14}$	
G	$C_2 - C_{18}$	
H	$C_4 - C_8$	
I	$C_4 - C_{12}$	
J	$C_4 - C_{16}$	
K	$C_6 - C_{10}$	
L	$C_6 - C_{14}$	
M	$C_8 - C_{12}$	

TABLE V A

SOLUTION OF A_η COEFFICIENTS

ANTI-SYMMETRICAL

	1	2	3a	3b	3c	4a	4b	4c
1	$(C_2 - C_6)\,A_4$							
2	$(C_4 - C_8)\,A_6$							
3	$(C_6 - C_{10})\,A_8$							
4	$(C_8 - C_{12})\,A_{10}$							
5	1 + 2 + 3 + 4							
6	$2B_2$ − 6	$2B_2$ = ________						
7	A_2 = 6 / $2P_2$	$2P_2$ = ________						
8	$(C_2 - C_6)\,A_2$							
9	$(C_2 - C_{10})\,A_6$							
10	$(C_4 - C_{12})\,A_8$							
11	$(C_6 - C_{14})\,A_{10}$							
12	8 + 9 + 10 + 11							
13	$2B_4$ − 12	$2B_4$ = ________						
14	A_4 = 13 / $2P_4$	$2P_4$ = ________						
15	$(C_4 - C_8)\,A_2$							
16	$(C_2 - C_{10})\,A_4$							
17	$(C_2 - C_{14})\,A_8$							
18	$(C_4 - C_{16})\,A_{10}$							
19	15 + 16 + 17 + 18							
20	$2B_6$ − 19	$2B_6$ = ________						
21	A_6 = 20 / $2P_6$	$2P_6$ = ________						
22	$(C_6 - C_{10})\,A_2$							
23	$(C_4 - C_{12})\,A_4$							
24	$(C_2 - C_{14})\,A_6$							
25	$(C_2 - C_{18})\,A_{10}$							
26	22 + 23 + 24 + 25							
27	$2B_8$ − 26	$2B_8$ = ________						
28	A_8 = 27 / $2P_8$	$2P_8$ = ________						
29	$(C_8 - C_{12})\,A_2$							
30	$(C_6 - C_{14})\,A_4$							
31	$(C_4 - C_{16})\,A_6$							
32	$(C_2 - C_{18})\,A_8$							
33	29 + 30 + 31 + 32							
34	$2B_{10}$ − 33	$2B_{10}$ = ________						
35	A_{10} = 34 / $2P_{10}$	$2P_{10}$ = ________						

TABLE VI A

SEMI-WING LIFT AND DRAG COEFFICIENTS ANTI-SYMMETRICAL

1	2	3	4	5
ODD CIRCULATION CONSTANTS FOR NO TWIST & C_L = 1.0				$3A_1A_2$ =
				$5A_2A_3$ =
				$7A_3A_4$ =
				$9A_4A_5$ =
A_1 =	A_2 =	$2(A_2)^2$ =	$\frac{2}{3}A_2$ =	$11A_5A_6$ =
A_3 =	A_4 =	$4(A_4)^2$ =	$-\frac{4}{15}A_4$ =	$13A_6A_7$ =
A_5 =	A_6 =	$6(A_6)^2$ =	$\frac{6}{35}A_6$ =	$15A_7A_8$ =
A_7 =	A_8 =	$8(A_8)^2$ =	$-\frac{8}{63}A_8$ =	$17A_8A_9$ =
A_9 =	A_{10} =	$10(A_{10})^2$ =	$\frac{10}{99}A_{10}$ =	$19A_9A_{10}$ =
		ΣnA_n^2 =	Σ =	$\Sigma(2n+1)A_nA_{n+1}$ =

LIFT COEFFICIENT OF SEMI-WING, $C_{L_b} = 4A\,u_0 \Sigma$ COL. 4 =

$$C_{D_{i_b}} = \pi A u_0^2 \sum nA_n^2 =$$

ROLLING MOMENT COEFFICIENT, $C'_l = \frac{\text{ROLLING MOMENT}}{qbS} = \frac{\pi}{4} A u_0 A_2 =$

YAWING MOMENT COEFFICIENT $\frac{C'_n}{C_L} = \frac{\text{YAWING MOMENT}}{qbS} = \frac{\pi A}{4} u_0^2 \sum (2n+1) A_n A_{n+1}$

$$\bar{y}_{L_b} = \left[\frac{\pi \quad A_2}{8 \Sigma \text{ COL. } 4}\right]\frac{b}{2} =$$

TABLE VII A

COMPUTATION OF LOAD DISTRIBUTIONS
ANTI-SYMMETRICAL

1	a	b	1	2	3	4	5	6	7	8	9
2	TABLE I LINE 3	θ	81	72	63	54	45	36	27	18	9
3		SIN 2θ	.3090	.5878	.8090	.9511	1.0000	.9511	.8090	.5878	.3090
4		SIN 4θ	-.5878	-.9511	-.9511	-.5878	0	.5878	.9511	.9511	.5878
5		SIN 6θ	.8090	.9511	.3090	-.5878	-1.0000	-.5878	.3090	.9511	.8090
6		SIN 8θ	-.9511	-.5878	.5878	.9511	0	-.9511	-.5878	.5878	.9511
7		SIN 10θ	1.0000	0	-1.0000	0	1.0000	0	-1.0000	0	1.0000
8	A_2=	A_2 SIN 2θ									
9	A_4 =	A_4 SIN 4θ									
10	A_6=	A_6 SIN 6θ									
11	A_8=	A_8 SIN 8θ									
12	A_{10}=	A_{10} SIN 10θ									
13	(8)+(9)+(10)+(11)+(12)	$\Sigma A_n \sin n\theta$									
14	TABLE I [(11)$_o$ X (13)]	$\frac{m_s c_s}{c}$									
15	(13) X (14)	c_{ℓ_b}									
16	TABLE I LINE 11	m_o									
17	TABLE I LINE 6	α_a									
18	(15) / (16)	α_o									
19	(17) − (18)	$\frac{W}{V}$									
20	(19) X (15)	$c_{d_{ib}}$									
21	TABLE I LINE 9	c									
22	(21) X (15)	cc_{ℓ_b}									
23	(22) X (3)	cc_{ℓ_b} SIN 2θ									
24		SIN θ	.9877	.9511	.8910	.8090	.7071	.5878	.4540	.3090	.1564
25	(20) X (21)	$cc_{d_{ib}}$									
26	(25) X (24)	$cc_{d_{ib}}$ SIN θ									

LINE 26: $\frac{\pi}{20}\frac{b}{S}\,\Sigma cc_{d_{ib}} \sin\theta = \frac{\pi}{20} c_{AVE}\left(\Sigma (26)\right) = C_{D_{ib}}$

LINE 23: $C_\ell' = \frac{\text{ROLLING MOM.}}{q b S} = \frac{\pi}{4} A u_o A_2 = \frac{\pi}{80}\frac{b}{S}\,\Sigma cc_{\ell_b} \sin 2\theta = \frac{\pi}{80} c_{AVE}\left(\Sigma (23)\right)$

CHAPTER 6

OTHER SECTION COEFFICIENTS

OTHER SECTION COEFFICIENTS

6.1 Profile Drag.

6.10 Division of Profile Drag - The section profile-drag coefficient c_{d_o} may be divided into minimum profile-drag coefficient $c_{d_{o_{min}}}$, which is independent of the section lift, and an additional profile-drag coefficient Δc_{d_o} which is a function of the lift coefficient.

6.11 Minimum Profile Drag.

6.110 The value of $c_{d_{o_{min}}}$ for any section may be assumed to be independent of the location of the section and independent of the lift which the section has. It does depend on the smoothness of the wing surface and on the effective Reynolds Number. Its value for aerodynamically smooth wings can be found under "Fundamental Section Characteristics" in all recent N.A.C.A. publications, such as reference 15. A formula is given in reference 18 by means of which $c_{d_{o_{min}}}$ may be estimated for the various sections of a tapered wing. It has the form

$$c_{d_{o_{min}}} = k + 0.0050 + 0.0033\left(\frac{t}{c}\right) + 0.1\left(\frac{t}{c}\right)^2 \qquad (6.1)$$

where k (which is approximately constant for sections having the same mean line) represents the increase in $c_{d_{o_{min}}}$ above that of the symmetrical section of corresponding thickness. For the 230 series of reference 15, k has a value of approximately 0.0003. Recent information indicates that this formula tends to give values too large for the thicker sections. (See reference 16).

6.111 For wings with flaps, the value of $c_{d_{o_{min}}}$ has a very much larger value. The amount which must be added to the $c_{d_{o_{min}}}$ for the plain wing in order to obtain the value for the wing with an ordinary or split flap may be found in reference 10 or 13, when a V.D.T. test of the section with flap is not available.

6.112 The values of $c_{d_{o_{min}}}$, usually given in N.A.C.A. publications, are for aerodynamically smooth airfoils at an effective Reynolds Number of about 8 million. When it is desired to find the value for some other Reynolds Number the following formula may be used -

$$c_{d_{o_{min}}} = \left(c_{d_{o_{min}}}\right)_{std} \left(\frac{R_{std}}{R}\right)^{0.11} \qquad (6.2)$$

in which R_{std} is the Reynolds Number given in the table containing $(c_{d_{o_{min}}})_{std}$ and is approximately 8,000,000. Since Reynolds Number is given by

$$R = \frac{\rho VL}{\mu} \qquad (6.3)$$

in which V is the velocity in ft/sec, L is the chord in feet, and μ is a function of temperature— ρ/μ being 6378 for standard air—it is seen that, in a tapered wing, the Reynolds Number at any speed for any section is proportional to the chord of that section. This means that, in a wing tapered 3 to 1, the relative value of $c_{d_{o_{min}}}$ is increased approximately 13 percent at the tip. Since the minimum profile drag has, in most cases, a relatively unimportant effect on the loads in the structure, this scale effect may usually be neglected. (See Paragraphs 7.18 and 7.192).

6.12 <u>Additional Profile Drag</u> - The increase in profile drag coefficient, Δc_{d_o}, accompanying an increase in lift coefficient may be given the same span distribution as that of either c_{d_i} or $c_{d_{o_{min}}}$, since its value is relatively small and, in accelerated flight, uncertain. It may, if desired, be calculated for several sections along the span for any given value of lift coefficient using the generalized variation of

$$c_{d_o} \text{ with } \frac{c_l - c_{l\,opt}}{c_{l\,max} - c_{l\,opt}}$$

presented in Figure 6-1, which has been taken from Figure 45 of reference 16, and in which $c_{l\,opt}$, the optimum lift coefficient, is the lift coefficient at which the profile drag coefficient is a minimum. This is illustrated in Paragraph 7.193 of the first example in Chapter 7.

6.2 Aerodynamic-Center Location.

6.20 It has been described how to obtain the magnitude and direction of all the resultant forces acting on the various sections along the span of a wing. Before these forces can be applied to the structure, it is necessary to know their points of application, which are assumed to be the aerodynamic centers of the sections. The location of the aerodynamic center is given under "Fundamental Section Characteristics" in all recent N.A.C.A. publications on airfoil characteristics, such as reference 15. In tapered wings where the location for only one thickness ratio is known, its location for other sections may be estimated by means of Figure 54 of reference 18, or from Figure 52 of reference 10. The location of the aerodynamic center for any particular section remains the same regardless of the location or angle of attack of the section, except that for some sections, it is necessary to use a slightly different location for the negative lift range. This can be avoided, however, by slightly modifying the moment coefficient in this range. (See Paragraph 6.32). The aerodynamic center of a section with flap is not necessarily the same as for the plain section, although it may be convenient to assume it to be the same and modify the moment coefficient to correct for the error involved.

6.3 Moment-Coefficient Distribution.

6.30 Source - Now after the resultant forces have all been determined and applied to the sections at their aerodynamic centers there remains a couple that must be applied to each section in addition. This is accomplished by means of the section moment coefficient, which is a characteristic of the section and is independent of the section location and angle of attack, since the aerodynamic center by definition is the point about which the moment coefficient is constant. Its value is given under "Fundamental Section Characteristics" in all recent N.A.C.A. publications on airfoil characteristics, such as reference 15. Where its value is known for only one section of a wing that tapers in thickness ratio, the value of the moment coefficient for the other thickness ratios may be estimated by means of Figure 88 of reference 6. When it is desired to estimate the moment coefficient for a section with flap, it may be found by adding to the moment coefficient for the plain section the additional amount produced by the flap as given in references 10 and 13.

6.31 Effect of Part-Span Flap - When part-span flaps are used, there is some tapering effect near the end of the flap as shown in Figure 11 of reference 9. The moment coefficient for the plain section is slightly larger and that for the section with flap is slightly smaller near the end of the flap. This appears to have only a local effect, the total moment on the wing being approximately the same as though the moment changed abruptly at the end of the flap.

6.32 Effect of Applying Forces at Point Other than at Aerodynamic Center - As previously explained, the moment coefficient about the aerodynamic center is independent of lift and, therefore, not affected by the distribution of the lift coefficient along the span. If for some reason it is desired to apply the forces at some point other than at the true aerodynamic center, the moment coefficient then becomes a function of lift and its distribution along the span is consequently affected by the span distribution of lift. Where it is desired to use an aerodynamic center only slightly different from the true location, as when the location for positive lift is used also for the negative lift range, or when the center for the plain section is used for the section with flap, it is not usually necessary to consider the effect on the distribution of moment coefficient although its effect on the magnitude of the moment should be considered.

6.33 Biplane Interference - There is one other exception to the statement that the moment coefficient of a section is independent of its location. In a biplane the moment on the upper wing is made more positive due to interference, while that of the lower wing is increased negatively an equal amount, so that the net result on the biplane is negligible. This increment is distributed along the entire span of the shorter wing but only along that portion of the span of the longer wing that lies within the projected span of the shorter wing. Its value is a linear function of the thickness-gap ratio and is given for the upper wing by

$$\Delta c_{m_{o}(t/G)_U} = 0.1 \left(\frac{t}{G} \right) \tag{6.4}$$

where t is the thickness of the lower wing and G is the average gap for the portion of the span affected. The increment for the lower wing is given by

$$\Delta c_{m_{o}} \left(\frac{t}{G}\right)_L = -\Delta c_{m_{o}} \left(\frac{t}{G}\right)_U \times \frac{S_U'}{S_L'} \times \frac{C_U'}{C_L'} \tag{6.5}$$

where S_U', C_U', S_L' and C_L' refer to the areas and chords of the portion of upper and lower wings involved, respectively.

6.34 Compressibility Effect - Although the moment coefficient is practically independent of Reynolds Number for normal speeds, it changes considerably at extremely high speeds—as does the profile drag—because of the compressibility effect. The speed at which this effect becomes appreciable depends on the local velocity over the section and, therefore, on the angle of attack. The effect of compressibility may usually be neglected unless the airplane is designed for a terminal velocity exceeding 400 m.p.h., in which case the latest information from the N.A.C.A. high speed wind tunnel should be consulted.

57882 O—38——10

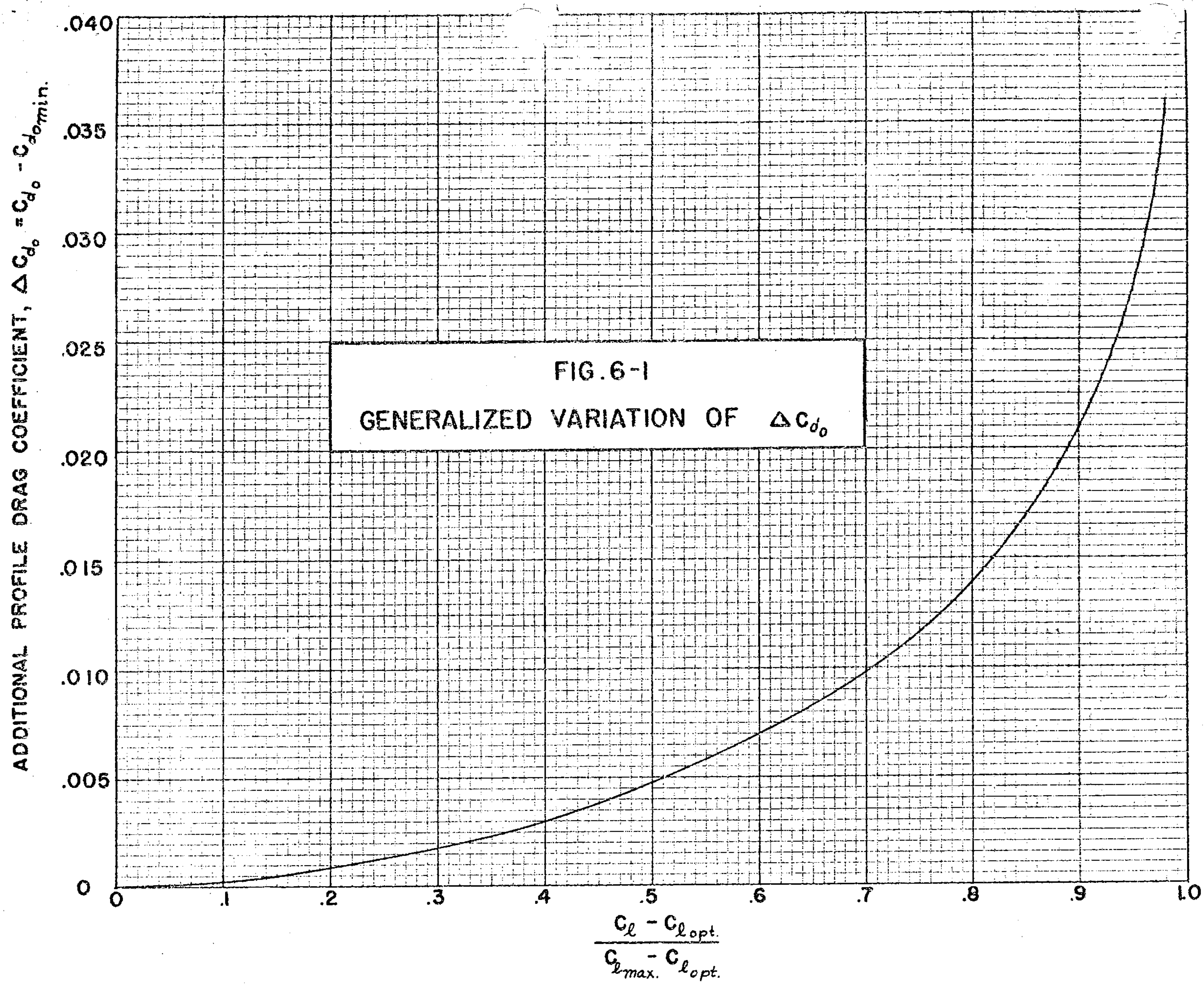

FIG. 6-1

GENERALIZED VARIATION OF Δc_{d_o}

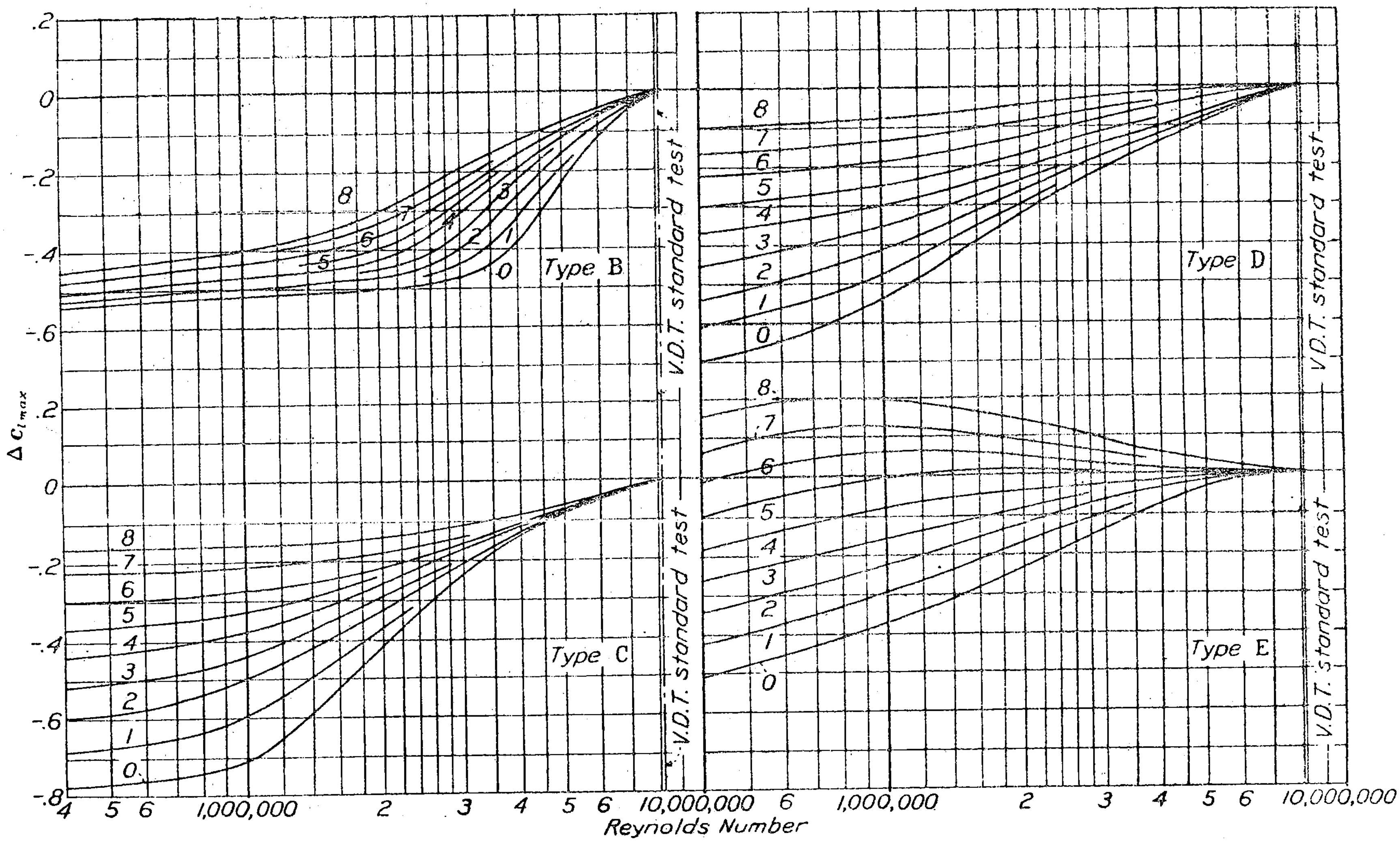

FIGURE 6-2.—SCALE EFFECT CORRECTIONS FOR $c_{l_{max}}$. IN ORDER TO OBTAIN THE SECTION MAXIMUM LIFT COEFFICIENT AT THE DESIRED REYNOLDS NUMBER, APPLY TO THE STANDARD-TEST VALUE THE INCREMENT INDICATED BY THE CURVE THAT CORRESPONDS TO THE SCALE-EFFECT DESIGNATION OF THE AIRFOIL.

CHAPTER 7

EXAMPLES

EXAMPLES.

7.0 General Description of Wing - In order to illustrate the methods described in this bulletin, a typical monoplane wing will be used as an example. The wing chosen is shown in Figure 7-1. The wing tapers linearly both in plan form and thickness. The N.A.C.A. Series 230 airfoil is used with a thickness ratio varying from approximately 18 to 8 percent. The section characteristics of this airfoil taken from reference 15 are plotted versus thickness ratio in Figure 7-2. The wing is so laid out that all zero lift chords, with flaps closed and ailerons neutral, are parallel. This means that under these conditions the wing has no aerodynamic twist. The wing is equipped with part-span simple split flaps that may be opened at any angle from zero to 60 degrees. Simple ailerons, differentially operated, extend from the end of the flaps to the tips, the maximum throw being 30 degrees up and 6 degrees down. The 10 percent chord points are on a line perpendicular to the plane of symmetry which gives the line of aerodynamic centers a negative sweepback of approximately 2 degrees. This and the dihedral are assumed to have a negligible effect on the distribution of loads along the span.

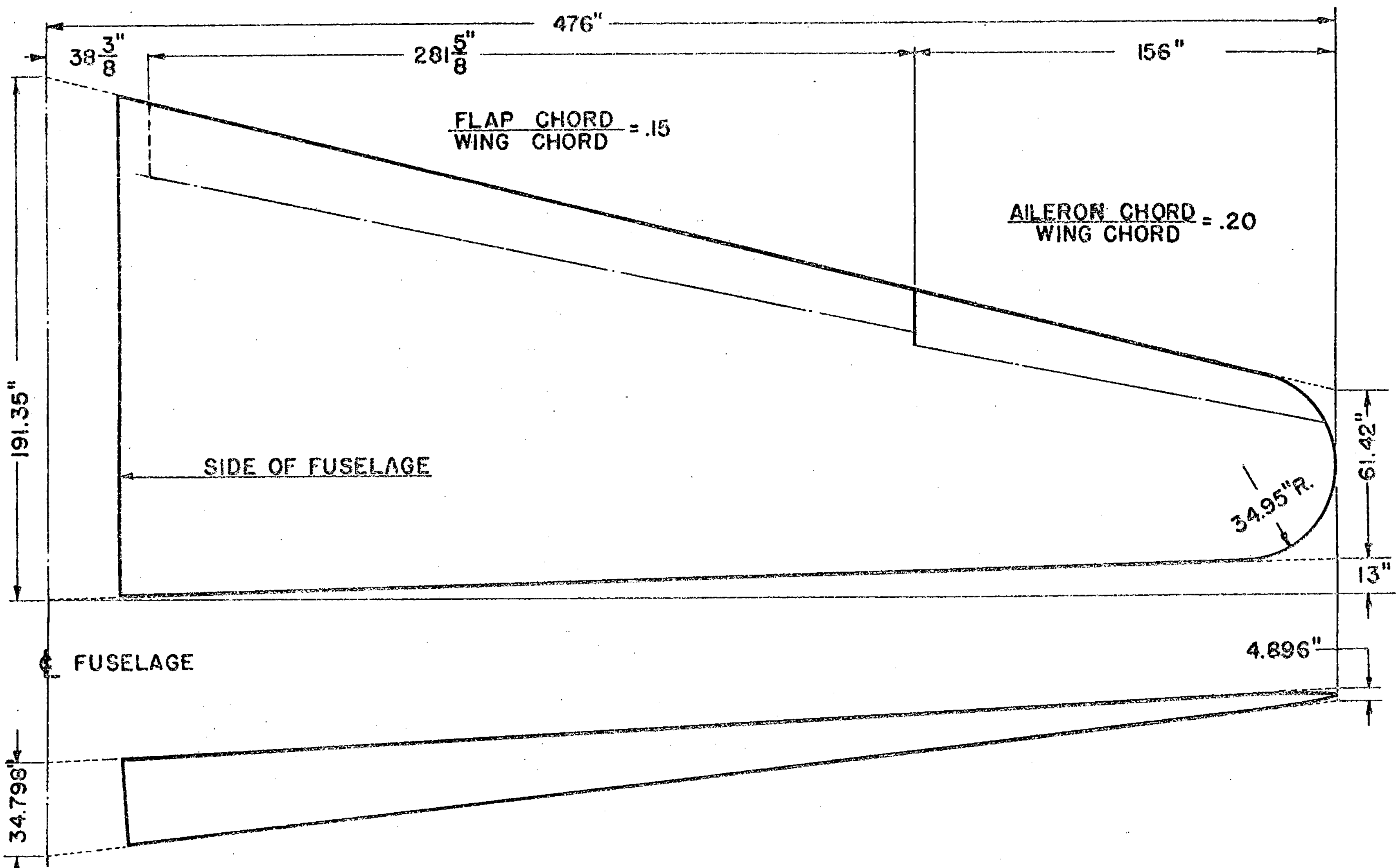

FIG. 7-1 SKETCH OF WING USED IN EXAMPLES

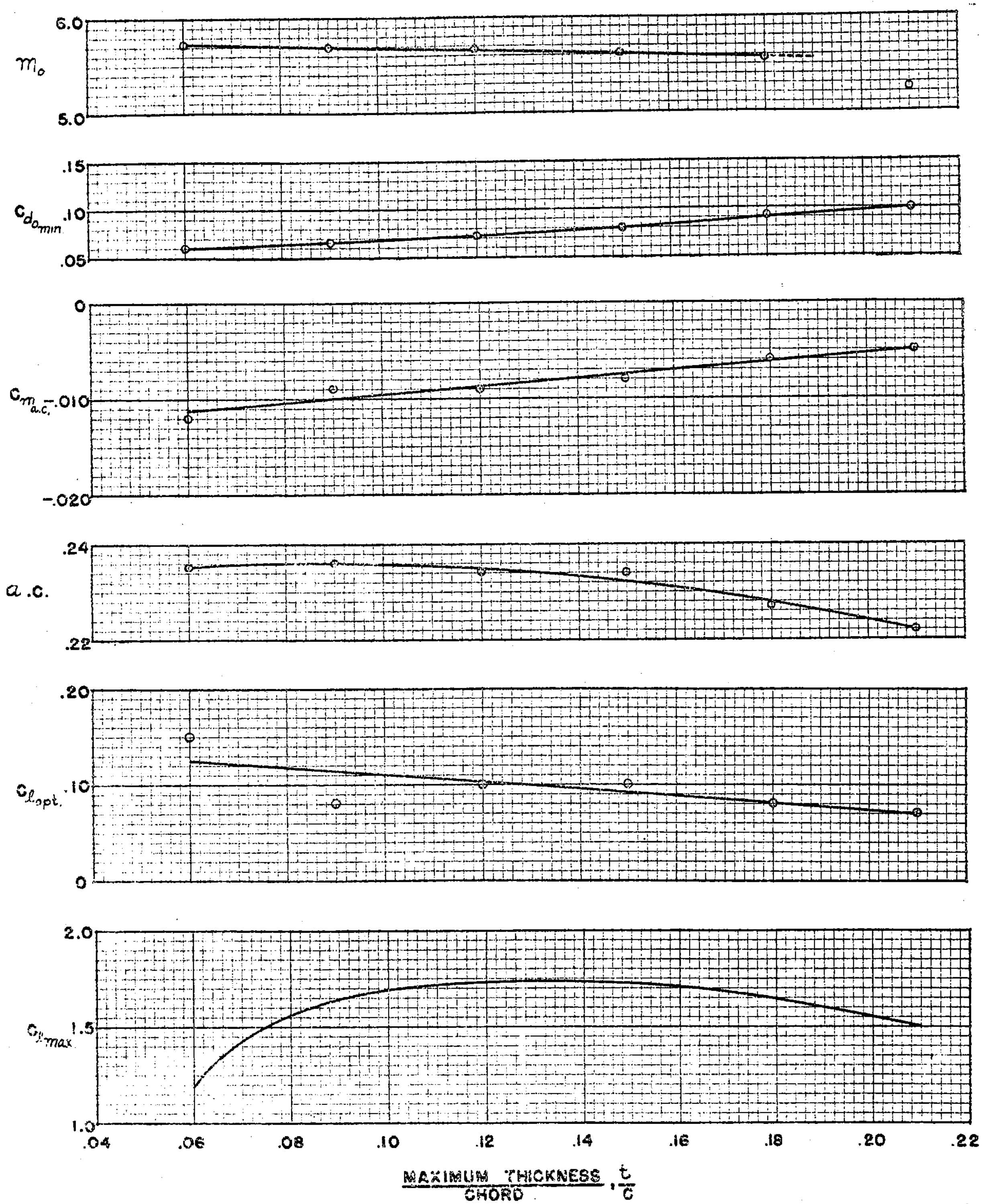

FIG. 7-2 AIRFOIL SECTION CHARACTERISTICS

7.1 EXAMPLE 1 - SPECIAL METHOD.

7.10 Description - It will at first be assumed that the distribution of the loads is desired for the basic wing only, with flaps closed and ailerons neutral. An examination will show that for this condition the wing meets the requirements of Section 2.0 (a) and (b) of Chapter 2, and the special method described in this chapter may, therefore, be used. Since the thickness ratio, and consequently m_0, varies along the span, the method of Section 2.2 will be followed, in which an equivalent wing having a constant m_0 is substituted for the actual wing for the purpose of reading $c_{l\,a_1}$ from the curves.

7.11 Tabulation of Wing Section Characteristics - The first step is to determine the characteristics of this equivalent wing. Table IX has been prepared as a convenient form in which to make most of the computations required by this method. Line 1 serves to designate the columns and represents several points chosen along the semispan so as to correspond to the points for which $c_{l\,a_1}$ curves are given in Figures 2-1, 2-2 and 2-3. Line 2 represents the actual distance of these points from the plane of symmetry in inches. Line 3 represents the maximum thickness and line 4 the chords of these sections in inches. It should be noted that, although the wing ends at the edge of the fuselage, it is assumed to extend to the plane of symmetry, holding the same taper both in thickness and plan. At points near the tip are given both the true thicknesses and chords, and the thicknesses and chords of a wing with square tip with the main taper both in thickness and plan extended to the extreme tip. In cases where the taper in thickness is not uniform as it is here, a curve may be extrapolated to the center and to the tip in order to determine these values for the extended wing. In line 5 is calculated the value of t/c for the several sections of the extended wing. In Figure 7-2, in the plot of m_o versus t/c, a straight line is seen to represent quite well the variation of m_o with t/c for the range of thickness ratios used in the wing. If the wing root had been much thicker, it would have been necessary to use a curve because m_o appears to decrease rather rapidly at thickness ratios greater than 18 percent. The values of m_o are read from this curve for the values of t/c in line 5 and entered in line 6. If this curve were not a straight line, it would be necessary to compute m_oc in line 7 for all points of the extended wing and on a plot of m_oc versus y approximate the curve with a straight line. Where this straight line intersected the center line and the tip would be read the value of $m_s'c_s$ and $m_t'c_t$, respectively. Since in this example m_o versus t/c is a straight line, and since c varies linearly with y, m_oc also varies linearly along the span and it is necessary to compute only $m_s c_s$ in column 0 and $m_t c_t$ in column 100. Each value in line 8 is found by dividing the corresponding value of m_o in line 6 by m_s' which is found by dividing $m_s' c_s$ by c_s. In this case, since $m_s' = m_s$, it means dividing by the value of m_o in column 0.

7.12 <u>Wing Dimensions</u> - Before the next line can be filled in, it is necessary to obtain and list the various dimensions of the wing in accordance with Section 2.2.

b = 952 inches.

S = 830.1 square feet or 119,535 square inches.

c_t = 61.42 inches.

c_s = 191.35 inches.

$m_s' = m_s = 5.574$

$m_t' = m_t = 5.706$

$\lambda = 61.42/191.35 = 0.3210$

$\lambda' = m_t\, c_t/m_s\, c_s = 350.5/1066.6 = 0.3286$

$A = (952)^2/119535 = 7.582$

$A' = 2b/c_s(1+\lambda') = 2 \times 952/191.35 \times 1.3286 = 7.489$

$A'/m_s' = 7.489/5.574 = 1.344$

$A'' = A \times (1+\lambda)/1 + \lambda') = 7.582 \times 1.3210/1.3286 = 7.539$

$A''/m_s' = 7.539/5.574 = 1.353$

$Y = \frac{(34.00 + 26.61)}{2}/476 = 30.305/476 = 0.0637$

7.13 <u>Lift Coefficient</u> - For $\lambda' = .3286$, and $A'/m_s = 1.344$

of the equivalent square-tip wing, the values of $c_l{}'_{a_1}$ are found from Figures 2-1, 2-2 and 2-3 and entered in line 9 of Table IX. These are made to apply to the actual extended wing by multiplying each value in line 9 by the corresponding value of m_o/m_s in line 8, which product is entered in line 10. Line 11 is next filled in by multiplying each c_{l_a} value in line 10 by the corresponding chord in line 4. The true chords of the actual rounded-tip wing are used in this case. In order to obtain the corresponding wing lift coefficient, it is necessary to find the integral of c c_{l_a} over the semispan, which is accomplished here by means of a modification of Simpson's Rule, the multiplying factors being listed in line 12. The values of c c_{l_a} in line 11 are multiplied by these factors in line 12 and the results entered in line 13. This integration may be performed graphically, if preferred, by plotting c c_{l_a} versus fraction of semispan. In either case the wing lift coefficient is found in line 14, the area under the curve being substituted for Σ (13)/3 in case the graphical method is used. The result shows that the c_{l_a} values of line 10 are slightly too large to give a C_L value of 1.0, and the true $c_{l_{a_1}}$ values are found in line 15 by dividing all the c_{l_a} values in line 10 by 1.0078. These $c_{l_{a_1}}$ values are plotted in Figure 7-3.

7.14 <u>Lateral Center of Pressure</u> - The lateral center of pressure of the square-tip wing is found from Figure 2-6, for $\lambda' = .3286$ and $A'/m_s' = 1.344$, to be $.418 \times 476 = 199.0$ inches from the center line. This is approximate, of course, because it does not allow for the effect of tip rounding or for the empirical tip correction. That it is sufficiently accurate for checking purposes, however, may be seen by comparing it with the value found in Table VI of example 2.

7.15 Angle of Attack - Before the induced-drag distribution can be found, it is necessary to determine the angle of attack corresponding to $C_L = 1.0$. For $\lambda' = .3286$ and $\gamma = .0637$ a point is located on Figure 2-4. From this point, a straight line is drawn to a point on the A/m_o scale equal to $A''/m_s' = 1.353$. This line crosses the τ scale at .019 which is the value of τ' for the equivalent round tip wing. Then

$$m' = \frac{m_s'}{1 + \frac{m_s'}{\pi A''}(1 + \tau')} = \frac{5.574}{1 + \frac{5.574 \times 1.019}{3.1416 \times 7.539}} = \frac{5.574}{1.2398} = 4.496$$

$$m = 4.496 \times \frac{1.3286}{1.3210} = 4.522$$

For $C_L = 1.0$, the absolute angle of attack in radians is

$$\alpha_a = \frac{1}{4.522} = .2212$$

7.16 Induced Drag - This α_a value is entered in all columns of line 16. The corresponding effective angles of attack are found in line 17 by dividing the $c_{l\,a_1}$ values in line 15 by the respective values of m_o in line 6. The downwash angle w/V is found in line 18 by subtraction. In line 19 is calculated the induced-drag coefficient $c_{d_{ia_1}} = \frac{w}{V} \; c_{l\,a_1}$ by multiplying line 15 by line 18. These values represent the induced-drag coefficient when the lift coefficient for the wing $C_L = 1.0$. If it is desired to know the distribution also when the wing induced-drag coefficient $C_{D_i} = 1.0$, this can be found by dividing the $c_{d_{ia_1}}$ values in line 19 by the wing coefficient $C_{D_{ia_1}}$ when $C_L = 1$, where

$$C_{D_{ia_1}} = \frac{(1 + \sigma)}{\pi A}$$

For $\lambda' = .3286$, $\gamma = .0637$ and $A''/m_s' = 1.353$, σ is found from Figure 2-5 to be approximately .012, in a manner similar to that used for finding τ' from Figure 2-4. Then

$$C_{D_{ia_1}} = \frac{1.012}{3.1416 \times 7.582} = 0.04249$$

7.17 Tip Correction.

7.170 Although this particular wing has sufficient taper to make the empirical tip correction negligible, the tip loading will be computed, nevertheless, in order to demonstrate the method.

$$C_{ave} = \frac{S}{b} = \frac{119535}{952} = 125.56$$

Affected tip distance = 0.4 x 125.56 = 50.22

The point at which the correction begins

$$y_o = \frac{b}{2} - 0.4\frac{S}{b} = 476 - 50.22 = 425.78$$

Then the percent of affected tip distance corresponding to each point in Table IX is calculated by

$$\frac{y - y_o}{0.4\ C_{ave}} \times 100 = \frac{y - 425.78}{50.22} \times 100$$

and entered in line 21. It is seen that only the last three points are affected.

Δc_{l_1} for each of these three points is read from Figure 2-7 and entered in line 22.

$F_2 = 1.0$ for $A' = 7.489$ from Figure 2-9

$F_3 = 0.080$ for $\lambda' = .3286$ from Figure 2-10

Line 22 can now be filled in and represents the Δc_l for this wing when the theoretical wing $C_L = 1.0$. This is plotted on Figure 7-3 and is seen to be very small for this wing.

7.171 In order to determine the total load on the wing, it is necessary to compute ΔC_{L_1} by means of equation 2.18. This is done by multiplying the values in line 23 by the actual chords in line 4 to obtain the values of $F_2 F_s \Delta c_{l_1} c$ in line 24, then multiplying the values in line 24 by the Simpson multipliers in line 25 to give the values of ΔC_{L_1} in line 26. The value of ΔC_L, in this case, is seen to be negligible.

7.172 The center of pressure of the additional tip load is taken from Figure 2-11 and listed in line 27 in percent of the chord from the leading edge.

7.18 Other Section Coefficients - In Figure 7-2, in addition to m_o, are plotted versus t/c the aerodynamic-center location, a.c., in percent of chord from the leading edge, the section moment coefficient, $c_{m_{a.c.}}$, and the minimum profile-drag coefficient, $c_{d_{o_{min}}}$; all taken from reference 15. The vertical location of the a.c. is given as 3 percent above the chord line for all thickness ratios. From these curves are read and tabulated in lines 28, 29 and 30 of Table IX the values of a.c., $c_{m_{a.c.}}$ and $c_{d_{o_{min}}}$, corresponding to the respective values of t/c given in line 5. The effective Reynolds Number at which these values are correct is about 8.3 millions. Although the effect of $c_{d_{o_{min}}}$ on the structure is usually too small to warrant taking the scale effect into account, it is done here to illustrate the method. The average chord 125.56 is divided by each chord in line 4 and the quotients raised to the 0.11 power and entered in line 31. The corrected distribution of $cd_{o_{min}}$ for a Reynolds Number of 8.3 millions based on the average chord is then found in line 32 by multiplying the values of $cd_{o_{min}}$, as taken from the curves in line 30 by the respective values in line 31.

7.19 APPLICATION TO A GIVEN ANGLE OF ATTACK:

7.190 Angle of Attack - Sufficient information is now available for obtaining the section coefficients for this wing at any angle of attack. In order to illustrate the method, it will be assumed that the distributions are desired for an absolute angle of attack of 15 degrees, corresponding, for this wing, to an angle of attack of 13.9° measured from the chord line.

7.191 Lift and Induced Drag - At this angle of attack

$$C_L = \frac{m}{57.3} \times 15 = \frac{4.522}{57.3} \times 15 = 1.184$$

$$F_1 = 1.400 \quad \text{from Figure 2-8 for } C_L = 1.184$$

$$\Delta C_L = F_1 \, \Delta C_{L_1} = 1.400 \times .00038 = .00053$$

$$C_{D_i} = C_{D_{ia_1}} \times C_L^2 = .04249 \times (1.184)^2$$

$$= .04249 \times 1.402 = .05957$$

Table X is used to make the section calculations. The first five lines are copied from Table IX as indicated. The section values of c_{l_a} corresponding to $C_L = 1.184$ are found in line 6 by multiplying the $c_{l_{a1}}$ values in line 2 by C_L. The section values of Δc_l for this C_L are found in line 7 by multiplying the values of $F_2 F_3 \Delta c_{l_1}$ of line 3 by F_1. The values of c_{d_i} for this C_L are found in line 8 by multiplying the values of $c_{d_{ia1}}$ in line 4 by $C_L{}^2$, or they could have been found by multiplying the relative values of $c_{d_{i1}}$ in line 20 of Table IX by $C_{Di} = .05957$.

7.192 Minimum Profile Drag - It will now be further assumed that a V-g diagram for this airplane indicates that the limit load for the angle of attack of 15 degrees occurs at a speed of 150 miles per hour. At this speed the Reynolds Number based on the average wing chord and in standard air is

$$R = \frac{\rho}{\mu} VL = 6378 \times 150 \times \frac{88}{60} \times \frac{125.56}{12} = 14,680,000$$

The minimum profile-drag coefficients for this condition are found in line 9 by multiplying the values of $c_{d_{o_{min}}}$ for a Reynolds Number of 8.3 millions in line 5 by

$$\left(\frac{R_{std}}{R}\right)^{0.11} = \left(\frac{8.3}{14.68}\right)^{0.11} = .939$$

If $C_{D_{o_{min}}}$ for the wing is desired, it may be found by multiplying these values in line 9 by their respective chords and integrating over the semispan as was done to find C_L'.

7.193 <u>Additional Profile Drag</u> - The foregoing calculations account for all the air loads except the additional profile drag. There is evidence to indicate that, during a rapid change in angle of attack, considerable time is required for this additional profile drag to reach the value indicated by static tests. In this example, however, static conditions will be assumed, in order to illustrate the method. The correct values for dynamic loading would be less than the values herein calculated, but sufficient data are not yet available for accurately determining the additional profile drag. In any case, it would probably vary with rate of change of angle of attack. Fortunately, the effect of this on the wing structure is not great, and any reasonable assumption may be made without appreciable error. At first the approximate method will be demonstrated. From reference 16, Table II, this airfoil series is seen to have the D_2 type of scale effect on maximum lift. From Figure 6-2 the $\Delta c_{l\,max}$ is found to be +.08 by extrapolation for R = 14.68 million. In Figure 7-2 are also plotted $c_{l\,opt}$ from reference 15 and $c_{l\,max}$ from reference 16, for a Reynolds Number of approximately 8.3 millions. It will be noted that the $c_{l\,max}$ values are slightly larger than the $C_{L_{max}}$ values given in reference 15. An average value of $c_{l_{opt}}$ will be assumed to be 0.10 and an average value for $c_{l_{max}}$, from the curve, of 1.70 which becomes 1.78 with the $\Delta c_{l_{max}}$ of 0.08 added. Then

$$\frac{C_L - C_{L_{opt}}}{C_{L_{max}} - C_{L_{opt}}} = \frac{1.184 - .10}{1.78 - .10} = \frac{1.084}{1.68} = .645$$

From Figure 6-1 $\Delta C_{D_o} = .0080$

$$C_{D_i} = \Delta C_{D_o} = .05957 + .0080 = .06757$$

$$(C_{D_i} + \Delta C_{D_o})/C_{D_i} = .06757/.05957 = 1.134$$

The values of c_{d_i} in line 8 are then multiplied by 1.134, and the products entered in line 10, to represent the sum of c_{d_i} and Δc_{d_o}. This is, of course, an approximation and in cases where the c_{d_i} distribution curve is very irregular as it some times is, it would be better to add Δc_{d_o} to the minimum profile drag instead, giving it either the same distribution as $c_{d_{o_{min}}}$ or keeping its value constant along the span, whichever is preferred. Line 11 represents the total section drag coefficients.

In order to illustrate a more precise method of handling the distribution of Δc_{d_o}, lines 12 to 22 have been added in which Δc_{d_o} is given a separate distribution. The ratio of the respective chords to the average chord is computed and entered in line 12. The Reynolds Number for each section is then found by multiplying these ratios in line 12 by the Reynolds Number computed for the average chord at the given speed, or 14.68 million. For these values of R the values of $\Delta c_{l_{max}}$ are read from Figure 6-2 and entered in line 14, extrapolating the curves where necessary. The $c_{l_{max}}$ values for the standard value of Reynolds Number are read from the curve in Figure 7-2 for the respective values of t/c and entered in line 15. The corrected values of $c_{l_{max}}$ in line 16 are found by adding the corrections in line 14 to the standard values in line 15. The $c_{l_{opt}}$ values are read from the curve in Figure 7-2 and entered in line 17. Line 18 is found by subtracting each value of $c_{l_{opt}}$ in line 17 from the corresponding value of c_l in line 6. Line 19 is the difference between the respective value of $c_{l_{max}}$ and $c_{l_{opt}}$. Line 20 is the ratio of $(c_l - c_{l_{opt}})$ to $(c_{l_{max}} - c_{l_{opt}})$ and is the value for which Δc_{d_o} is read from Figure 6-1, and entered in line 21. Line 22 represents the total values of c_{d_i} and when these are compared with line 11 it is seen that, for all practical purposes, the approximate method is satisfactory. It should be noted, however, that the true shape of the Δc_{d_o} distribution in line 21 is quite different from that of c_{d_i} of line 8 and neither does it agree very well with the $c_{d_{o_{min}}}$ distribution. In cases where extreme accuracy is desired, it would be better to use the more refined method as illustrated by lines 12 to 22. Where curves of c_{d_o} versus c_l for the wing section used are available, $\Delta c_{d_o} = c_{d_o} - c_{d_{o_{min}}}$ may be taken directly from the curves, if desired.

TABLE IX
APPLICATION OF SPECIAL METHOD
UNIT DISTRIBUTIONS

1	PERCENT OF SEMISPAN		0	20	40	60	80	90	95	97.5	100
2	(1) x b/200	y	0	95.2	190.4	285.6	380.8	428.4	452.2	464.1	476
3	FIG. 7-1	t	34.80	28.82	22.84	16.86	10.88	7.89	6.39 6.23	5.64 4.59	4.896 --
4	FIG. 7-1	c	191.35	165.36	139.38	113.39	87.41	74.41	67.92 66.25	64.67 52.54	61.42 0
5	(3)/(4)	t/c	.1819	.1743	.1639	.1487	.1245	.1060	.0941	.0873	.0797
6	FIG. 7-2	m_o	5.574	5.583	5.597	5.617	5.648	5.672	5.688	5.696	5.706
7	(4) x (6)	$m_o c$	1066.6								350.5
8	(6)/m'_s	m_o / m'_s	1.0000	1.0016	1.0041	1.0077	1.0133	1.0176	1.0205	1.0219	1.0237
9	FIGS. 2-1 TO 2-3	c'_{ℓ_a}	.900	.987	1.035	1.069	1.050	.968	.835	.665	0
10	(8) x (9)	c_{ℓ_a}	.900	.989	1.039	1.077	1.064	.985	.852	.680	0
11	(4) x (10)	cc_{ℓ_a}	172.2	163.5	144.8	122.1	93.0	73.3	56.4	35.7	0
12		f	.2	.8	.4	.8	.3125	.3375	.025	.10	.025
13	(11) x (12)	fcc_{ℓ_a}	34.44	130.80	57.92	97.68	29.06	24.74	1.41	3.57	0
14	$C'_L = \frac{2}{S}\int_0^{b/2} cc_{\ell_a} dy = \frac{b}{S}\frac{\Sigma (13)}{3} = \frac{952}{119535} \times \frac{379.62}{3} = 1.0078$										
15	(10) / C'_L	$c_{\ell_{a_1}}$	.893	.981	1.031	1.069	1.056	.977	.845	.675	0
16	$\frac{1}{m}$	α_a	.2212	.2212	.2212	.2212	.2212	.2212	.2212	.2212	.2212
17	(15)/(6)	α_o	.1602	.1757	.1842	.1903	.1870	.1722	.1486	.1185	0
18	(16) - (17)	w/V	.0610	.0455	.0370	.0309	.0342	.0490	.0726	.1027	-
19	(15) X (18)	$c_{d_{i a_1}}$	.0545	.0446	.0381	.0330	.0361	.0479	.0613	.0693	-
20	(19) / $C_{D_{i a_1}}$	$c_{d_{i_1}}$	1.283	1.050	.897	.777	.850	1.127	1.443	1.631	-
21	PERCENT OF AFFECTED TIP DISTANCE							5.22	52.61	76.30	100
22	FIG. 2-7	Δc_{ℓ_1}						.002	.140	.300	-
23	$F_2 F_3$ x (22)	$F_2 F_3 \Delta c_{\ell_1}$						.000	.011	.024	-
24	(4) x (23)	$F_2 F_3 \; c \Delta c_{\ell_1}$						.000	.729	1.261	-
25	(12) x (24)	$F_2 F_3 f c \Delta c_{\ell_1}$						.000	.0182	.1261	-
26	$\Delta C_{L_1} = \frac{2}{S}\int_0^{b/2} F_2 F_3 \; c \Delta c_{\ell_1} dy = \frac{b}{S}\frac{\Sigma (25)}{3} = \frac{952}{119535} \times \frac{.1443}{3} = .00038$										
27	FIG. 2-11	C.P.						.94	.74	.64	.55
28	FIG. 7-2	a.c.	.2275	.2288	.2302	.2323	.2347	.2357	.2360	.2360	.2359
29	FIG. 7-2	$c_{m_{a.c.}}$	-.0062	-.0065	-.0070	-.0078	-.0086	-.0094	-.0099	-.0101	-.0105
30	FIG. 7-2	$c_{d_{o_{min(STD)}}}$	.0092	.0088	.0085	.0080	.0073	.0069	.0066	.0065	--
31		$(C_{AVE}/c)^{.11}$	.954	.969	.988	1.011	1.041	1.059	1.073	1.101	--
32	(30) x (31)	$c_{d_{o_{min.}}}$	.0088	.0085	.0084	.0081	.0076	.0073	.0071	.0072	--

TABLE X

APPLICATION OF SPECIAL METHOD

DISTRIBUTIONS FOR A GIVEN ANGLE OF ATTACK

C_L = 1.184 $\qquad\qquad$ F_1 = 1.400

1	PERCENT OF SEMISPAN		0	20	40	60	80	90	95	97.5	100
2	TABLE IX LINE 15	$c_{\ell a_1}$	.893	.981	1.031	1.069	1.056	.977	.845	.675	0
3	TABLE IX LINE 23	$F_2F_3\Delta c_{\ell_1}$						.000	.011	.024	-
4	TABLE IX LINE 19	$c_{d_{i a_1}}$	.0545	.0446	.0381	.0330	.0361	.0479	.0613	.0693	0
5	TABLE IX LINE 27	$c_{d_{o\,min}}$	.0088	.0085	.0084	.0081	.0076	.0073	.0071	.0072	-
6	C_L X (2)	c_ℓ	1.057	1.162	1.221	1.266	1.250	1.157	1.000	.799	0
7	F_1 X (3)	Δc_ℓ						.000	.015	.034	-
8	C_L^2 X (4)	c_{d_i}	.0764	.0625	.0534	.0463	.0506	.0672	.0859	.0972	0
9	.939 X (5)	$c_{d_{o\,min}}$	.0083	.0080	.0079	.0076	.0071	.0069	.0067	.0068	-
10	1.134 X (8)	$c_{d_i} + \Delta c_{d_o}$	.0866	.0709	.0606	.0525	.0574	.0762	.0974	.1102	-
11	(9) + (10)	c_d	.0949	.0789	.0685	.0601	.0645	.0831	.1041	.1170	-
	ALTERNATIVE METHOD OF DETERMINING Δc_{d_o}										
12	TABLE IX (4)/$C_{AVE.}$	c / C_{AVE}	1.524	1.317	1.110	.903	.696	.593	.528	.418	0
13	R_{AVE} X (12)	$R / 10^6$	22.37	19.33	16.29	13.26	10.22	8.71	7.75	6.14	-
14	FIG. 6-2	$\Delta c_{\ell max.}$	.10	.10	.09	.07	.03	.01	-.01	-.05	-
15	FIG. 7-2	$c_{\ell max.}$	1.64	1.67	1.70	1.73	1.73	1.70	1.66	1.62	-
16	(14) + (15)	$c_{\ell max (corrected)}$	1.74	1.77	1.79	1.80	1.76	1.71	1.65	1.57	-
17	FIG. 7-2	$c_{\ell\,opt}$	.079	.082	.086	.091	.100	.107	.110	.114	-
18	(6) - (17)	$c_\ell - c_{\ell opt}$	.978	1.080	1.135	1.175	1.150	1.050	.890	.685	-
19	(16) - (17)	$c_{\ell max.} - c_{\ell opt.}$	1.66	1.69	1.70	1.71	1.66	1.60	1.54	1.46	-
20	(18) / (19)	$\dfrac{c_\ell - c_{\ell opt.}}{c_{\ell max.} - c_{\ell opt.}}$	.589	.639	.668	.687	.693	.656	.578	.469	-
21	FIG. 6-1	Δc_{d_o}	.0067	.0080	.0088	.0094	.0096	.0085	.0064	.0041	-
22	(8) + (9) + (21)	c_d	.0914	.0785	.0701	.0633	.0673	.0826	.0990	.1081	-

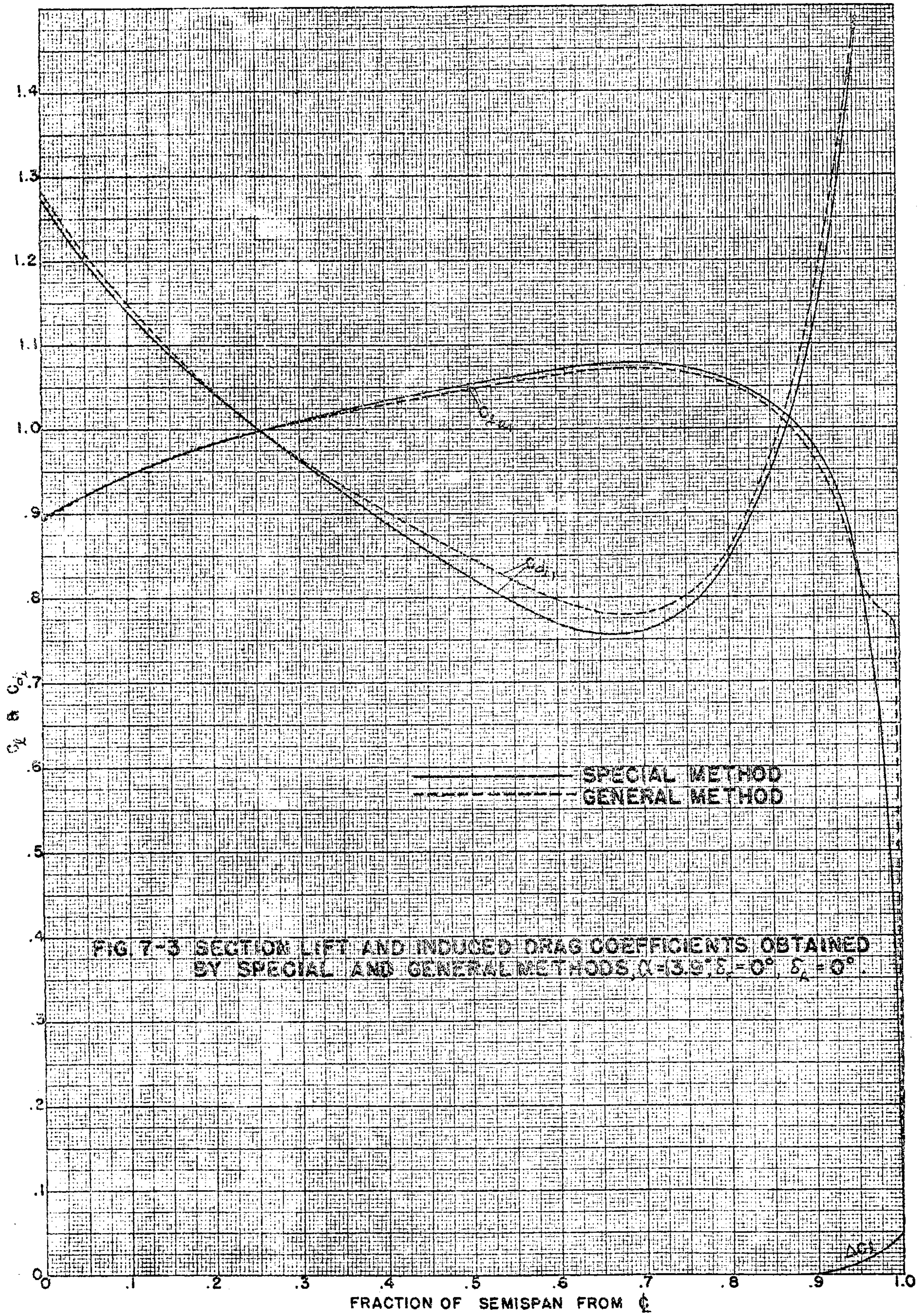

FIG. 7-3 SECTION LIFT AND INDUCED DRAG COEFFICIENTS OBTAINED BY SPECIAL AND GENERAL METHODS, $\alpha = 13.9°, \delta_f = 0°, \delta_A = 0°$.

7.2 EXAMPLE 2 - GENERAL METHOD - UNTWISTED WING.

7.20 Description - It will again be assumed that the distribution of the loads is desired for the basic wing only, with flaps closed and ailerons neutral. This time, however, instead of using the special method, as is permissible for this wing, the general method will be employed; since the illustration will be just as good as though it were applied to a wing of such characteristics that the special method could not be used, and it will also enable the results by the two methods to be compared.

7.21 Determination of Unit Coefficients - It is believed unnecessary to make many comments regarding this example, as Tables I to VII are computed in accordance with the procedure outlined in Chapter 3. The computations are made for an angle of attack of one radian. The necessary dimensions required in Table I are taken from Figure 7-1. Since λ is less than 0.5, the tip correction will not be applied and the true chords are used in line 9 of Table I. Since the basic distributions of this wing are zero, the c_{l_1} values in line 22 of Table VII are the $c_{l_{a_1}}$ values, and are plotted in Figure 7-3 in order to compare with the similar values obtained by the special method. It will be noted that the general method gives higher values at the extreme tip because of the use of the actual chords, while the curves of the special method were based on square tips. The relative c_{d_i} values in line 25 are also plotted in Figure 7-3 in order to compare with the relative values obtained by the special method.

7.22 Application to a Given Angle of Attack - The procedure for finding the distributions at a given angle of attack is similar to that illustrated by the special method (ref. 4.1). As before, it will be assumed that the c_l and c_{d_i} distributions are desired for an absolute angle of attack of 15 degrees. In Table VI the wing lift coefficient for an angle of attack of one radian is found to be 4.5215. Therefore,

$$m = \frac{C_L}{\alpha_a} = \frac{4.5215}{1.0} = 4.5215$$

Then at 15 degrees,

$$C_L = \frac{4.5215}{57.296} \times 15 = 4.5215 \times .26180 = 1.1837$$

The section values for $\alpha_a = 15^\circ$ can, therefore, be found by multiplying the values in line 22 of Table VII for $C_L = 1.0$ by 1.1837. Since the induced-drag coefficient for the wing when $C_L = 1.0$ may be found in Table VI as $\frac{(1 + \sigma)}{\pi A} = C_{D_{i_{a1}}} = .042478$, the coefficient for $\alpha_a = 15^\circ$ is

$$C_{D_i} = C_{D_{i_{a1}}} \times C_L^2 = .042478 \times \overline{1.1837}^2 = .05952$$

The section values for $\alpha_a = 15^\circ$ can now be found by multiplying the values in line 25 for $C_{D_i} = 1.0$ by .05952.

7.23 Tip Correction and Other Coefficients - The distribution of all other coefficients is accomplished in the same way as previously demonstrated for the special method. Since no tip correction is required, nothing more is needed in this simple example.

TABLE I.

GEOMETRIC CHARACTERISTICS OF WING

BASIC WING (NO TWIST)

1	a	b	0	1	2	3	4	5	6	7	8	9	10
2	FRACTION SEMI-SPAN	$\frac{2y}{b} = \cos\theta$	0	.1564	.3090	.4540	.5878	.7071	.8090	.8910	.9511	.9877	1.0000
3		θ	90	81	72	63	54	45	36	27	18	9	0
4		SIN θ	1.0000	.9877	.9511	.8910	.8090	.7071	.5878	.4540	.3090	.1564	. 0
5	(2) x $\frac{b}{2}$	y	0	74.45	147.08	216.10	279.79	336.58	385.08	424.12	452.72	470.15	476
6	ABSOLUTE IN RADIANS	α_a	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
7	(6) x (4)	α_a SIN θ	1.0000	.9877	.9511	.8910	.8090	.7071	.5878	.4540	.3090	.1564	0
8	MAXIMUM THICKNESS	t	34.798	30.122	25.560	21.225	17.225	13.658	10.612	8.160	6.187	3.236	0
9	CHORD	c	191.35	171.03	151.20	132.36	114.98	99.48	86.24	75.58	65.89	38.71	
10	(8)/(9)	$\frac{t}{c}$	.1819	.1761	.1690	.1605	.1498	.1373	.1231	.1080	.0939	.0836	
11	$dc_l/d\alpha_o$	m_o	5.574	5.581	5.590	5.601	5.615	5.632	5.650	5.670	5.688	5.701	
12	$(11)_o/(11)_n$	m_s/m_o	1.0	.9987	.9971	.9952	.9927	.9897	.9865	.9831	.9800	.9777	
13	$(9)_o/(9)_n$	c_s/c	1.0	1.1188	1.2655	1.4457	1.6642	1.9235	2.2188	2.5318	2.9041	4.9432	
14	(12) x (13) x (4)	$\frac{m_s c_s}{m_o c}$ SIN θ	1.0	1.1036	1.2001	1.2819	1.3365	1.3461	1.2866	1.1300	.8794	.7559	

$b = 952'' = 12 \times 79.33$

$S = 119535 = 144 \times 830.1$

$A = \frac{b^2}{S} = 7.582$

$C_{AVE.} = S/b = 125.56$

$\lambda = \frac{61.42}{191.35} = .3210$

TABLE III
COMPUTATION OF PLAN FORM COEFFICIENTS, C_{2n}

1	TABLE I			y_9	.7559	y_8	.8794	y_7	1.1300	y_6	1.2866	y_5	1.3461	v_0	.5000	v_1	1.8595	v_2	2.0795
2	LINE 14	$\frac{y_0}{2}$	.5000	y_1	1.1036	y_2	1.2001	y_3	1.2819	y_4	1.3365			v_5	1.3461	v_4	2.6231	v_3	2.4119
3	①+②	v_0	.5000	v_1	1.8595	v_2	2.0795	v_3	2.4119	v_4	2.6231	v_5	1.3461	p_0	1.8461	p_1	4.4826	p_2	4.4914
4	①−②	w_0	-.5000	w_1	-.3477	w_2	-.3207	w_3	-.1519	w_4	-.0499			q_0	-.8461	q_1	-.7636	q_2	-.3224

5	MULTIPLIER	0	20	2 − 18 a	2 − 18 b	4	16	6 − 14 a	6 − 14 b	8	12	10
6	.3090			w_4 -.0154		q_2 -.1027	p_2 1.3878	$-w_2$ .0991		p_1 1.3851	$-q_1$ .2360	
7	.5878				w_3 -.0893				w_1 -.2044			
8	.8090			w_2 -.2594		q_1 -.6178	$-p_1$ -3.6264	$-w_4$ .0404		$-p_2$ -3.6335	$-q_2$ .2689	
9	.9511				w_1 -.3307				$-w_3$ .1445			
10	1.0000	$p_0+p_1+p_2$ 10.8201	$q_0-q_1+q_2$ -.4149	w_0 -.5000		q_0 -.8461	p_0 1.8461	w_0 -.5000		p_0 1.8461	q_0 -.8461	$w_0-w_2+w_4$ -.2292
11	TOTAL	10.8201	-.4149	-.7748	-.4200	-1.5666	-.3925	-.3605	-.0599	-.4023	-.3412	-.2292
12	a + b			-1.1948				-.4204				
13	a − b				-.3548				-.3006			
14	DIVIDE BY 5	2.1640	-.0830	-.2390	-.0710	-.3133	-.0785	-.0841	-.0601	-.0805	-.0682	-.0458
15	C_{2n}	$2C_0$	$2C_{20}$	C_2	C_{18}	C_4	C_{16}	C_6	C_{14}	C_8	C_{12}	C_{10}

CHECK: $\frac{2C_0 + 2C_{20}}{2} + C_2 + C_4 + C_6 + C_8 + C_{10} + C_{12} + C_{14} + C_{16} + C_{18} = 0$

TABLE IV

CONSTANTS REQUIRED IN SOLUTION OF A_n COEFFICIENTS

ITEM		
U_0	$\frac{m_s C_s}{4\,b}$	$\frac{5.574 \times 191.35}{4 \times 952} =$ .2801
$2P_1$	$2C_0 - C_2 + 2U_0$	2.1640 + .2390 + .5602 = 2.9632
$2P_3$	$2C_0 - C_6 + 6U_0$	2.1640 + .0841 + 1.6806 = 3.9287
$2P_5$	$2C_0 - C_{10} + 10U_0$	2.1640 + .0458 + 2.8010 = 5.0108
$2P_7$	$2C_0 - C_{14} + 14U_0$	2.1640 + .0601 + 3.9214 = 6.1455
$2P_9$	$2C_0 - C_{18} + 18U_0$	2.1640 + .0710 + 5.0418 = 7.2768
D	$C_2 - C_4$	-.2390 + .3133 = .0743
E	$C_2 - C_8$	-.2390 + .0805 = -.1585
F	$C_2 - C_{12}$	-.2390 + .0682 = -.1708
G	$C_2 - C_{16}$	-.2390 + .0785 = -.1605
H	$C_4 - C_6$	-.3133 + .0841 = -.2292
I	$C_4 - C_{10}$	-.3133 + .0458 = -.2675
J	$C_4 - C_{14}$	-.3133 + .0601 = -.2532
K	$C_6 - C_8$	-.0841 + .0805 = -.0036
L	$C_6 - C_{12}$	-.0841 + .0682 = -.0159
M	$C_8 - C_{10}$	-.0805 + .0458 = -.0347

TABLE V.
SOLUTION OF A_n COEFFICIENTS

	1	2	3a	3b	3c	4a	4b	4c
1	$(C_2 - C_4)A_3$	+.0743	---	-.0128	-.0115	---	-.0010	-.0009
2	$(C_4 - C_6)A_5$	-.2292	---	.0305	.0309	---	-.0070	-.0071
3	$(C_6 - C_8)A_7$	-.0036	---	.0007	.0009	---	---	---
4	$(C_8 - C_{10})A_9$	-.0347	---	.0043	.0043	---	-.0001	-.0001
5	1+2+3+4					---	-.0081	-.0081
6	$2B_1$ - 5	$2B_1$ = 2.0				2.0000	2.0081	2.0081
7	A_1 = 6/$2P_1$	$2P_1$ = 2.9632				.6749	.6777	.6777
8	$(C_2 - C_4)A_1$	+.0743	.6749	.6777	.6777	.0501	.0504	.0504
9	$(C_2 - C_8)A_5$	-.1585	---	.0305	.0309	---	-.0048	-.0049
10	$(C_4 - C_{10})A_7$	-.2675	---	.0007	.0009	---	-.0002	-.0002
11	$(C_6 - C_{12})A_9$	-.0159	---	.0043	.0043	---	-.0001	-.0001
12	8+9+10+11					.0501	.0453	.0452
13	$2B_3$ - 12	$2B_3$ = 0				-.0501	-.0453	-.0452
14	A_3 = 13/$2P_3$	$2P_3$ = 3.9287				-.0128	-.0115	-.0115
15	$(C_4 - C_6)A_1$	-.2292	.6749	.6777	.6777	-.1547	-.1553	-.1553
16	$(C_2 - C_6)A_3$	-.1585	-.0128	-.0115	-.0115	.0020	.0018	.0018
17	$(C_2' - C_{12})A_7$	-.1708	---	.0007	.0009	---	-.0001	-.0002
18	$(C_4 - C_{14})A_9$	-.2532	---	.0043	.0043	---	-.0011	-.0011
19	15+16+17+18					-.1527	-.1547	-.1548
20	$2B_5$ - 19	$2B_5$ = 0				.1527	.1547	.1548
21	A_5 = 20/$2P_5$	$2P_5$ = 5.0108				.0305	.0309	.0309
22	$(C_6 - C_8)A_1$	-.0036	.6749	.6777	.6777	-.0024	-.0024	-.0024
23	$(C_4 - C_{10})A_3$	-.2675	-.0128	-.0115	.0115	.0034	.0031	.0031
24	$(C_2 - C_{12})A_5$	-.1708	.0305	.0309	.0309	-.0052	-.0053	-.0053
25	$(C_2 - C_{16})A_9$	-.1605	---	.0043	.0043	---	-.0007	-.0007
26	22+23+24+25					-.0042	-.0053	-.0053
27	$2B_7$ - 26	$2B_7$ = 0				.0042	.0053	.0053
28	A_7 = 27/$2P_7$	$2P_7$ = 6.1455				.0007	.0009	.0009
29	$(C_8 - C_{10})A_1$	-.0347	.6749	.6777	.6777	-.0234	-.0235	-.0235
30	$(C_6 - C_{12})A_3$	-.0159	-.0128	-.0115	-.0115	.0002	.0002	.0002
31	$(C_4 - C_{14})A_5$	-.2532	.0305	.0309	.0309	-.0077	-.0078	-.0078
32	$(C_2 - C_{16})A_7$	-.1605	.0007	.0009	.0009	-.0001	-.0001	-.0001
33	29+30+31+32					-.0310	-.0312	-.0312
34	$2B_9$ - 33	$2B_9$ = 0				+.0310	.0312	.0312
35	A_9 = 34/$2P_9$	$2P_9$ = 7.2768				.0043	.0043	.0043

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TABLE VI

COMPUTATION OF WING LIFT AND DRAG COEFFICIENTS

1	2	3
$A_1 =$ .6777	$(A_1)^2 =$ .4593	$A_1/3 =$ .2259
$A_3 =$ -.0115	$3(A_3)^2 =$ .0004	$A_3/5 =$ -.0023
$A_5 =$ .0309	$5(A_5)^2 =$ .0048	$-A_5/21 =$ -.0015
$A_7 =$ .0009	$7(A_7)^2 =$ .0000	$A_7/45 =$ .0000
$A_9 =$ .0043	$9(A_9)^2 =$ .0002	$-A_9/77 =$ -.0001
	$\Sigma n A_n^2 =$ .4647	$\Sigma =$ .2220

$$1 + \sigma = \frac{\Sigma n A_n^2}{A_1^2} = \frac{.4647}{.4593} = 1.0118$$

$$C_L = \pi A u_0 A_1 = 3.1416 \times 7.582 \times .2801 \times .6777 = 4.5215$$

OR

$$\frac{\pi}{4} \frac{m_s c_s}{C_{AVE.}} A_1 = .7854 \; \frac{5.574 \times 191.35}{125.56} \times .6777 = 4.5214$$

$$C_{D_i} = \left(\frac{1+\sigma}{\pi A}\right) C_L^2 = C_{D_{i_1}} \times C_L^2 = \frac{1.0118}{3.1416 \times 7.582} \times \overline{4.5215}^2 = .042478 \times 20.444 = .8684$$

FOR WING WITH NO AERODYNAMIC TWIST, $C_{D_{i_1}} = C_{D_{i_{a_1}}}$

$$\bar{y}_L = \left(\frac{4}{\pi} \times \frac{\Sigma \text{COL. } 3}{A_1}\right)\frac{b}{2} = \frac{4 \times .2220}{3.1416 \times .6777} \times \frac{952}{2} = .41709 \times 476 = 198.53$$

TABLE VII

COMPUTATION OF LOAD DISTRIBUTIONS

$C_{D_i} = 0.8684$ $C_L = 4.5215$

1	a	b	0	1	2	3	4	5	6	7	8	9
2	TABLE I LINE 3	θ	90	81	72	63	54	45	36	27	18	9
3	TABLE I LINE 4	SIN θ	1.0000	.9877	.9511	.8910	.8090	.7071	.5878	.4540	.3090	.1564
4		SIN 3θ	-1.0000	-.8910	-.5878	-.1564	.3090	.7071	.9511	.9877	.8090	.4540
5		SIN 5θ	1.0000	.7071	0	-.7071	-1.0000	-.7071	0	.7071	1.0000	.7071
6		SIN 7θ	-1.0000	-.4540	.5878	.9877	.3090	-.7071	-.9511	-.1564	.8090	.8910
7		SIN 9θ	1.0000	.1564	-.9511	-.4540	.8090	.7071	-.5878	-.8910	.3090	.9877
8	$A_1 =$ .6777	A_1 SIN θ	.6777	.6694	.6446	.6038	.5483	.4792	.3984	.3077	.2094	.1060
9	$A_3 =$ - .0115	A_3 SIN 3θ	.0115	.0102	.0068	.0018	-.0036	-.0081	-.0109	-.0114	-.0093	-.0052
10	$A_5 =$ .0309	A_5 SIN 5θ	.0309	.0218	0	-.0218	-.0309	-.0218	0	.0218	.0309	.0218
11	$A_7 =$ .0009	A_7 SIN 7θ	-.0009	-.0004	.0005	.0009	.0003	-.0006	-.0009	-.0001	.0007	.0008
12	$A_9 =$ .0043	A_9 SIN 9θ	.0043	.0007	-.0041	-.0020	.0035	.0030	-.0025	-.0038	.0013	.0042
13	(8)+(9)+(10)+(11)+(12)	$\sum A_n \sin n\theta$	.7235	.7017	.6478	.5827	.5176	.4517	.3841	.3142	.2330	.1276
14	TABLE I [(11)$_0$ x (13)]	$\frac{m_s C_s}{C}$	5.574	6.236	7.054	8.058	9.276	10.722	12.368	14.112	16.187	27.553
15	(13) x (14)	c_ℓ	4.0328	4.3758	4.5696	4.6954	4.8013	4.8431	4.7505	4.4340	3.7716	3.5158
16	TABLE I LINE 11	m_0	5.574	5.581	5.590	5.601	5.615	5.632	5.650	5.670	5.688	5.701
17	TABLE I LINE 6	α_a	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
18	(15) / (16)	α_o	.7235	.7841	.8175	.8383	.8551	.8599	.8408	.7820	.6631	.6167
19	(17) - (18)	$\frac{w}{V}$	.2765	.2159	.1825	.1617	.1449	.1401	.1592	.2180	.3369	.3833
20	(19) x (15)	c_{d_i}	1.1151	.9447	.8340	.7592	.6957	.6785	.7563	.9666	1.2707	1.3476
21	TABLE I LINE 9	c	191.35	171.03	151.20	132.36	114.98	99.48	86.24	75.58	65.89	38.71
22	(15) / C_L	c_{ℓ_1}	.8919	.9678	1.0106	1.0385	1.0619	1.0711	1.0506	.9806	.8341	.7776
23	(22) x (21)	$c\,c_{\ell_1}$	170.67	165.52	152.80	137.46	122.10	106.55	90.60	74.11	54.96	30.10
24	(23) x (3)	cc_{ℓ_1} SIN θ	170.67	163.48	145.33	122.48	98.78	75.34	53.25	33.65	16.98	4.71
25	(20) / C_{D_i}	$c_{d_{i_1}}$	1.2841	1.0879	.9604	.8742	.8011	.7813	.8709	1.1131	1.4633	1.5518
26	(25) x (21)	$cc_{d_{i_1}}$	245.71	186.06	145.21	115.71	92.11	77.72	75.11	84.13	96.42	60.07
27	(26) x (3)	$cc_{d_{i_1}}$ SIN θ	245.71	183.77	138.11	103.10	74.52	54.96	44.15	38.20	29.79	9.39

CHECKS:

LINE 24: $\frac{\pi}{20}\frac{b}{S}\sum cc_{\ell_1}\sin\theta = \frac{\pi}{20 C_{AVE}}\left(\sum (24) - \frac{1}{2}(24)_0\right) = 1.0;$ .001251 (799.34) = 1.0000

LINE 27: $\frac{\pi}{20}\frac{b}{S}\sum cc_{d_{i_1}}\sin\theta = \frac{\pi}{20 C_{AVE}}\left(\sum (27) - \frac{1}{2}(27)_0\right) = 1.0;$ .001251 (798.85) = .9994

7.3 EXAMPLE 3 - WING WITH PART-SPAN FLAP

7.31 Untwisted Wing - It will next be assumed that the distribution of the loads is desired for the same wing with the part-span split flaps open. The normal procedure would be to first calculate the distributions for the wing with no twist, but since this has already been done in the preceding section (7.2) for the normal wing, it is not necessary to repeat it here. (See paragraph 4.211).

7.32 Twisted Wing - Since the ratio of flap chord to wing chord is constant throughout the span of the flap, it is convenient to assume an angle of attack of one radian ($k\delta = 1$) for the portion of the span covered by the flap and zero for the remainder of the span. If there were no air leakage between the inboard end of the flap and the fuselage on a high wing monoplane, it might be possible to assume that the flap extended across without a break. Since this is decidedly not the case here and since it is usually conservative for the wing to assume no flap across the fuselage, the central portion of the wing, including the fuselage, is assumed to have the characteristics of the basic wing. In filling in line 6 of Table I, the value of α_a is modified in columns 1 and 5 by interpolating between the midpoints at the bottom of the page. (See Paragraph 4.253). The remainder of the calculations are in accordance with the usual procedure. The B_n's are found in Table II. In computing the A_n's in Table V, the C_{2n} and $2P_n$ values are taken from Table IV for the normal untwisted wing. Table VI is computed as usual. In Table VII, only the c_l distribution is computed and checked, since the induced drag is more conveniently handled in Table VIII. (See Paragraph 4.212).

7.33 Unit Distributions - Table VIII is completely filled in in accordance with the procedure outlined in Paragraph 4.213. In taking information from previous computations, care must be exercised in distinguishing between those for the twisted and those for the normal or untwisted wing. (To assist in this, a letter (T) has been placed after the table number at the head of each table for the twisted wing). It should be noted that the value of C_{Di}, found at the bottom of the table by integrating the distributions, checks exactly the value computed in Table VI.

7.34 Application to a Given Dondition.

7.341 Determination of $k\delta$ - Sufficient information is now available for computing the lift and induced-drag distributions for any flap angle and for any angle of attack. It will be assumed that the wing and section characteristics are desired for this wing with the flap open 60 degrees and for an angle of attack of 13.9 degrees measured from the chord line of the normal wing section. The value of $k\delta$ for the flap will be obtained from the curves of Figure 4-1, in order to illustrate their use. Using the curves for split flap, for $E = 0.15$ and for $\delta_F = 60$,

$$k\delta = F_E \, F_{\delta_F} = .17 \times 1.02 = .1734$$

(See Sections 4.23 and 4.25)

7.342 Modification of Unit Distributions - The first step is to modify the basic distributions obtained in Table VIII for $k\delta = 1.0$, so that they will be correct for $k\delta = 0.1734$. This is accomplished in Table XI in lines 5, 6 and 7 by multiplying the values of c_{l_b} and $c_{d_{i_{a_1 b}}}$ by 0.1734, the ratio of the respective values of $k\delta$, and $c_{d_{i_b}}$ by $\overline{0.1734}^2 = 0.03007$—after first copying the values for $k\delta = 1.0$ in lines 2, 3 and 4 from Table VIII. The values for $c_{d_{i_{a_1}}}$ and $c_{l_{a_1}}$ are copied into lines 8 and 9, since they are independent of flap angle.

7.343 Wing Coefficients - Before the section values can be found for the angle of attack of 13.9 degrees, it is necessary to determine the wing lift coefficient for this angle with the flap open 60 degrees. Since the zero-lift angle for the normal section of this wing is -1.1 degrees, this corresponds to an angle of attack of 15 degrees when measured from the zero-lift chord. If the section at the plane of symmetry is used as a reference, the true absolute angle of attack of this section is 15 degrees or 15/57.3

= 0.2618 radian, since there is no flap at this section. In Table VIII, line 8, column 0, the value of α_s for zero lift for a flap $k\delta$ of 1.0 is -.6785, which becomes -.6785 x .1734 = -.1177 for a flap angle of 60 degrees. Then by Equation 4.17

$$C_L = 4.5215 \left[.2618 - (-.1177)\right] = 4.5215 \times .3795 = 1.716$$

which is the lift coefficient for this wing with flap open 60 degrees at this angle of attack. Also, by Equation 4.16

$$C_{D_i} = .1984 \times \overline{.1734}^2 - .00595 \times .1734 \times 1.716 + .042478$$

$$\times \overline{1.716}^2 = .0060 - .0018 + .1251 = .1293$$

7.344 Total Distributions.- The section values of c_l are now found in line 11 by adding to the values of c_{l_b} in line 5 the values of c_{l_a} found in line 10 for $C_L = 1.716$. Likewise, the values of $c_{d_{i_a}}$ are found in line 12 by multiplying the $c_{l_{a_1}}$ values in line 9 by C_L^2, and $c_{d_{i_{ab}}}$ in line 13 by multiplying $c_{d_{i_{a_1 b}}}$ of line 7 by C_L. The three partial distributions are then added to obtain the total c_{d_i} in line 14. These distributions can be found for any other angle of attack by first computing C_L and then repeating lines 10 to 14.

7.345 Alternative Method for Induced Drag Distribution -

The remainder of Table XI has been added to show an alternative method of obtaining the induced-drag distribution which may be preferred by many. It is especially advantageous where several types of twist are present in the same wing, as will be demonstrated later in another example. Line 15 is copied from Table I, line 9, while line 16 is copied from Table VIII, line 8. The angle of attack for zero lift for this particular flap angle is found in line 17 as indicated. The absolute angle of attack for each section is found in line 18 by adding an angle sufficient to give the reference section the specified value. Lines 19, 20 and 21 are used to compute the effective angle, induced angle, and induced-drag coefficient in the usual way. To obtain the c_{d_i} values for any other angle of attack for the same flap angle requires a repetition of lines 18 to 21, after first obtaining the correct values of c_l . When this method is used to compute the induced-drag distribution for each desired angle of attack, it is possible to omit all computations involving induced drag in Tables VI, VII and VIII.

7.346 Plot of Results - In Figure 7-4 are plotted the basic, additional, and total section lift, and the total induced-drag coefficients. It will be noted that all c_l curves are continuous, while the c_{d_i} curve is discontinuous at the ends of the flap. If more points had been coefficients is accomplished in the same manner as that used in Sections 7.18 and 7.19 for the special method. For that portion of the span covered by the flap, an additional minimum profile-drag coefficient must be added to that for the normal wing. From Figure 5 of reference 13 for $\delta_f = 60$ and $E = .15$

$$\Delta c_{d_{o_{min}}} = 0.112$$

Likewise, for this portion of the span, an additional moment coefficient must be added. In the absence of better data, this may be taken from Figure 7 of reference 13, where it is seen to vary with c_l . After an inspection of line 11 of Table XI, its value may be read for $\delta_F = 60^o$ and $C_L = 1.9$ as

$$\Delta c_{m_{a.c.}} = -0.21$$

TABLE I. (T)

GEOMETRIC CHARACTERISTICS OF WING

FLAP $k\delta = 1.0$

1	a	b	0	1	2	3	4	5	6	7	8	9	10
2	FRACTION SEMI-SPAN	$\frac{2y}{b} = \cos\theta$	0	.1564	.3090	.4540	.5878	.7071	.8090	.8910	.9511	.9877	1.0000
3		θ	90	81	72	63	54	45	36	27	18	9	0
4		SIN θ	1.0000	.9877	.9511	.8910	.8090	.7071	.5878	.4540	.3090	.1564	. 0
5		$k\delta$	0	1.0	1.0	1.0	1.0	0	0	0	0	0	0
6	ABSOLUTE IN RADIANS	α_a	0	.9856*	1.0000	1.0000	1.0000	.1944**	0	0	0	0	0
7	(6) x (4)	α_a SIN θ	0	.9735	.9511	.8910	.8090	.1375	0	0	0	0	0
8	MAXIMUM THICKNESS	t											
9	CHORD	c											
10	(8)/(9)	$\frac{t}{c}$											
11	$dc_l/d\alpha_o$	m_o											
12	$(11)_o/(11)_\eta$	m_s/m_o											
13	$(9)_o/(9)_\eta$	c_s/c											
14	(12)x(13)x(4)	$\frac{m_s c_s}{m_o c}$ SIN θ											

b = S = $C_{AVE.} = S/b =$

*Flap ends at $\theta = \cos^{-1}(38.375/476) = 85.37^\circ$; $(6)_1 = k\delta \; (85.37 - 76.5)/(85.5 - 76.5) = .9856 \; k\delta$

**Flap ends at $\theta = \cos^{-1}(320/476) = 47.75^\circ$; $(6)_5 = k\delta \; (49.5 - 47.75)/(49.5 - 40.5) = .1944 \; k\delta$

TABLE II. (T)

COMPUTATION OF ANGLE COEFFICIENTS, B_η

FLAP $k\delta$ = 1.0

TABLE I. LINE 7	$y_0/2$	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9
	0	.9735	.9511	.8910	.8090	.1375	0	0	0	0

$y_1 - y_3 - y_5 + y_7 + y_9 = \eta_1$	$y_0/2 - y_4 + y_8 = \eta_2$
.9735 − .8910 − .1375 + 0 + 0 = −.0550	0 − .8090 + 0 = − .8090

	MULTI-PLIER	1 − 19 a	1 − 19 b	3 − 17 a	3 − 17 b	5 − 15 a	5 − 15 b	7 − 13 a	7 − 13 b	9 − 11 a	9 − 11 b
1	.1564	y_9 0		$-y_3$ −.1394				$-y_7$ 0		y_1 .1523	
2	.3090		y_8 0		y_4 .2500				y_4 .2500		y_8 0
3	.4540	y_7 0		y_9 0				$-y_1$ −.4420		$-y_3$ −.4045	
4	.5878		y_6 0		$-y_2$ −.5591				y_2 .5591		$-y_6$ 0
5	.7071	y_5 .0972		y_5 .0972		η_1 −.0389		$-y_5$ −.0972		y_5 .0972	
	.8090		y_4 .6545		y_8 0				y_8 0		y_4 .6545
7	.8910	y_3 .7939		$-y_1$ −.8674				y_9 0		$-y_7$ 0	
8	.9511		y_2 .9046		y_6 0				$-y_6$ 0		$-y_2$ −.9046
9	.9877	y_1 .9615		y_7 0				y_3 .8800		y_9 0	
10	1.0000		$y_0/2$ 0		$-y_0/2$ 0		η_2 −.8090		$-y_0/2$ 0		$y_0/2$ 0
11	TOTAL	1.8526	1.5591	−.9096	−.3091	−.0389	−.8090	.3408	.8091	−.1550	−.2501
12	a + b	3.4117		−1.2187		−.8479		1.1499		−.4051	
13	a − b		.2935		−.6005		.7701		−.4683		.0951
14	(12) & (13) / 2.5	1.3647	.1174	−.4875	−.2402	−.3392	.3080	.4600	−.1873	−.1620	.0380
15	$2B_\eta$	$2B_1$	$2B_{19}$	$2B_3$	$2B_{17}$	$2B_5$	$2B_{15}$	$2B_7$	$2B_{13}$	$2B_9$	$2B_{11}$

CHECK: $2B_1 - 2B_3 + 2B_5 - 2B_7 + 2B_9 - 2B_{11} + 2B_{13} - 2B_{15} + 2B_{17} - 2B_{19} = 2y_0 = 2\alpha_s$

NOTE: IF α_a IS CONSTANT ALONG THE SPAN, $B_1 = \alpha_s$ & B_3 TO B_{19} ARE ZERO.

CHECK:
1.3647 + .4875 − .3392 − .4600 − .1620 − .0380 − .1873 − .3080 − .2402 − .1174 = .0001

TABLE V. (T)

SOLUTION OF A_n COEFFICIENTS

FLAP kδ = 1.0

	1	2	3a	3b	3c	4a	4b	4c
1	$(C_2 - C_4)A_3$	.0743	---	-.1328	.1303	---	-.0099	-.0097
2	$(C_4 - C_6)A_5$	-.2292	---	-.0508	-.0495	---	.0116	.0113
3	$(C_6 - C_8)A_7$	-.0036	---	.0679	.0675	---	-.0002	-.0002
4	$(C_8 - C_{10})A_9$	-.0347	---	-.0206	-.0206	---	.0007	.0007
5	1 + 2 + 3 + 4					---	.0022	.0021
6	$2B_1$ - 5	$2B_1$ = 1.3647				1.3647	1.3625	1.3626
7	A_1 = 6 / $2P_1$	$2P_1$ = 2.9632				.4605	.4598	.4598
8	$(C_2 - C_4)A_1$	.0743	.4605	.4598	.4598	.0342	.0342	.0342
9	$(C_2 - C_8)A_5$	-.1585	---	-.0508	-.0495	---	.0081	.0078
10	$(C_4 - C_{10})A_7$	-.2675	---	.0679	.0675	---	-.0182	-.0181
11	$(C_6 - C_{12})A_9$	-.0159	---	-.0206	-.0206	---	.0003	.0003
12	8 + 9 + 10 + 11					.0342	.0244	.0242
13	$2B_3$ - 12	$2B_3$ = -.4875				-.5217	-.5119	-.5117
14	A_3 = 13 / $2P_3$	$2P_3$ = 3.9287				-.1328	-.1303	-.1302
15	$(C_4 - C_6)A_1$	-.2292	.4605	.4598	.4598	-.1055	-.1054	-.1054
16	$(C_2 - C_8)A_3$	-.1585	-.1328	-.1303	-.1302	.0210	.0207	.0206
17	$(C_2 - C_{12})A_7$	-.1708	---	.0679	.0675	---	-.0116	-.0115
18	$(C_4 - C_{14})A_9$	-.2532	---	-.0206	-.0206	---	.0052	.0052
19	15 + 16 + 17 + 18					-.0845	-.0911	-.0911
20	$2B_5$ - 19	$2B_5$ = -.3392				-.2547	-.2481	-.2481
21	A_5 = 20 / $2P_5$	$2P_5$ = 5.0106				-.0508	-.0495	-.0495
22	$(C_6 - C_8)A_1$	-.0036	.4605	.4598	.4598	-.0017	-.0017	-.0017
23	$(C_4 - C_{10})A_3$	-.2675	-.1328	-.1303	-.1302	.0355	.0349	.0348
24	$(C_2 - C_{12})A_5$	-.1708	-.0508	-.0495	-.0495	.0087	.0085	.0085
25	$(C_2 - C_{16})A_9$	-.1605	---	-.0206	-.0206	---	.0033	.0033
26	22 + 23 + 24 + 25					.0425	.0450	.0449
27	$2B_7$ - 26	$2B_7$ = .4600				.4175	.4150	.4151
28	A_7 = 27 / $2P_7$	$2P_7$ = 6.1455				.0679	.0675	.0675
29	$(C_8 - C_{10})A_1$	-.0347	.4605	.4598	.4598	-.0160	-.0160	-.0160
30	$(C_6 - C_{12})A_3$	-.0159	-.1328	-.1303	-.1302	.0021	.0021	.0021
31	$(C_4 - C_{14})A_5$	-.2532	-.0508	-.0495	-.0495	.0129	.0125	.0125
32	$(C_2 - C_{16})A_7$	-.1605	.0679	.0675	.0675	-.0109	-.0108	-.0108
33	29 + 30 + 31 + 32					-.0119	-.0122	-.0122
34	$2B_9$ - 33	$2B_9$ = -.1620				-.1501	-.1498	-.1498
35	A_9 = 34 / $2P_9$	$2P_9$ = 7.2768				-.0206	-.0206	-.0206

TABLE VI (T)

COMPUTATION OF WING LIFT AND DRAG COEFFICIENTS

FLAP k_δ = 1.0

1	2	3
A_1 = .4598	$(A_1)^2$ = .2114	$A_1/3$ = .1533
A_3 = -.1302	$3(A_3)^2$ = .0509	$A_3/5$ = -.0260
A_5 = -.0495	$5(A_5)^2$ = .0123	$-A_5/21$ = .0024
A_7 = .0675	$7(A_7)^2$ = .0319	$A_7/45$ = .0015
A_9 = -.0206	$9(A_9)^2$ = .0038	$-A_9/77$ = .0003
	$\Sigma\, nA_n^2$ = .3103	Σ = .1315

$$1+\sigma = \frac{\Sigma n A_n^2}{A_1^2} = \frac{.3103}{.2114} = 1.4678$$

$$C_L = \pi A u_0 A_1 = 3.1416 \times 7.582 \times .2801 \times .4598 = 3.0677$$

OR

$$\frac{\pi}{4}\frac{m_s c_s}{C_{AVE.}} A_1 = \frac{.7854 \times 5.574 \times 191.35}{125.56} \times .4598 = 3.0676$$

$$C_{D_i} = \left(\frac{1+\sigma}{\pi A}\right) C_L^2 = C_{D_{i_1}} \times C_L^2 = \frac{1.4678}{3.1416 \times 7.582} \times 3.0677^2 = .5799$$

FOR WING WITH NO AERODYNAMIC TWIST, $C_{D_{i_1}} = C_{D_{i a_1}}$

$$\bar{y}_L = \left(\frac{4}{\pi} \times \frac{\Sigma \text{COL. 3}}{A_1}\right)\frac{b}{2} = \frac{4 \times .1315}{3.1416 \times .4598} \times \frac{952}{2} = .36414 \times 476 = 173.33$$

TABLE VII (T)

COMPUTATION OF LOAD DISTRIBUTIONS

C_{D_i} =   FLAP k_δ = 1.0 C_L = 3.0677

1	a	b	0	1	2	3	4	5	6	7	8	9
2	TABLE I LINE 3	θ	90	81	72	63	54	45	36	27	18	9
3	TABLE I LINE 4	SIN θ	1.0000	.9877	.9511	.8910	.8090	.7071	.5878	.4540	.3090	.1564
4		SIN 3θ	-1.0000	-.8910	-.5878	-.1564	.3090	.7071	.9511	.9877	.8090	.4540
5		SIN 5θ	1.0000	.7071	0	-.7071	-1.0000	-.7071	0	.7071	1.0000	.7071
6		SIN 7θ	-1.0000	-.4540	.5878	.9877	.3090	-.7071	-.9511	-.1564	.8090	.8910
7		SIN 9θ	1.0000	.1564	-.9511	-.4540	.8090	.7071	-.5878	-.8910	.3090	.9877
8	A_1 = .4598	A_1 SIN θ	.4598	.4541	.4373	.4097	.3720	.3251	.2703	.2087	.1421	.0719
9	A_3 = -.1302	A_3 SIN 3θ	.1302	.1160	.0765	.0204	-.0402	-.0921	-.1238	-.1286	-.1053	-.0591
10	A_5 = -.0495	A_5 SIN 5θ	-.0495	-.0350	0	.0350	.0495	.0350	0	-.0350	-.0495	-.0350
11	A_7 = .0675	A_7 SIN 7θ	-.0675	-.0306	.0397	.0667	.0209	-.0477	-.0642	-.0106	.0546	.0601
12	A_9 = -.0206	A_9 SIN 9θ	-.0206	-.0032	.0196	.0094	-.0167	-.0146	.0121	.0184	-.0064	-.0203
13	(8)+(9)+(10)+(11)+(12)	$\sum A_n \text{SIN } n\theta$	.4524	.5013	.5731	.5412	.3855	.2057	.0944	.0529	.0355	.0176
14	[TABLE I, (11)$_0$ x (13)]	$\frac{m_s c_s}{c}$	5.574	6.236	7.054	8.058	9.276	10.722	12.368	14.112	16.187	27.553
15	(13) x (14)	c_ℓ	2.5217	3.1261	4.0426	4.3610	3.5759	2.2055	1.1675	.7465	.5746	.4849
16	TABLE I LINE 11	m_0										
17	TABLE I LINE 6	α_a										
18	(15) / (16)	α_o										
19	(17) - (18)	$\frac{w}{V}$										
20	(19) x (15)	c_{di}										
21	TABLE I LINE 9	c	191.35	171.03	151.20	132.36	114.98	99.48	86.24	75.58	65.89	38.71
22	(15) / C_L	c_{ℓ_1}	.8220	1.0190	1.3178	1.4216	1.1657	.7189	.3806	.2433	.1873	.1581
23	(22) x (21)	$c\,c_{\ell_1}$	157.29	174.28	199.25	188.16	134.03	71.52	32.82	18.39	12.34	6.12
24	(23) x (3)	cc_{ℓ_1} SIN θ	157.29	172.14	189.51	167.65	108.43	50.57	19.29	8.35	3.81	.96
25	(20) / C_{D_i}	$c_{d_{i_1}}$										
26	(25) x (21)	$cc_{d_{i_1}}$										
27	(26) x (3)	$cc_{d_{i_1}}$ SIN θ										

CHECKS:

LINE 24: $\frac{\pi}{20}\frac{b}{S}\sum cc_{\ell_1} \text{ SIN } \theta = \frac{\pi}{20 c_{AVE}}\left(\sum (24) - \frac{1}{2}(24)_0\right) = 1.0$ .001251 x 799.36 = 1.0000

LINE 27: $\frac{\pi}{20}\frac{b}{S}\sum cc_{d_{i_1}} \text{ SIN } \theta = \frac{\pi}{20 c_{AVE}}\left(\sum (27) - \frac{1}{2}(27)_0\right) = 1.0$

TABLE VIII

COMPUTATION OF UNIT LOAD DISTRIBUTIONS

SINGLE SYMMETRICAL TWIST

$C_{L(T)} = 3.0677$ FLAP $k\delta = 1.0$ $C_L = m = 4.5215$

1	a	b	0	1	2	3	4	5	6	7	8	9
2	TABLE VII LINE 22	$C_{\ell a_1}$	.8919	.9678	1.0106	1.0385	1.0619	1.0711	1.0506	.9806	.8341	.7776
3	TABLE VII(T) LINE 15	C_ℓ	2.5217	3.1261	4.0426	4.3610	3.5759	2.2055	1.1675	.7465	.5746	.4849
4	(2) × $C_{L(T)}$	$C_{\ell a}$	2.7361	2.9689	3.1002	3.1858	3.2576	3.2858	3.2229	3.0082	2.5588	2.3854
5	(3) - (4)	$C_{\ell b}$	-.2144	.1572	.9424	1.1752	.3183	-1.0803	-2.0554	-2.2617	-1.9842	-1.9005
6	TABLE I LINE 11	m_o	5.574	5.581	5.590	5.601	5.615	5.632	5.650	5.670	5.688	5.701
7	TABLE I(T) LINE 6	$\alpha_a(C_{L(T)} = 3.0677)$	0	.9856	1.0	1.0	1.0	.1944	0	0	0	0
8	(7) - $C_{L(T)}/m$	$\alpha_a(C_{L(T)} = 0)$	-.6785	.3071	.3215	.3215	.3215	-.4841	-.6785	-.6785	-.6785	-.6785
9	(5) / (6)	α_o	-.0385	.0282	.1686	.2098	.0567	-.1918	-.3638	-.3989	-.3488	-.3334
10	(8) - (9)	$\frac{w}{V}$	-.6400	.2789	.1529	.1117	.2648	-.2923	-.3147	-.2796	-.3297	-.3451
11	(8) × (10)	$C_{d_{i_b}}$	.1372	.0438	.1441	.1313	.0843	.3158	+.6468	.6324	.6542	.6559
12	TABLE VII LINE 20	C_{d_i}	1.1151	.9447	.8340	.7592	.6957	.6785	.7563	.9666	1.2707	1.3476
13	(12) / C_L^2	$C_{d_{i a_1}}$	.05454	.04621	.04079	.03714	.03403	.03319	.03699	.04728	.06216	.06592
14	(5) / (2)	$C_{\ell_b} / C_{\ell a_1}$	-.2404	.1624	.9325	1.1316	.2997	-1.0086	-1.9564	-2.3064	-2.3789	-2.4441
15	(13) × (14)	$\frac{C_{d_{i a_1}} \times C_{\ell b}}{C_{\ell a_1}}$	-.0131	.0075	.0380	.0420	.0102	-.0335	-.0724	-.1090	-.1479	-.1611
16	(11) / (14)	$\frac{C_{d_{i b}} \times C_{\ell a_1}}{C_{\ell b}}$	-.5707	.2697	.1545	.1160	.2813	-.3131	-.3306	-.2742	-.2750	-.2684
17	(15) + (16)	$C_{d_{i a,b}}$	-.5838	.2772	.1925	.1580	.2915	-.3466	-.4030	-.3832	-.4229	-.4295
18		SIN θ	1.0000	.9877	.9511	.8910	.8090	.7071	.5878	.4540	.3090	.1564
19	TABLE I LINE 9	C	191.35	171.03	151.20	132.36	114.98	99.48	86.24	75.58	65.89	38.71
20	(11) × (19)	$cC_{d_{i_b}}$	26.25	7.49	21.79	17.38	9.69	31.42	55.78	47.80	43.11	25.39
21	(18) × (20)	$cC_{d_{i_b}}$ SIN θ	26.25	7.40	20.72	15.49	7.84	22.22	32.79	21.70	13.32	3.97
22	(17) × (19)	$cC_{d_{i a,b}}$	-111.71	47.41	29.11	20.91	33.52	-34.48	-34.75	-28.96	-27.86	-16.63
23	(18) × (22)	$cC_{d_{i a,b}}$ SIN θ	-111.71	46.83	27.69	18.63	27.12	-24.38	-20.43	-13.15	-8.61	-2.60

LINE 21: $\frac{\pi}{20}\frac{b}{S}\sum cC_{d_{i_b}} \sin\theta = \frac{\pi}{20 C_{AVE}}\left(\Sigma (21) - \frac{1}{2}(21)_0\right) = C_{D_{i_b}} = .001251 \times 158.58 = .1984$

LINE 23: $\frac{\pi}{20}\frac{b}{S}\sum cC_{d_{i_{a,b}}} \sin\theta = \frac{\pi}{20 C_{AVE}}\left(\Sigma (23) - \frac{1}{2}(23)_0\right) = C_{D_{i_{a,b}}} = .001251 \times (-4.76) = -.00595$

CHECK: $C_{D_i} = C_{D_{i_b}} + C_{D_{i_{a,b}}} \times C_{L(T)} + C_{D_{i_{a_1}}} \times C_{L(T)}^2 = .1984 - .00595 \times 3.0677 + .042478 \times (3.0677)^2$
$= .1984 - .0183 + .3998 = .5799$

TABLE XI

USE OF UNIT VALUES IN DETERMINING DISTRIBUTIONS FOR

α = 13.9° δ_F = 60° & δ_A = 0°

C_L = 1.716

1	a	b	kδ	C_L	0	1	2	3	4	5	6	7	8	9
2	TABLE VIII LINE 5	C_{l_b}	1.0	—	-.2144	.1572	.9424	1.1752	.3183	-1.0803	-2.0554	-2.2617	-1.9842	-1.9005
3	TABLE VIII LINE 11	C_{di_b}	1.0	—	.1372	.0438	.1441	.1313	.0843	.3158	.6468	.6324	.6542	.6559
4	TABLE VIII LINE 17	$C_{di_{a,b}}$	1.0	1.0	-.5838	.2772	.1925	.1580	.2915	-.3466	-.4030	-.3832	-.4229	-4295
5	.1734 × (2)	C_{l_b}	.1734	—	-.0372	.0273	.1634	.2038	.0552	-.1873	-.3564	-.3922	-.3441	-.3295
6	$(.1734)^2$ × (3)	C_{di_b}	.1734	—	+.0041	.0013	.0043	.0039	.0025	.0095	.0194	.0190	.0197	.0197
7	.1734 × (4)	$C_{di_{a,b}}$	.1734	1.0	-.10123	.04807	.03338	.02740	.05055	-.06010	-.06988	-.06645	-.07333	-.07448
8	TABLE VIII LINE 13	$C_{di_{a_1}}$	—	1.0	.05454	.04621	.04079	.03714	.03403	.03319	.03699	.04728	.06216	.06592
9	TABLE VIII LINE 2	$C_{l_{a_1}}$	—	1.0	.8919	.9678	1.0106	1.0385	1.0619	1.0711	1.0506	.9806	.8341	.7776
10	C_L × (9)	C_{l_a}	—	1.716	1.5305	1.6607	1.7342	1.7821	1.8222	1.8380	1.8028	1.6827	1.4313	1.3344
11	(5) + (10)	C_l	.1734	1.716	1.4933	1.6880	1.8976	1.9859	1.8774	1.6507	1.4464	1.2905	1.0872	1.0049
12	C_L^2 × (8)	C_{di_a}	—	1.716	.1606	.1361	.1201	.1094	.1002	.0977	.1089	.1392	.1830	.1941
13	C_L × (7)	$C_{di_{ab}}$	.1734	1.716	-.1737	.0825	.0573	.0470	.0867	-.1031	-.1199	-.1140	-.1258	-.1278
14	(6)+(12)+(13)	C_{di}	.1734	1.716	-.0090	.2199	.1817	.1603	.1894	+.0041	.0084	.0442	.0769	.0860
					ALTERNATIVE METHOD OF DETERMINING C_{di}									
15	TABLE I LINE 11	m_o	—	—	5.574	5.581	5.590	5.601	5.615	5.632	5.650	5.670	5.688	5.701
16	TABLE VIII LINE 8	α_a	1.0	0	-.6785	.3071	.3215	.3215	.3215	-.4841	-.6785	-.6785	-.6785	-.6785
17	.1734 × (16)	α_a	.1734	0	-.1177	.0533	.0557	.0557	.0557	-.0839	-.1177	-.1177	-.1177	-.1177
18	.3795 + (17)	α_a	.1734	1.716	.2618	.4328	.4352	.4352	.4352	.2956	.2618	.2618	.2618	.2618
19	(11) / (15)	α_o	.1734	1.716	.2679	.3025	.3395	.3546	.3344	.2931	.2560	.2276	.1911	.1762
20	(18) − (19)	W/V	.1734	1.716	-.0061	.1303	.0957	.0806	.1008	.0025	.0058	.0342	.0707	.0855
21	(11) × (20)	C_{di}	.1734	1.716	-.0091	.2199	.1816	.1601	.1892	.0041	.0034	.0441	.0769	.0859

57882 O—38——13

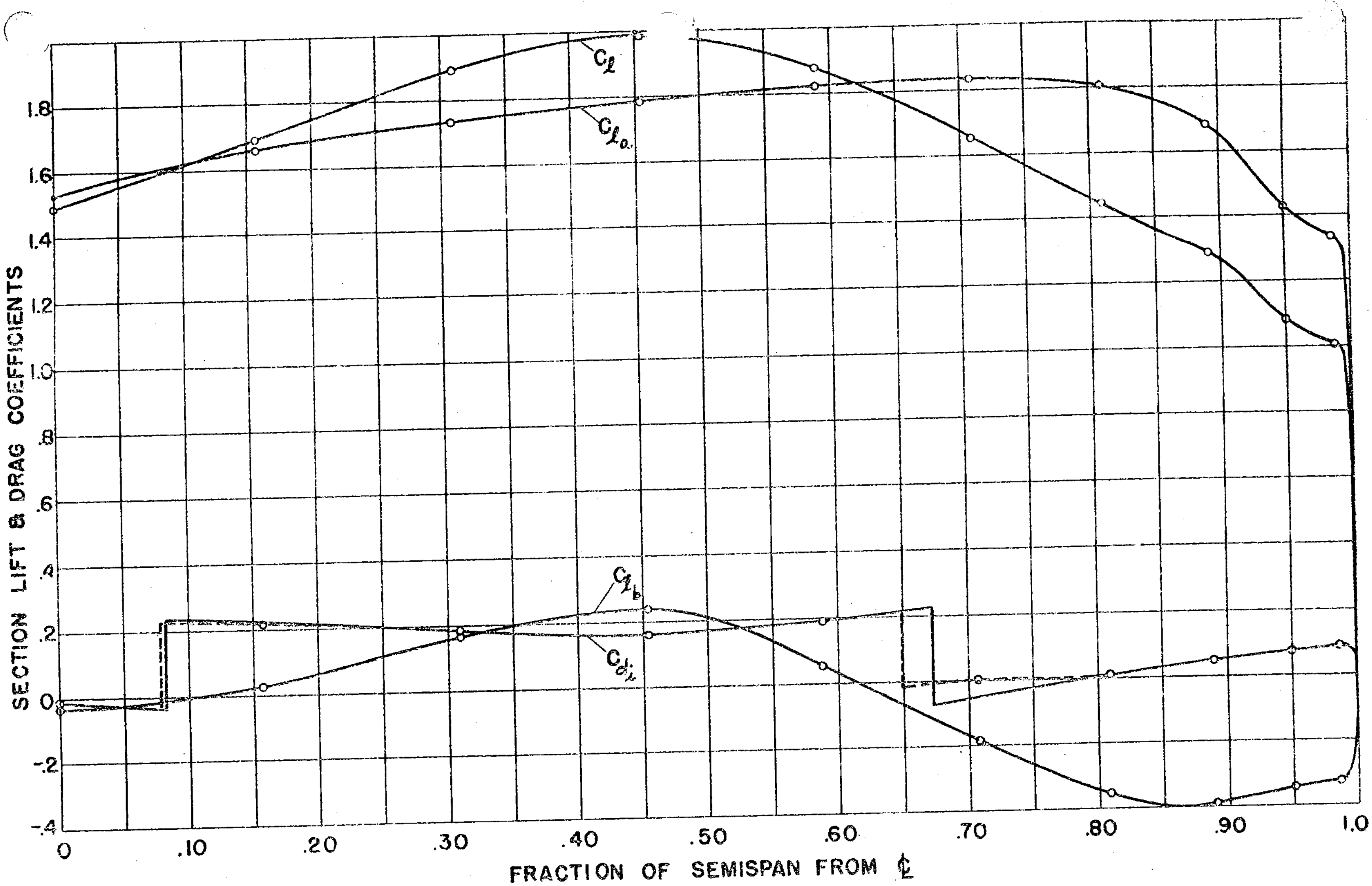

FIG. 7-4

SECTION LIFT & INDUCED DRAG FOR WING WITH PART-SPAN FLAP

$\alpha = 13.9°, \delta_F = 60° \ \delta_A = 0°$

7.4 EXAMPLE 4 - WING WITH DIFFERENTIALLY DISPLACED AILERONS.

7.40 Method of Attack - In order to demonstrate the procedure when the loading is not symmetrical, an example will be taken in which the ailerons are fully deflected. The same wing as used in the previous examples and shown in Figure 7-1 will be used. The inboard flaps will be assumed closed. The normal procedure would be to first calculate the distributions for the wing with no twist, but since this has already been done in Section 7.2 for the normal wing, it is not necessary to repeat it here. If the aileron displacement were equal and opposite on the two sides and the value of k were the same for positive and negative displacements, the effective twist would be completely antisymmetrical. In order to show how to handle a wing with two twists existing simultaneously, however, the ailerons will be assumed to be displaced differentially, which is the same as antisymmetrical displacement combined with a symmetrical displacement or droop. Three simultaneous twists could be illustrated if the inboard flaps also were assumed displaced, but it is believed that the following example will sufficiently demonstrate the method so that no difficulty will be experienced in combining any desired number of twists.

7.41 Symmetrical Twist - The tables for symmetrical aileron displacement will be computed first as though this were the only twist present. These computations are similar to those made in the preceding section for the wing with inboard flap open. The absolute angle of attack for the inboard portion of the wing is taken as zero in line 6 of Table I. A slightly different procedure is used in obtaining the $k\delta$ or α_a values for the part of the wing containing the aileron than was used in the case of the inboard flap. Since, because of tip rounding, the ratio of aileron chord to wing chord is not constant along the span, it is not convenient to use a constant $k\delta$ of 1.0. In the absence of better information, the value of $k\delta$ is assumed to vary with chord ratio E in accordance with F_E, read from the curve for ordinary flap in Figure 4-1 for the values of E at each span location. These values of F_E are used as $k\delta$ in Table I, which is the same as assuming an aileron displacement of one radian, or $F\delta_F = 1.0$. As in the case of the inboard flap, the value of $k\delta$ or α_a at the inboard end is modified, because the end of the aileron does not fall midway between the two adjacent points. Since the values of α_a are so much smaller than in the previous examples where one radian was used, it has been found desirable to carry one more place in the computations, not because the accuracy warrants such action, but because better checks are obtained at the various places where checks are provided. Some work could be saved by using

α_a values ten times as great, which would eliminate many zeros necessary only for locating the decimal point. This would be the same as using a value of $F_{\delta_F} = 10.0$. (See Section 4.21).

7.42 Antisymmetrical Twist - The antisymmetrical computations are made in a similar manner, using the same Table I as for the symmetrical case. Table IV-A is necessary because the combination of C_{2n} coefficients are not the same as those for the symmetrical computations, although the plan-form coefficients themselves are, of course, the same. It should be noted that values of C_{D_i} and $C_l{}'$ found at the bottom of Table VII-A and the value of $C_n{}'/C_L$, found at the bottom of Table VIII-A by integrating the distributions, check the respective values computed in Table VI-A. (See Section 4.22).

7.43 APPLICATION TO A GIVEN CONDITION.

7.431 Determination of kδ - Sufficient information is now available for computing the lift and induced-drag distributions for any aileron displacement and for any angle of attack. It will be assumed that the wing and section characteristics are desired for this wing with one aileron up 30 degrees and the other one down 6 degrees, and for an angle of attack of 13.9 degrees measured from the chord line of the normal-wing section. It will be assumed that the aileron is equally effective whether displaced upward or downward, and that $k\delta$ may be found from the ordinary flap curves of Figure 4-1, as

$$k\delta = F_E \, F_{\delta_F}$$

These curves are based on tests of ordinary or plain flaps with no leakage at the hinge. Ailerons may have characteristics quite different, and may be more effective in one direction than the other. Where possible, therefore, the section characteristics of the actual aileron-wing combination should be used. From Figure 4-1

$$F_{\delta_F} = -.738 \quad \text{for } \delta_F = -30^\circ$$

and

$$F_{\delta_F} = +.230 \quad \text{for } \delta_F = +6^\circ$$

These values may be divided into symmetrical and antisymmetrical parts,

as $\quad F_{\delta_A} = -.254 -.484 \quad \text{for } \delta_A = -30^\circ$

and $\quad F_{\delta_A} = -.254 +.484 \quad \text{for } \delta_A = +6^\circ$

7.432 Modification of Unit Distributions - The next step is to modify the basic distributions obtained in Tables VIII and VIII-A for $F_{\delta_F} = 1.0$, so that they will be correct for $F_{\delta_A} = -.254$ and $\pm.484$ for the symmetrical and antisymmetrical cases, respectively. This is accomplished in Table XII, where b_1 is used to indicate the basic symmetrical distribution, b_2 the antisymmetrical, and b without a subscript a combination of both. The values, for $F_{\delta_A} = 1.0$, of $c_{l\,b_1}$ and $c_{d_{i\,a_1 b_1}}$ are multiplied by -0.254, and $c_{d_{i\,b_1}}$ by $(-.254)^2$; those of $c_{l\,b_2}$ and $c_{d_{i\,a_1 b_2}}$ are multiplied by .484, and $c_{d_{i\,b_2}}$ by $(.484)^2$—in lines 8 to 13—after first copying the values for $F_{\delta_A} = 1.0$ in lines 2 to 7 from Tables VIII and VIII-A. The values of $c_{d_{i\,a_1}}$ and $c_{l\,a_1}$ are copied into lines 14 and 15 since they are independent of aileron displacement.

7.433 Wing Coefficients - Before the section values can be found for the angle of attack of 13.9 degrees, it is necessary to determine the wing lift coefficient for this angle with the ailerons displaced -30° and +6°. The zero-lift angle for any section of the normal wing is -1.1 degrees, which corresponds to an angle of attack of 15 degrees when measured from the zero-lift chord. If the section at the plane of symmetry is used as a reference, the true absolute angle of attack of this section is 15 degrees, or 15/57.3 = 0.2618 radian. In Table VIII, line 8, column 0, the value of α_s for zero lift for a symmetrical aileron displacement of one radian ($F_{\delta_A} = 1.0$) is -.04290, which becomes $\alpha_{s_{o_1}} = -.254 \times (-.04290) = +.01090$ for $F_{\delta_A} = -.254$. Since antisymmetrical displacement produces no total wing lift, the value of $\alpha_{s_{o_2}} = 0$. Then, by Equation 4.22,

$$C_L = m\left[\alpha_s - (\alpha_{s_{o1}} + \alpha_{s_{o2}})\right]$$

$$= 4.5215 \left[0.2618 - (.01090 + 0)\right] = 4.5215 \times .2509 = 1.1344$$

which is the lift coefficient for this wing with ailerons displaced -30 degrees and +6 degrees at this angle of attack. Also, by Equation 4.16

$$C_{D_i} = .00806 \times (-.254)^2 + .01002 \times (.484)^2 + (-.254 \times (-.000387) \times 1.1344 + .042478 \times (1.1344)^2 = .00052 + + .00235 + .00011 + .05466 = .05764$$

and from Table VI-A

$$C_l' = .06590 \times .484 = .03190$$

7.434 Total Distributions - The section values of c_l for the two sides are now found in lines 18 and 19 by first adding in line 17 to the value $c_{l_{b1}}$ in line 8 the additional lift found in line 16 for $C_L = 1.1344$, and then adding and subtracting the value of $c_{l_{b2}}$ in line 11. For this particular aileron displacement the distributions of c_l can be found for any other angle of attack by first computing C_L and then repeating lines 16, 17, 18 and 19. Lines 20, 21, 22 and 24 are used to compute $c_{d_{ib_1b_2}}$, the portion of $c_{d_{i_b}}$ caused by the interaction of the basic lifts. It should be noted that this portion has a different sign on the two sides because of the change in sign of $c_{l_{b_2}}$ and, therefore, introduces an additional yawing moment that would not be present if there were no symmetrical twist. In lines 25 and 26 the total values of $c_{d_{i_b}}$ are found for each side. These two lines represent the portion of the induced-drag coefficient that is independent of angle of attack and depends only on the aileron displacement. In lines 27 and 28 the values of $c_{d_{i_{a_1 b}}}$, which represent the total portion of induced-drag coefficient that varies directly with C_L for a wing $C_L = 1.0$, are found by simply adding these values for the two separate twists. These are multiplied by the value of wing C_L in lines 29 and 30, to find the correct values for this particular angle of attack. The same is done for $c_{d_{i_a}}$ in line 31 and the final totals for the two sides are found in lines 32 and 33 by addition. For this particular aileron displacement, the distributions of c_{d_i} can be found for any other angle of attack by first computing C_L and then repeating lines 29 to 33. (See Section 4.24).

7.435 Induced-Drag Distribution by Alternative Method - It is seen that the computations for unit distributions of induced drag become rather long in such an example and would be even more so if another twist, such as open inboard flaps, had been assumed. The remainder of Table XII has been added, therefore, as was done in the previous example in Table XI to show an alternative method of obtaining the induced-drag distribution. This method requires lines 41 to 48 to be repeated for each angle of attack for which the distributions are desired, but it makes it unnecessary to compute any of the first 30 lines in Table XII, except lines 1, 2, 5, 8, 11, 15, 16, 17, 18 and 19, and makes it possible to omit all computations involving induced drag in Tables VI, VII, VIII, VI-A and VII-A, while Table VIII-A may be omitted entirely. The alternative method does not separate the various portions of the induced drag, but simply calculates the total value for a given condition. The calculations in lines 34 to 48 of Table XII are self-explanatory, and are twice as long as they would be if it were not necessary to calculate each side separately because of the unsymmetrical twist.

7.436 Yawing Moment - Lines 49 to 52 have been added to calculate the yawing moment produced by the interaction of the two basic-lift distributions. This is done by integrating the values of $c_{d_{i_{b_1 b_2}}}$ in line 24 as was done for $c_{d_{i_{a_1 b}}}$ in Table VIII-A. This could be done by the method of Table VI-A, if the odd A_n coefficients were known for the wing with symmetrical twist at zero lift. The yawing-moment coefficient found in Table VI-A and checked at the bottom of Table VIII-A is for an antisymmetrical aileron displacement of one radian, on the wing without other twist, at a lift coefficient of 1.0. The yawing moment coefficient for this particular example can be found by adding to the basic value of -.00083, found at the bottom of Table XII, that caused by the additional lift, which is found by modifying the value of .00834 from Table VI-A or VIII-A, for F_{δ_A} = .484 and C_L = 1.1344, or

$$C_n' = -.00083 + .00834 \times .484 \times 1.1344$$
$$= -.00083 + .00458 = .00375$$

This can be checked, if desired, by subtracting the total c_{d_i} values on one wing, line 33 or 48, from those on the other wing, line 32 or 44, and then integrating the difference in the same manner as was done with the $c_{d_{ib_1b_2}}$ values at the bottom of Table XII, except that if the same formula is used, the answer must be divided by two.

7.437 <u>Other Coefficients</u> - The determination of the other coefficients is accomplished in the same manner as that used in Sections 7.18 and 7.19 for the special method. For that portion of the span covered by the aileron, an additional minimum profile-drag coefficient must be added to that for the normal wing. From Figure 4 of reference 13

$$\Delta c_{d_{o_{min}}} = 0.055 \quad \text{for } \delta_A = -30^\circ \text{ and } E = .20$$

and

$$\Delta c_{d_{o_{min}}} = 0.004 \text{ for } \delta_A = +6^\circ \text{ and } E = .20$$

Where better information is available for the particular type of aileron it should be used. It will be noted by comparing lines 32 and 33 of Table XII that, in the region of the aileron, there is a difference in section c_{d_i} of approximately .095, causing an adverse yawing moment. This would be increased slightly by the additional profile drag, if taken into account. It is apparent, therefore, that the difference in minimum profile drag produced by the large differential is only about half the amount required to balance the induced yawing moment. Since these values are based on a simple hinged flap, the advantage of using a device, such as a Frise balance, to increase the drag on the up aileron side is shown. Likewise, for this portion of the span, an additional moment coefficient must be added. In the absence of better data, this may be taken from Figure 6 of reference 13 where it is seen to vary with c_l . After an inspection of lines 18 and 19 of Table XII, the values may be read as

$$\Delta c_{m_{a.c.}} = +.17 \text{ for } \delta_A = -30^\circ \text{ and } C_L = 0.70$$

and

$$\Delta c_{m_{a.c.}} = -.04 \text{ for } \delta_A = +6^\circ \text{ and } C_L = 1.40$$

7.438 Plot of Results - The values of c_l and c_{d_i} are plotted for the full span wing in Figure 7-5.

TABLE I. (T)

GEOMETRIC CHARACTERISTICS OF WING

AILERON FLAP ANGLE = 1.0 RADIAN

1	a	b	0	1	2	3	4	5	6	7	8	9	10
2	FRACTION SEMI-SPAN	$\frac{2y}{b} = \cos\theta$	0	.1564	.3090	.4540	.5878	.7071	.8090	.8910	.9511	.9877	1.0000
3		θ	90	81	72	63	54	45	36	27	18	9	0
4		SIN θ	1.0000	.9877	.9511	.8910	.8090	.7071	.5878	.4540	.3090	.1564	0
5	Fig. 7-1	E						.2000	.2000	.2000	.2028	.0806	
	Fig. 4-1	$k\delta$						.2045	.2045	.2045	.2060	.1285	
6	ABSOLUTE IN RADIANS	α_a	0	0	0	0	0	.1647*	.2045	.2045	.2060	.1285	
7	(6) x (4)	α_a SIN θ	0	0	0	0	0	.11646	.12021	.09284	.06365	.02010	
8	MAXIMUM THICKNESS	t											
9	CHORD	c											
10	(8)/(9)	$\frac{t}{c}$											
11	$dc_\ell/d\alpha_o$	m_o											
12	$(11)_o/(11)_n$	m_s/m_o											
13	$(9)_o/(9)_n$	c_s/c											
14	(12)x(13)x(4)	$\frac{m_s c_s}{m_o c}$ SIN θ											

b = S = $C_{AVE.} = S/b =$

*Flap Ends at $\theta = \cos^{-1}(320/476) = 47.75°$; $(6)_5 = k\delta(47.75 - 40.5)/(49.5 - 40.5) = .8056\ k\delta$

TABLE II. (T)

COMPUTATION OF ANGLE COEFFICIENTS, B_n

AILERON FLAP ANGLE = 1.0 RADIAN

TABLE I.	$y_0/2$	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9
LINE 7	0	0	0	0	0	.11646	.12021	.09284	.06365	.02010

$y_1 - y_3 - y_5 + y_7 + y_9 = r_1$	$y_0/2 - y_4 + y_8 = r_2$
0 -0 -.11646 +.09289 + 02010=-.00352	0 - 0 +.06365 = .06365

	MULTI-PLIER	1 - 19		3 - 17		5 - 15		7 - 13		9 - 11	
		a	b	a	b	a	b	a	b	a	b
1	.1564	y_9 .00314		$-y_3$ 0				$-y_7$ -.01452		y_1 0	
2	.3090		y_8 .01967		y_4 0				y_4 0		y_8 .01967
3	.4540	y_7 .04215		y_9 .00913				$-y_1$ 0		$-y_3$ 0	
4	.5878		y_6 .07066		$-y_2$ 0				y_2 0		$-y_6$ -.07066
5	.7071	y_5 .08235		y_5 .08235		r_1 -.00249		$-y_5$ -.08235		y_5 .08235	
	.8090		y_4 0		y_8 .05149				y_8 .05149		y_4 0
7	.8910	y_3 0		$-y_1$ 0				y_9 .01791		$-y_7$ -.08272	
8	.9511		y_2 0		y_6 .11433				$-y_6$ -.11433		$-y_2$ 0
9	.9877	y_1 0		y_7 .09170				y_3 0		y_9 .01985	
10	1.0000		$y_0/2$ 0		$-y_0/2$ 0		r_2 .06365		$-y_0/2$ 0		$y_0/2$ 0
11	TOTAL	.12764	.09033	.18318	.16582	-.00249	.06365	-.07896	-.06284	.01948	-.05099
12	a + b	.21797		.34900		.06116		-.14180		-.03151	
13	a - b		.03731		.01736		-.06614		-.01612		.07047
14	$\frac{(12)\ \&\ (13)}{2.5}$	.08719	.01492	.13960	.00694	.02446	-.02646	-.05672	-.00645	-.01260	.02819
15	$2B_n$	$2B_1$	$2B_{19}$	$2B_3$	$2B_{17}$	$2B_5$	$2B_{15}$	$2B_7$	$2B_{13}$	$2B_9$	$2B_{11}$

CHECK: $2B_1 - 2B_3 + 2B_5 - 2B_7 + 2B_9 - 2B_{11} + 2B_{13} - 2B_{15} + 2B_{17} - 2B_{19} = 2y_0 = 2\alpha_s$

NOTE: IF α_a IS CONSTANT ALONG THE SPAN, $B_1 = \alpha_s$ & B_3 TO B_{19} ARE ZERO.

.08719-.13960+.02446 + .05672 - .01260 -.02819 -.00645 +.02646 +.00694 -.01492 = .00001

TABLE V. (T)

SOLUTION OF A_n COEFFICIENTS

AILERON FLAP ANGLE = 1.0 RADIAN

	1	2	3a	3b	3c	4a	4b	4c
1	$(C_2 - C_4)A_3$	.0743	---	.03498	.03476	---	.00260	.00258
2	$(C_4 - C_6)A_5$	-.2292	---	.00733	.00698	---	-.00168	-.00160
3	$(C_6 - C_8)A_7$	-.0036	---	-.00749	-.00754	---	.00003	.00003
4	$(C_8 - C_{10})A_9$	-.0347	---	-.00142	-.00144	---	.00005	.00005
5	1 + 2 + 3 + 4					---	.00100	.00106
6	$2B_1$ - 5	$2B_1$ = .08719				.08719	.08619	.08613
7	A_1 = 6 / $2P_1$	$2P_1$ = 2.9632				.02942	.02909	.02907
8	$(C_2 - C_4)A_1$	.0743	.02942	.02909	.02907	.00219	.00216	.00216
9	$(C_2 - C_8)A_5$	-.1585	---	.00733	.00698	---	-.00116	-.00111
10	$(C_4 - C_{10})A_7$	-.2675	---	-.00749	-.00754	---	.00200	.00202
11	$(C_6 - C_{12})A_9$	-.0159	---	-.00142	-.00144	---	.00002	.00002
12	8 + 9 + 10 + 11					.00219	.00302	.00309
13	$2B_3$ - 12	$2B_3$ = .13960				.13741	.13658	.13651
14	A_3 = 13 / $2P_3$	$2P_3$ = 3.9287				.03498	.03476	.03475
15	$(C_4 - C_6)A_1$	-.2292	.02942	.02909	.02907	-.00674	-.00667	-.00666
16	$(C_2 - C_8)A_3$	-.1585	.03498	.03476	.03475	-.00554	-.00551	-.00551
17	$(C_2 - C_{12})A_7$	-.1708	---	-.00749	-.00754	---	.00128	.00129
18	$(C_4 - C_{14})A_9$	-.2532	---	-.00142	-.00144	---	.00036	.00036
19	15 + 16 + 17 + 18					-.01228	-.01054	-.01052
20	$2B_5$ - 19	$2B_5$ = .02446				.03674	.03500	.03498
21	A_5 = 20 / $2P_5$	$2P_5$ = 5.0108				.00733	.00698	.00698
22	$(C_6 - C_8)A_1$	-.0036	.02942	.02909	.02907	-.00011	-.00010	-.00010
23	$(C_4 - C_{10})A_3$	-.2675	.03498	.03476	.03475	-.00936	-.00930	-.00930
24	$(C_2 - C_{12})A_5$	-.1708	.00733	.00698	.00698	-.00125	-.00119	-.00119
25	$(C_2 - C_{16})A_9$	-.1605	---	-.00142	-.00144	---	.00023	.00023
26	22 + 23 + 24 + 25					-.01072	-.01036	-.01036
27	$2B_7$ - 26	$2B_7$ = -.05672				-.04600	-.04636	-.04636
28	A_7 = 27 / $2P_7$	$2P_7$ = 6.1455				-.00749	-.00754	-.00754
29	$(C_8 - C_{10})A_1$	-.0347	.02442	.02909	.02907	-.00102	-.00101	-.00101
30	$(C_6 - C_{12})A_3$	-.0159	.03498	.03476	.03475	-.00056	-.00055	-.00055
31	$(C_4 - C_{14})A_5$	-.2532	.00733	.00698	.00698	-.00186	-.00177	-.00177
32	$(C_2 - C_{18})A_7$	-.1605	-.00749	.00754	-.00754	.00120	.00121	.00121
33	29 + 30 + 31 + 32					-.00224	-.00212	-.00212
34	$2B_9$ - 33	$2B_9$ = -.01260				-.01036	-.01048	-.01048
35	A_9 = 34 / $2P_9$	$2P_9$ = 7.2768				-.00142	-.00144	-.00144

TABLE VI (T)

COMPUTATION OF WING LIFT AND DRAG COEFFICIENTS

AILERON FLAP ANGLE = 1.0 RADIAN

1	2	3
$A_1 = .02907$	$(A_1)^2 = .000845$	$A_1/3 = .00969$
$A_3 = .03475$	$3(A_3)^2 = .003623$	$A_3/5 = .00695$
$A_5 = .00698$	$5(A_5)^2 = .000244$	$-A_5/21 = -.00033$
$A_7 = -.00754$	$7(A_7)^2 = .000398$	$A_7/45 = -.00017$
$A_9 = -.00144$	$9(A_9)^2 = .000019$	$-A_9/77 = .00002$
	$\Sigma n A_n^2 = .005129$	$\Sigma = .01616$

$$1 + \sigma = \frac{\Sigma n A_n^2}{A_1^2} = \frac{.005129}{.000845} = 6.069$$

$$C_L = \pi A u_0 A_1 = 3.1416 \times 7.582 \times .2801 \times .02907 = .19395$$

OR

$$\frac{\pi}{4} \frac{m_s c_s}{C_{AVE.}} A_1 = .7854 \ \frac{5.574 \times 191.35}{125.56} \times .02907 = .19395$$

$$C_{D_i} = \left(\frac{1+\sigma}{\pi A}\right) C_L^2 = C_{D_{i_1}} \times C_L^2 = \frac{6.069}{3.1416 \times 7.582} \times .19395^2 = .25479 \times .037617$$
$$= .00958$$

FOR WING WITH NO AERODYNAMIC TWIST, $C_{D_{i_1}} = C_{D_{i_{a_1}}}$

$$\bar{y}_L = \left(\frac{4}{\pi} \times \frac{\Sigma \text{Col. } 3}{A_1}\right) \frac{b}{2} = \frac{4}{3.1416} \times \frac{.01616}{.02907} \times \frac{952}{2} = .7078 \times \frac{952}{2} = 336.91$$

TABLE VII (T)

COMPUTATION OF LOAD DISTRIBUTIONS

C_{D_i} =

AILERON FLAP ANGLE = 1.0 RADIAN

C_L = .19395

1	a	b	0	1	2	3	4	5	6	7	8	9
2	TABLE I LINE 3	θ	90	81	72	63	54	45	36	27	18	9
3	TABLE I LINE 4	SIN θ	1.0000	.9877	.9511	.8910	.8090	.7071	.5878	.4540	.3090	.1564
4		SIN 3θ	-1.0000	-.8910	-.5878	-.1564	.3090	.7071	.9511	.9877	.8090	.4540
5		SIN 5θ	1.0000	.7071	0	-.7071	-1.0000	-.7071	0	.7071	1.0000	.7071
6		SIN 7θ	-1.0000	-.4540	.5878	.9877	.3090	-.7071	-.9511	-.1564	.8090	.8910
7		SIN 9θ	1.0000	.1564	-.9511	-.4540	.8090	.7071	-.5878	-.8910	.3090	.9877
8	A_1= .02907	A_1 SIN θ	.02907	.02871	.02765	.02590	.02352	.02056	.01709	.01320	.00898	.00455
9	A_3= .03475	A_3 SIN 3θ	-.03475	-.03096	-.02043	-.00543	.01074	.02457	.03305	.03432	.02811	.01578
10	A_5= .00698	A_5 SIN 5θ	.00698	.00494	0	-.00494	-.00698	-.00494	0	.00494	.00698	.00494
11	A_7= -.00754	A_7 SIN 7θ	.00754	.00342	-.00443	-.00745	-.00233	.00533	.00717	.00118	-.00610	-.00672
12	A_9= -.00144	A_9 SIN 9θ	-.00144	-.00023	.00137	.00065	-.00116	-.00102	.00085	.00128	-.00044	-.00142
13	(8)+(9)+(10)+(11)+(12)	$\sum A_n \text{SIN } n\theta$	.00740	.00588	.00416	.00873	.02379	.04450	.05816	.05492	.03753	.01713
14	TABLE I [(11)$_0$ x (13)]	$\frac{m_s c_s}{c}$	5.574	6.236	7.054	8.058	9.276	10.722	12.368	14.112	16.187	27.553
15	(13) x (14)	c_ℓ	.04125	.03667	.02934	.07035	.22068	.47713	.71932	.77503	.60750	.47198
16	TABLE I LINE 11	m_0										
17	TABLE I LINE 6	α_a										
18	(15) / (16)	α_0										
19	(17) - (18)	$\frac{w}{V}$										
20	(19) x (15)	c_{di}										
21	TABLE I LINE 9	c	191.35	171.03	151.20	132.36	114.98	99.48	86.24	75.58	65.89	38.71
22	(15) / C_L	c_{ℓ_1}	.21268	.18907	.15128	.36272	1.1378	2.4601	3.7088	3.9960	3.1323	2.4335
23	(22) x (21)	$c c_{\ell_1}$	40.696	32.337	22.874	48.010	130.824	244.731	319.847	302.018	206.337	94.201
24	(23) x (3)	cc_{ℓ_1} SIN θ	40.696	31.939	21.755	42.777	105.837	173.049	188.006	137.116	63.774	14.733
25	(20) / C_{D_i}	c_{di_1}										
26	(25) x (21)	cc_{di_1}										
27	(26) x (3)	cc_{di_1} SIN θ										

CHECKS:

LINE 24: $\frac{\pi}{20}\frac{b}{S}\sum cc_{\ell_1}\text{ SIN }\theta = \frac{\pi}{20 C_{AVE}}\left(\sum (24) - \frac{1}{2}(24)_0\right) = 1.0$; .0012510 x 799.334 = .99997

LINE 27: $\frac{\pi}{20}\frac{b}{S}\sum cc_{d_{i_1}}\text{ SIN }\theta = \frac{\pi}{20 C_{AVE}}\left(\sum (27) - \frac{1}{2}(27)_0\right) = 1.0$

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TABLE VIII

COMPUTATION OF UNIT LOAD DISTRIBUTIONS

SINGLE SYMMETRICAL TWIST

$C_{L(T)}$ = .19395 AILERON FLAP ANGLE = 1.0 RADIAN $C_L = m$ = 4.5215

1	a	b	0	1	2	3	4	5	6	7	8	9
2	TABLE VII LINE 22	$C_{\ell_{a_1}}$	.8919	.9678	1.0106	1.0385	1.0619	1.0711	1.0506	.9806	.8341	.7776
3	TABLE VII(T) LINE 18	C_ℓ	.04125	.03667	.02934	.07035	.22068	.47713	.71932	.77593	.60750	.47198
4	(2) × $C_{L(T)}$	C_{ℓ_a}	.17298	.18770	.19601	.20142	.20596	.20774	.20376	.19019	.16177	.15082
5	(3) - (4)	C_{ℓ_b}	-.13173	-.15103	-.16607	-.13107	.01472	.26939	.51556	.58484	.44573	.32116
6	TABLE I LINE 11	m_o	5.574	5.581	5.590	5.601	5.615	5.632	5.650	5.670	5.688	5.701
7	TABLE I(T) LINE 6	$\alpha_a(C_{L(T)} = .19395)$	0	0	0	0	0	.1647	.2045	.2045	.2060	.1285
8	(7) - $C_{L(T)}/m$	$\alpha_a(C_L = 0)$	-.04290	-.04290	-.04290	-.04290	-.04290	.12180	.16160	.16160	.16310	.08560
9	(5) / (6)	α_o	-.02363	-.02706	-.02983	-.02340	.00262	.04783	.09125	.10315	.07836	.05633
10	(8) - (9)	$\frac{w}{V}$	-.01927	-.01584	-.01308	-.01950	-.04552	.07397	.07035	.05845	.08474	.02927
11	(5) x (10)	$C_{d_{i_b}}$	.00254	.00239	.00218	.00256	-.00067	.01993	.03627	.03418	.03777	.00940
12	TABLE VII LINE 20	C_{d_i}	1.1151	.9447	.8340	.7592	.6957	.6785	.7563	.9666	1.2707	1.3476
13	(12) / C_L^2	$C_{d_{i_{a_1}}}$	.05454	.04621	.04079	.03714	.03403	.03319	.03699	.04728	.06216	.06592
14	(5) / (2)	$C_{\ell_b} / C_{\ell_{a_1}}$	-.14770	-.15605	-.16492	-.12621	.01386	.25151	.49073	.59641	.53438	.41301
15	(13) × (14)	$\frac{C_{d_{i_{a_1}}} \times C_{\ell_b}}{C_{\ell_{a_1}}}$	-.00806	-.00721	-.00673	-.00469	.00047	.00835	.01815	.02820	.03322	.02723
16	(11) / (14)	$\frac{C_{d_{i_b}} \times C_{\ell_{a_1}}}{C_{\ell_b}}$	-.01720	-.01532	-.01322	-.02028	-.04834	.07924	.07391	.05731	.07068	.02276
17	(15) + (16)	$C_{d_{i_{a,b}}}$	-.02526	-.02253	-.01995	-.02497	-.04787	.08759	.09206	.08551	.10390	.04999
18		**SIN θ**	**1.0000**	**.9877**	**.9511**	**.8910**	**.8090**	**.7071**	**.5878**	**.4540**	**.3090**	**.1564**
19	TABLE I LINE 9	C	191.35	171.03	151.20	132.36	114.98	99.48	86.24	75.58	65.89	38.71
20	(11) × (19)	$CC_{d_{i_b}}$	.4860	.4088	.3296	.3388	-.0770	1.9826	3.1279	2.5833	2.4887	.3639
21	(18) × (20)	$CC_{d_{i_b}}$ SIN θ	.4860	.4038	.3135	.3019	-.0623	1.4019	1.8386	1.1728	.7690	.0569
22	(17) × (19)	$CC_{d_{i_{a,b}}}$	-4.834	-3.853	-3.016	-3.305	-5.504	8.714	7.939	6.463	6.846	1.935
23	(18) × (22)	$CC_{d_{i_{a,b}}}$ SIN θ	-4.834	-3.806	-2.869	-2.945	-4.453	6.162	4.667	2.934	2.115	.303

LINE 21: $\frac{\pi}{20}\frac{b}{S}\sum CC_{d_{i_b}} \sin\theta = \frac{\pi}{20 C_{AVE}}\left(\sum (21) - \frac{1}{2}(21)_0\right) = C_{D_{i_b}}$ = .001251 x 6.4391 = .00806

LINE 23: $\frac{\pi}{20}\frac{b}{S}\sum CC_{d_{i_{a,b}}} \sin\theta = \frac{\pi}{20 C_{AVE}}\left(\sum (23) - \frac{1}{2}(23)_0\right) = C_{D_{i_{a,b}}}$ = .001251 x(-0.309)=-.000387

CHECK: $C_{D_i} = C_{D_{i_b}} + C_{D_{i_{a,b}}} \times C_{L(T)} + C_{D_{i_{a_1}}} \times C_{L(T)}^2$ = .00806 - .000387 x .19395 + .042472 x (.19395)^2

= .00806 - .00008 + .00160 = .00958

TABLE II A

COMPUTATION OF ANGLE COEFFICIENTS, B_n

ANTI-SYMMETRICAL

AILERON ANGLE = 1.0 RADIAN

1	TABLE I	y_9	.02010	y_8	.06365	y_7	.09284	y_6	.12021	y_5	.11646
2	LINE 7	y_1	0	y_2	0	y_3	0	y_4	0		0
3	① + ②	V_1	.02010	V_2	.06365	V_3	.09284	V_4	.12021	V_5	.11646
4	① − ②	W_1	.02010	W_2	.06365	W_3	.09284	W_4	.12021		

5	MULTI-PLIER	2 − 18		4 − 16		6 − 14		8 − 12		10
		a	b	a	b	a	b	a	b	
6	.3090	V_1 .00621				V_3 .02869				
7	.5878		V_2 .03741	W_1 .01181	W_4 .07066		$-V_4$ −.07066	$-W_3$ −.05457	W_2 .03741	
8	.8090	V_3 .07511				V_1 .01626				
9	.9511		V_4 .11433	W_3 .08830	W_2 .06054		V_2 .06054	W_1 .01912	$-W_4$ −.11433	
10	1.0000	V_5 .11646				$-V_5$ −.11646				$V_1-V_3+V_5$ .04372
11	TOTAL	.19778	.15174	.10011	.13120	−.07151	−.01012	−.03545	−.07692	.04372
12	a + b	.34952		.23131		−.08163		−.11237		
13	a − b		.04604		−.03109		−.06139		.04147	
14	⑫ OR ⑬ / 2.5	.13981	.01842	.09252	−.01244	−.03265	−.02456	−.04495	.01659	.01749
15	$2B_n$	$2B_2$	$2B_{18}$	$2B_4$	$2B_{16}$	$2B_6$	$2B_{14}$	$2B_8$	$2B_{12}$	$2B_{10}$

CHECKS:

$2B_2 - 2B_6 + 2B_{10} - 2B_{14} + 2B_{18} = 2y_5 =$

$.5878(2B_2 + 2B_8 - 2B_{12} - 2B_{18}) + .9511(2B_4 + 2B_6 - 2B_{14} - 2B_{16}) = 2y_8 =$

CHECKS:

.13981 + .03265 + .01749 + .02456 + .01842 = .23293; $2Y_5$ = .23292

.5878(.13981 - .04495 - .01659 - .01842) + .9511(.09252 - .03265 + .02456 + .01244)

= .03518 + .09213 = .12731; $2Y_8$ = .12730

TABLE IV A

CONSTANTS REQUIRED IN SOLUTION OF A_n COEFFICIENTS

ANTI-SYMMETRICAL

AILERON ANGLE = 1 RADIAN

ITEM			
U_0	$\frac{m_5 C_5}{4\ b}$	$\frac{5.574 \times 191.35}{4 \times 952}$ =	.2801
$2P_2$	$2C_0 - C_4 + 4U_0$	2.1640 + .3133 + 1.1204 =	3.5977
$2P_4$	$2C_0 - C_8 + 8U_0$	2.1640 + .0805 + 2.2408 =	4.4853
$2P_6$	$2C_0 - C_{12} + 12\ U_0$	2.1640 + .0682 + 3.3612 =	5.5934
$2P_8$	$2C_0 - C_{16} + 16\ U_0$	2.1640 + .0785 + 4.4816 =	6.7241
$2P_{10}$	$2C_0 - C_{20} + 20U_0$	2.1640 + .0415 + 5.6020 =	7.8075
D	$C_2 - C_6$	-.2390 + .0841 =	-.1549
E	$C_2 - C_{10}$	-.2390 + .0458 =	-.1932
F	$C_2 - C_{14}$	-.2390 + .0601 =	-.1789
G	$C_2 - C_{18}$	-.2390 +.0710 =	-.1680
H	$C_4 - C_8$	-.3133 + .0805 =	-.2328
I	$C_4 - C_{12}$	-.3133 + .0682 =	-.2451
J	$C_4 - C_{16}$	-.3133 + .0785 =	-.2348
K	$C_6 - C_{10}$	-.0841 + .0458 =	-.0383
L	$C_6 - C_{14}$	-.0841 + .0601 =	-.0240
M	$C_8 - C_{12}$	-.0805 + .0682 =	-.0123

TABLE VI A
SOLUTION OF A_n COEFFICIENTS
ANTI-SYMMETRICAL

AILERON ANGLE = 1.0 RADIAN

	1	2	3a	3b	3c	4a	4b	4c
1	$(C_2 - C_6)A_4$	-.1549	---	.02197	.02154	---	-.00340	-.00334
2	$(C_4 - C_8)A_6$	-.2328	---	-.00346	-.00354	---	.00081	.00082
3	$(C_6 - C_{10})A_8$	-.0383	---	-.00575	-.00572	---	.00022	.00022
4	$(C_8 - C_{12})A_{10}$	-.0123	---	.00214	.00214	---	-.00003	-.00003
5	1 + 2 + 3 + 4					---	-.00240	-.00233
6	$2B_2$ - 5	$2B_2$ = .13981				.13981	.14221	.14214
7	A_2 = 6 / $2P_2$	$2P_2$ = 3.5977				.03886	.03953	.03951
8	$(C_2 - C_6)A_2$	-.1549	.03886	.03953	.03951	-.00602	-.00612	-.00612
9	$(C_2 - C_{10})A_6$	-.1932	---	-.00346	-.00354	---	.00067	.00068
10	$(C_4 - C_{12})A_8$	-.2451	---	-.00575	-.00572	---	.00141	.00140
11	$(C_6 - C_{14})A_{10}$	-.0240	---	.00214	.00214	---	-.00005	-.00005
12	8 + 9 + 10 + 11					-.00602	-.00409	-.00409
13	$2B_4$ - 12	$2B_4$ = .09252				.09854	.09661	.09661
14	A_4 = 13 / $2P_4$	$2P_4$ = 4.4853				.02197	.02154	.02154
15	$(C_4 - C_8)A_2$	-.2328	.03886	.03953	.03951	-.00905	-.00920	-.00920
16	$(C_2 - C_{10})A_4$	-.1932	.02197	.02154	.02154	-.00424	-.00416	-.00416
17	$(C_2 - C_{14})A_8$	-.1789	---	-.00575	-.00572	---	.00103	.00102
18	$(C_4 - C_{16})A_{10}$	-.2348	---	.00214	.00214	---	-.00050	-.00050
19	15 + 16 + 17 + 18					-.01329	-.01283	-.01284
20	$2B_6$ - 19	$2B_6$ = -.03265				-.01936	-.01982	-.01981
21	A_6 = 20 / $2P_6$	$2P_6$ = 5.5934				-.00346	-.00354	-.00354
22	$(C_6 - C_{10})A_2$	-.0383	.03886	.03953	.03951	-.00149	-.00151	-.00151
23	$(C_4 - C_{12})A_4$	-.2451	.02197	.02154	.02154	-.00538	-.00528	-.00528
24	$(C_2 - C_{14})A_6$	-.1789	-.00346	-.00354	-.00354	.00062	.00063	.00063
25	$(C_2 - C_{18})A_{10}$	-.1680	---	.00214	.00214	---	-.00036	-.00036
26	22 + 23 + 24 + 25					-.00625	-.00652	-.00652
27	$2B_8$ - 26	$2B_8$ = -.04495				-.03870	-.03843	-.03843
28	A_8 = 27 / $2P_8$	$2P_8$ = 6.7241				-.00575	-.00572	-.00572
29	$(C_8 - C_{12})A_2$	-.0123	.03886	.03953	.03951	-.00048	-.00049	-.00049
30	$(C_6 - C_{14})A_4$	-.0240	.02197	.02154	.02154	-.00053	-.00052	-.00052
31	$(C_4 - C_{16})A_6$	-.2348	-.00346	-.00354	-.00354	.00081	.00083	.00083
32	$(C_2 - C_{18})A_8$	-.1680	-.00575	-.00572	-.00572	.00097	.00096	.00096
33	29 + 30 + 31 + 32					.00077	.00075	.00078
34	$2B_{10}$ - 33	$2B_{10}$ = .01749				.01672	.01671	.01671
35	A_{10} = 34 / $2P_{10}$	$2P_{10}$ = 7.8075				.00214	.00214	.00214

TABLE VI A
SEMI-WING LIFT AND DRAG COEFFICIENTS
ANTI-SYMMETRICAL

1	2	3	4	5
ODD CIRCULATION CONSTANTS FOR NO TWIST & $C_L = 1.0$	Even circulation constants for zero angle of attack of basic wing and aileron angle of one radian.			$3A_1A_2 = .01777$ $5A_2A_3 = -.00050$ $7A_3A_4 = -.00036$ $9A_4A_5 = .00132$
$A_1 = .14988$	$A_2 = .03951$	$2(A_2)^2 = .003122$	$\frac{2}{3}A_2 = .026340$	$11A_5A_6 = -.00027$
$A_3 = -.00254$	$A_4 = .02154$	$4(A_4)^2 = .001856$	$-\frac{4}{15}A_4 = -.005744$	$13A_6A_7 = -.00001$
$A_5 = .00683$	$A_6 = -.00354$	$6(A_6)^2 = .000075$	$\frac{6}{35}A_6 = -.000607$	$15A_7A_8 = -.00002$
$A_7 = .00020$	$A_8 = -.00572$	$8(A_8)^2 = .000262$	$-\frac{8}{63}A_8 = .000726$	$17A_8A_9 = -.00009$
$A_9 = .00095$	$A_{10} = .00214$	$10(A_{10})^2 = .000046$	$\frac{10}{99}A_{10} = .000216$	$19A_9A_{10} = .00004$
		$\Sigma nA_n^2 = .005361$	$\Sigma = .020931$	$\Sigma(2n+1)A_nA_{n+1} = .01786$

LIFT COEFFICIENT OF SEMI-WING, $C_{L_b} = 4A\,U_0 \Sigma$ COL. 4 $= 4 \times 7.582 \times .2801 \times .020931 = .1778$

$$C_{D_{i_b}} = \pi A U_0^2 \Sigma nA_n^2 = 3.1416 \times 7.582 \times \overline{.2801}^2 \times .005361 = .01002$$

ROLLING MOMENT COEFFICIENT, $C_l' = \dfrac{\text{ROLLING MOMENT}}{qbS} = \dfrac{\pi}{4} A U_0 A_2 = .7854 \times 7.582 \times .2801 \times .03951$
$= .06590$

YAWING MOMENT COEFFICIENT $\dfrac{C_n'}{C_L} = \dfrac{\text{YAWING MOMENT}}{qbS} = \dfrac{\pi A}{4} U_0^2 \Sigma (2n+1) A_n A_{n+1}$

$= .7854 \times 7.582 \times \overline{.2801}^2 \times .01786 = .00834$

$$\bar{y}_{L_b} = \left[\frac{\pi \quad A_2}{8 \Sigma \text{ COL. } 4}\right]\frac{b}{2} = \frac{3.1416 \times .03951}{8 \times .020931} \times \frac{952}{2} = .74127 \times 476 = 352.84''$$

TABLE VII A

COMPUTATION OF LOAD DISTRIBUTIONS

ANTI-SYMMETRICAL

AILERON ANGLE = 1.0 RADIAN

1	a	b	1	2	3	4	5	6	7	8	9
2	TABLE I LINE 3	θ	81	72	63	54	45	36	27	18	9
3		SIN 2θ	.3090	.5878	.8090	.9511	1.0000	.9511	.8090	.5878	.3090
4		SIN 4θ	-.5878	-.9511	-.9511	-.5878	0	.5878	.9511	.9511	.5878
5		SIN 6θ	.8090	.9511	.3090	-.5878	-1.0000	-.5878	.3090	.9511	.8090
6		SIN 8θ	-.9511	-.5878	.5878	.9511	0	-.9511	-.5878	.5878	.9511
7		SIN 10θ	1.0000	0	-1.0000	0	1.0000	0	-1.0000	0	1.0000
8	A_2 = .03951	A_2 SIN 2θ	.01221	.02322	.03196	.03758	.03951	.03758	.03196	.02322	.01221
9	A_4 = .02154	A_4 SIN 4θ	-.01266	-.02049	-.02049	-.01266	0	.01266	.02049	.02049	.01266
10	A_6 = -.00354	A_6 SIN 6θ	-.00286	-.00337	-.00109	.00208	.00354	.00208	-.00109	-.00337	-.00286
11	A_8 = -.00572	A_8 SIN 8θ	.00544	.00336	-.00336	-.00544	0	.00544	.00336	-.00336	-.00544
12	A_{10} = .00214	A_{10} SIN 10θ	.00214	0	-.00214	0	.00214	0	-.00214	0	.00214
13	(8)+(9)+(10)+(11)+(12)	$\sum A_n \sin n\theta$	.00427	.00272	.00488	.02156	.04519	.05776	.05258	.03698	.01871
14	TABLE I [(11)$_a$ × (13)]	$\frac{m_s c_s}{c}$	6.236	7.054	8.058	9.276	10.722	12.368	14.112	16.187	27.553
15	(13) × (14)	c_{ℓ_b}	.02663	.01919	.03932	.19999	.48453	.71438	.74201	.59860	.51552
16	TABLE I LINE 11	m_o	5.581	5.590	5.601	5.615	5.632	5.650	5.670	5.688	5.701
17	TABLE I LINE 6	α_a	0	0	0	0	.1647	.2045	.2045	.2060	.1285
18	(15) / (16)	α_o	.00477	.00343	.00702	.03562	.08603	.12644	.13087	.10524	.09043
19	(17) − (18)	$\frac{W}{V}$	-.00477	-.00343	-.00702	-.03562	.07867	.07806	.07363	.10076	.03807
20	(19) × (15)	$c_{d_{ib}}$	-.00013	-.00006	-.00028	-.00712	.03812	.05576	.05463	.06031	.01963
21	TABLE I LINE 9	c	171.03	151.20	132.36	114.98	99.48	86.24	75.58	65.89	38.71
22	(21) × (15)	cc_{ℓ_b}	4.555	2.902	5.204	22.995	48.201	61.608	56.081	39.442	19.956
23	(22) × (3)	cc_{ℓ_b} SIN 2θ	1.407	1.706	4.210	21.871	48.201	58.595	45.370	23.184	6.166
24		SIN θ	.9877	.9511	.8910	.8090	.7071	.5878	.4540	.3090	.1564
25	(20) × (21)	$cc_{d_{ib}}$	-.022	-.009	-.037	-.819	3.792	4.809	4.129	3.974	.760
26	(25) × (24)	$cc_{d_{ib}}$ SIN θ	-.022	-.009	-.033	-.663	2.681	2.827	1.875	1.228	.119

LINE 26: $\frac{\pi}{20}\frac{b}{S}\sum cc_{d_{ib}} \sin\theta = \frac{\pi}{20 c_{AVE}}\left(\sum (26)\right) = C_{D_{ib}} = \frac{3.1416}{2511.2}(8.003) = .01001$

LINE 23: $C_\ell' = \frac{\text{ROLLING MOM.}}{q\,b\,S} = \frac{\pi}{4} A u_o A_2 = \frac{\pi}{80}\frac{b}{S}\sum cc_{\ell_b} \sin 2\theta = \frac{\pi}{80 c_{AVE}}\left(\sum (23)\right)$

$= \frac{3.1416}{10044.8}(210.710) = .06590$

.BLE VIII A

UNIT LOAD DISTRIBUTIONS

ANTI-SYMMETRICAL

AILERON ANGLE = 1.0 RADIAN

C_L = 4.5215

1	a	b	1	2	3	4	5	6	7	8	9
2	TABLE VIIA LINE 15	C_{ℓ_b}	.02663	.01919	.03932	.19999	.48453	.71438	.74201	.59860	.51552
3	TABLE VII LINE 22	$C_{\ell_{a_1}}$	.9678	1.0106	1.0385	1.0619	1.0711	1.0506	.9806	.8341	.7776
4	TABLE VIIA LINE 20	$C_{d_{i_b}}$	-.00013	-.00006	-.00028	-.00712	.03812	.05576	.05463	.06031	.01963
5	TABLE VII LINE 20	C_{d_i}	.9447	.8340	.7592	.6957	.6785	.7563	.9666	1.2707	1.3476
6	(5) / C_L^2	$C_{d_{i_{a_1}}}$	.04621	.04079	.03714	.03403	.03319	.03699	.04728	.06216	.06592
7	(2) / (3)	$C_{\ell_b} / C_{\ell_{a_1}}$	.02752	.01899	.03786	.18833	.45237	.67997	.75669	.71766	.66296
8	(6) × (7)	$C_{d_{i_{a_1}}} \times C_{\ell_b} / C_{\ell_{a_1}}$	.00127	.00077	.00141	.00641	.01501	.02515	.03578	.04461	.04370
9	(4) / (7)	$C_{d_{i_b}} \times C_{\ell_{a_1}} / C_{\ell_b}$	-.00472	-.00316	-.00740	-.03781	.08427	.08200	.07220	.08404	.02961
10	(8) + (9)	$C_{d_{i_{a,b}}}$	-.00345	-.00239	-.00599	-.03140	.09928	.10715	.10798	.12865	.07331
11		SIN 2θ	.3090	.5878	.8090	.9511	1.0000	.9511	.8090	.5878	.3090
12	TABLE I LINE 9	C	171.03	151.20	132.36	114.98	99.48	86.24	75.58	65.89	38.71
13	(10) × (12)	$C\,C_{d_{i_{a,b}}}$	-.590	-.361	-.793	-3.610	9.876	9.241	8.161	8.477	2.838
14	(11) × (13)	$C C_{d_{i_{a,b}}} \times \text{SIN } 2\theta$	-.182	-.212	-.642	-3.433	9.876	8.789	6.602	4.983	.877

$$\frac{C_n'}{C_L} = \frac{\pi}{80 C_{AVE}} \Sigma (14) = .00031276(26.658) = .00834$$

TABLE XII

USE OF UNIT VALUES IN DETERMINING DISTRIBUTIONS FOR

$\alpha = 12.2°$, $\delta_F = 0°$, $\delta_A = -30°, +6°$

1	a	b	$r\delta_A$	C_L	0	1	2	3	4	5	6	7	8	9
2	TABLE VIII LINE 5	$c_{l_{b_1}}$	1.0	-	-.11173	.15193	-.16667	-.13127	.01472	.26939	.51556	.58484	.44573	.29116
3	TABLE VIII LINE 11	$c_{d_{i_{b_1}}}$	1.0	-	.00254	.00230	.00218	.00256	-.00067	.01993	.03627	.03418	.03777	.00940
4	TABLE VIII LINE 17	$c_{d_{i_{a_1 b_1}}}$	1.0	1.0	-.02526	-.02253	-.01935	-.02497	-.04787	.08759	.09206	.08551	.10390	.04999
5	TABLE VIIIA LINE 2	$c_{l_{b_2}}$	1.0	-	0	.02663	.01919	.03932	.19997	.48453	.71438	.74201	.59860	.51557
6	TABLE VIIIA LINE 4	$c_{d_{i_{b_2}}}$	1.0	-	0	.00013	-.00006	-.00028	-.00712	.03812	.05576	.05463	.06031	.01963
7	TABLE VIIIA LINE 10	$c_{d_{i_{a_2 b_2}}}$	1.0	1.0	0	-.00345	-.00239	-.00599	-.03142	.09928	.10715	.10798	.12865	.07331
8	-.254 × (2)	$c_{l_{b_1}}$	-.254	-	.03346	.03836	.04233	+.03329	-.00374	-.06843	-.13095	-.14855	-.11322	-.08157
9	$(.254)^2$ × (3)	$c_{d_{i_{b_1}}}$	-.254	-	+.00016	+.00015	+.00014	+.00017	-.00004	+.00129	+.00234	-.00221	+.00244	+.00061
10	-.254 × (4)	$c_{d_{i_{a_1 b_1}}}$	-.254	1.0	.00642	.00572	.00507	.00634	.01216	-.02225	-.02338	-.02172	-.02639	-.01270
11	.484 × (5)	$c_{l_{b_2}}$	.484	-	0	.01289	.00929	.01903	.09680	.23451	.34576	.35913	.28972	.24951
12	$(.484)^2$ × (6)	$c_{d_{i_{b_2}}}$	.484	-	0	.00003	-.00001	-.00007	-.00167	.00893	.01306	.01280	.01413	.00460
13	.484 × (7)	$c_{d_{i_{a_2 b_2}}}$	.484	1.0	0	-.00167	-.00116	-.00290	-.01520	.04805	.05186	.05226	.06227	.03548
14	TABLE VIII LINE 13	$c_{d_{i_{a_1}}}$	-	1.0	.05454	.04621	.04079	.03714	.03403	.03319	.03699	.04728	.06216	.06592
15	TABLE VIII LINE 2	$c_{l_{a_1}}$	-	1.0	.8939	.9678	1.0106	1.0385	1.0619	1.0711	1.0506	.9806	.8341	.7776
16	1.1344 × (15)	c_{l_a}	-	1.1344	1.0118	1.0979	1.1464	1.1781	1.2046	1.2151	1.1918	1.1124	.9462	.8821
17	(16) + (8)	$c_{l_a} + c_{l_{b_1}}$	-.254	1.1344	1.0453	1.1363	1.1887	1.2114	1.2009	1.1467	1.0608	.9639	.8330	.8005
18	(17) + (11)	c_l	.230	1.1344	1.0453	1.1492	1.1980	1.2304	1.2977	1.3812	1.4066	1.3230	1.1227	1.0500
19	(17) - (11)	c_l	-.738	1.1344	1.0453	1.1234	1.1794	1.1924	1.1041	.9122	.7150	.6048	.5433	.5510
20	(11) / (8)	$c_{l_{b_2}} / c_{l_{b_1}}$		-	0	.32603	.21947	.57164	-25.882	-3.4270	-2.6404	-2.4176	-2.5589	-3.0588
21	(9) × (20)	$c_{d_{i_{b_1}}} \times c_{l_{b_2}} / c_{l_{b_1}}$		-	0	+.00005	+.00003	+.00010	+.00104	-.00442	-.00618	-.00534	-.00624	-.00187
22	(12) / (20)	$c_{d_{i_{b_2}}} \times c_{l_{b_1}} / c_{l_{b_2}}$		-	0	-.00009	-.00005	.00012	.00006	-.00261	-.00495	-.00529	-.00552	-.00150
23	(9) + (12)	$c_{d_{i_{b_1}}} + c_{d_{i_{b_2}}}$		-	.00016	.00012	.00013	.00010	-.00171	.01022	.01540	.01501	.01657	.00521
24	(21) + (22)	$c_{d_{i_{b_1 b_2}}}$		-	0	-.00004	-.00002	-.00002	.00110	-.00703	-.01113	-.01063	.01176	-.00337
25	(23) + (24)	$c_{d_{i_b}}$	.230	-	.00016	.00008	.00011	.00008	-.00061	.00319	.00427	.00438	.00481	.00184
26	(23) - (24)	$c_{d_{i_b}}$	-.738	-	.00016	.00016	.00015	.00012	-.00281	.01725	.02653	.02564	.02833	.00858
27	(10) + (13)	$c_{d_{i_{a,b}}}$	.230	1.0	.00642	.00405	.00391	.00344	-.00304	.02580	.02848	.03054	.03588	.02278
28	(10) - (13)	$c_{d_{i_{a,b}}}$	-.738	1.0	.00642	.00739	.00623	.00924	.02736	-.07030	-.07524	-.07398	-.08866	-.04818
29	1.1344 × (27)	$c_{d_{i_{ab}}}$	.230	1.1344	.00728	.00459	.00444	.00390	-.00345	.02927	.03231	.03464	.04070	.02584
30	1.1344 × (28)	$c_{d_{i_{ab}}}$	-.738	1.1344	.00728	.00838	.00707	.01048	.03104	-.07975	-.08535	-.08392	-.10058	-.05466
31	1.1344^2 × (14)	$c_{d_{i_a}}$	-	1.1344	.07019	.05947	.05249	.04779	.04379	.04271	.04760	.06084	.07999	.08483
32	(25) + (29) + (31)	c_{d_i}	.230	1.1344	.07763	.06414	.05704	.05177	.03973	.07517	.08418	.09986	.12550	.11251
33	(26) + (30) + (31)	c_{d_i}	-.738	1.1344	.07763	.06801	.05971	.05839	.07202	-.01979	-.01122	.00256	.00774	.03875
					ALTERNATIVE METHOD OF DETERMINING c_{d_i}									
34	TABLE I LINE 11	m_o	-	-	5.574	5.581	5.590	5.601	5.615	5.632	5.650	5.670	5.688	5.701
35	TABLE VIII LINE 8	α_{a_1}	1.0	0	-.04290	-.04290	-.04290	-.04290	-.04290	.12180	.16160	.16160	.16210	.08560
36	TABLE VIIA LINE 17	α_{a_2}	1.0	0	0	0	0	0	0	.1647	.2045	.2045	.2060	.1285
37	-.254 × (35)	α_{a_1}	-.254	0	.01090	.01090	.01090	.01090	.01090	-.03094	-.04105	-.04105	-.04143	-.02174
38	.484 × (36)	α_{a_2}	.484	0	0	0	0	0	0	+.07971	+.09898	+.09898	+.09970	+.06219
39	(37) + (38)	α_a	.230	0	.01090	.01090	.01090	.01090	.01090	.04877	.05793	.05793	.05827	.04045
40	(37) - (38)	α_a	-.738	0	.01090	.01090	.01090	.01090	.01090	-.11065	-.14003	-.14003	-.14113	-.08393
41	.2509 + (39)	α_a	.230	1.1344	.2618	.2618	.2618	.2618	.2618	.29967	.30883	.30883	.30917	.29135
42	(18) / (34)	α_o	.230	1.1344	.18753	.20591	.21431	.21968	.23111	.24524	.24896	.23333	.19738	.18418
43	(41) - (42)	$\frac{w}{V}$	.230	1.1344	.07427	.05589	.04749	.04212	.03069	.05443	.05987	.07550	.11179	.10717
44	(18) × (43)	c_{d_i}	.230	1.1344	.07763	.06423	.05689	.05182	.03983	.07518	.08421	.09989	.12551	.11253
45	.2509 + (40)	α_a	-.738	1.1344	.2618	.2618	.2618	.2618	.2618	.14025	.11087	.11087	.10977	.16697
46	(19) / (34)	α_o	-.738	1.1344	.18753	.20129	.21098	.21289	.19663	.16197	.12655	.10667	.09552	.09665
47	(45) - (46)	$\frac{w}{V}$	-.738	1.1344	.07427	.06051	.05082	.04891	.06517	-.02172	-.01568	.00420	.00425	.07032
48	(19) × (47)	c_{d_i}	-.738	1.1344	.07763	.06798	.05994	.05832	.07195	-.01981	-.01121	.00254	.00774	.03875
49		SIN 2θ			0	.3090	.5878	.8090	.9511	1.000	.9511	.8090	.5878	.3090
50	TABLE I LINE 9	c			0	171.03	151.20	132.36	114.98	99.48	86.24	75.58	65.89	28.71
51	(24) × (50)	$c\, c_{d_{i_{b_1 b_2}}}$			0	-.0068	-.0030	-.0026	+.1265	-.6993	-.9599	-.8034	-.7749	-.1305
52	(50) × (51)	$c\, c_{d_{i_{b_1 b_2}}}$ SIN 2θ			0	-.0021	-.0018	-.0021	+.1203	-.6993	-.9130	-.6500	-.4555	-.0403

$$C'_n = \frac{\pi}{80\, C_{AVE}} \Sigma (52) = .00031276(-2.6438) = -.000827$$

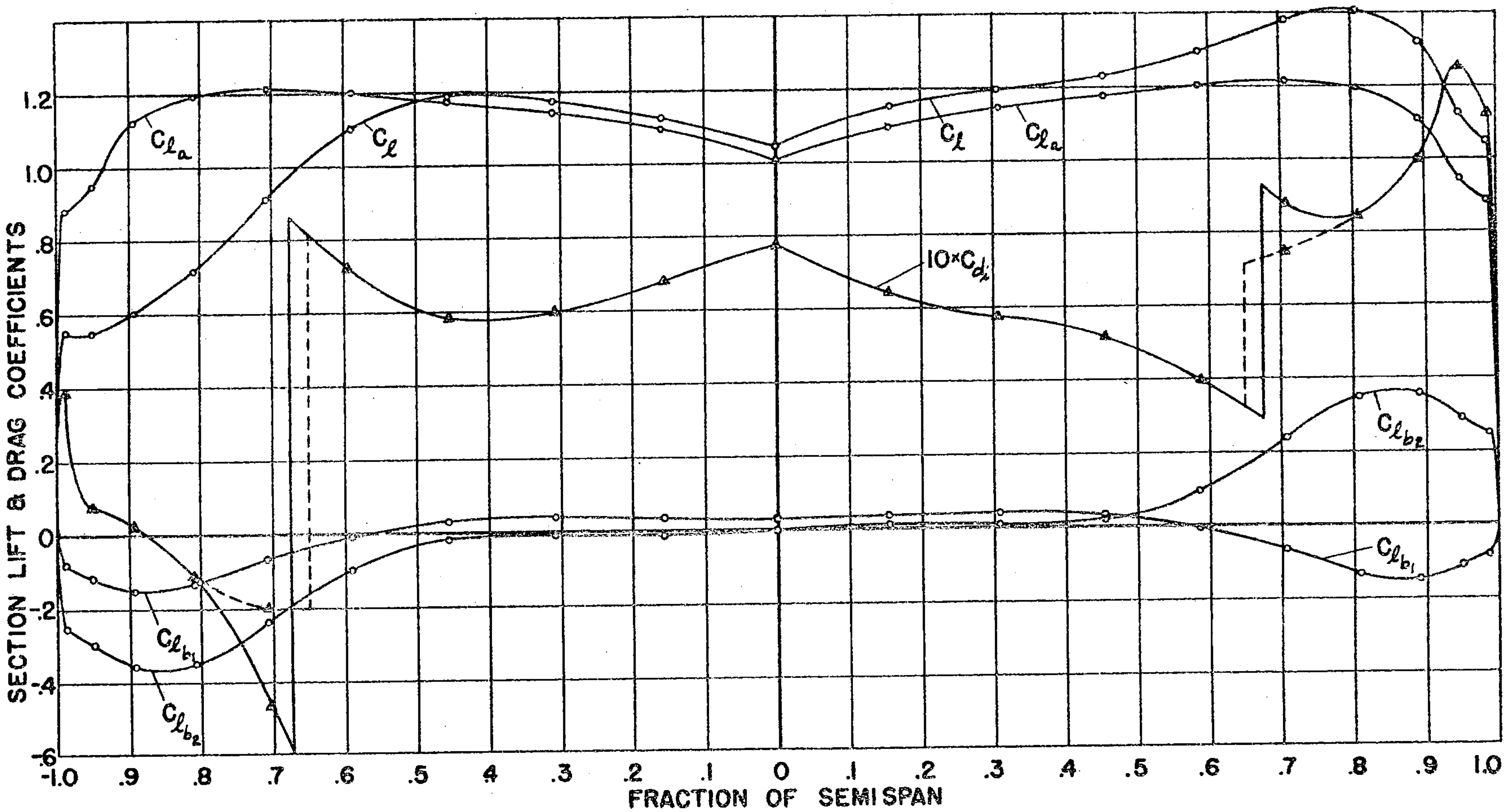

FIG.7-5 SECTION LIFT & INDUCED DRAG COEFFICIENTS FOR WING WITH DISPLACED AILERONS. $\alpha = 13.9^\circ$, $\delta_F = 0^\circ$ $\delta_A = -30^\circ, +6^\circ$

7.5 EXAMPLE 5 - WING WITH ANGULAR VELOCITY IN ROLL.

7.50 Introduction - It has been shown in the preceding section how the lift and induced-drag distributions may be obtained for a wing with displaced ailerons. It should be emphasized that these conditions exist only for an instant after the ailerons are assumed to be thrown hard over, before the airplane has time to obtain an angular velocity in roll. With the ailerons in this position, the resulting rolling moment gives the airplane an angular acceleration which causes it to roll. As soon as a rolling velocity develops, a damping moment occurs which opposes the roll. This damping moment is produced by a change in angle of attack, caused by a combination of the forward velocity with a velocity normal to the flight path, that varies linearly along the span from zero at the center to maximum positive and negative values at the respective wing tips. The purpose of this example is to demonstrate the calculation of the distributions of lift and induced-drag coefficients resulting from the roll and their combination with the other distributions.

7.51 Damping Distributions - For the purpose of computing the damping distributions, the wing is first assumed to have an antisymmetrical linear twist in which the angle of attack at each section is given by $p'y/V$, in which

p'/V is the ratio of the angular velocity about the wind axis to the forward velocity. The antisymmetrical tables are used, in which the angle of attack at the extreme tip, $p'b/2V$, is taken as one radian, in which case the angles of attack at the other points are given by line 2 of Table I. It can be shown that for a wing with such a twist and zero angle of attack at the center, all of the B_n coefficients are zero, except B_2 which is equal to 0.5. It is, therefore, unnecessary to fill in Table II-A and, since the same wing will be used as in the previous examples, Table I does not need to be repeated. The A_n coefficients are found in Table V-A, using $2B_2 = 1.0$, and obtaining the values of the other constants from Table IV-A of the preceding section. Table VI-A is filled in except for columns 1, 3 and 5. The section values of c_{l_b} for a wing with this twist are computed in Table VII-A in the usual manner, and a check of C_l' made by comparing the integrated value at the bottom of this table with the value calculated in Table VI-A. All calculations involving induced drag may be omitted.

7.52 <u>Difference Between Damping and Linear Twist Distributions</u> - The reason for omitting the induced drag in the computations for the damping distribution is that such a distribution, if computed, could not be combined in the usual manner with other loadings. This is because the directions in which the section lifts act in a uniformly twisted wing are not the same as for a wing during roll; since they act normal to the local wind at all times, and this direction changes when the wing is rolling. This difficulty is overcome, as will be shown later, by using the alternative method of computing the c_{d_i} distributions for the combined loadings. The same procedure would be necessary in the case of an unsymmetrical gust, where the effective twist is likewise caused by a twist in the relative wind, rather than by a twist in the wing itself.

7.53 <u>Evaluation of the Damping Moment</u> - The damping distribution will be combined with the loadings determined in the previous example for the wing with flap displacement zero, ailerons up 30 degrees and down 6 degrees, at an angle of attack of 13.9 degrees with the chord line. Before this can be accomplished, it is necessary to decide on the magnitude of the damping loads to be used, since the damping distribution was calculated for a unit value of $p'b/2V$. For this purpose, it will be assumed that the right aileron is displaced down and the left up, which will start the airplane rolling to the left. The angular velocity will continue to increase until the damping moment becomes equal to the rolling moment applied by the displaced ailerons before rolling started, at which angular velocity the airplane will continue to roll at a uniform rate as long as the ailerons are held in this position. The rolling-moment coefficient applied by the ailerons was found to be 0.03190 in Paragraph 7.433. Since the damping moment coefficient for $(p'b/2V) = 1.0$ was found in Table VI-A to be 0.46571, the value of $p'b/2V$ for maximum rolling velocity is

$$\frac{p'b}{2V} = \frac{.03190}{.46571} = .06850$$

The total distributions for each wing will be computed, both for the condition when the rolling velocity has reached its maximum, and for the same rolling velocity with the ailerons fully reversed.

7.54 Computation of the Combined Distributions - The computation of the total loadings is accomplished in Table XIII and, since four different distributions of both c_l and c_{d_i} are necessary, the work will be described in detail in order to avoid confusion.

(1) Line 1 gives the usual column headings, except that a column for $p'b/2V$ has been added to distinguish between unit value and the value indicating the actual damping loads used; and a column for F_{δ_A} indicating by the use of +.230 and -.738 whether the aileron is down or up, respectively.

(2) Line 2 gives the distribution of the damping c_l , here designated as c_{l_b}, for $p'b/2V$ as

copied from Table VII-A, line 15.

(3) Line 3 gives the value of the section slopes as originally given in Table I, line 11.

(4) Line 4 gives the value of c_{l_b} for a value of $p'b/2V$ = .06850, found by multiplying line 2 by .06850.

(5) In line 5 is copied from Table XII, line 41, the absolute angles of attack for the wing with aileron down before rolling starts.

(6) Line 6 gives the c_l distribution for the wing with aileron down as given in Table XII, line 18, before rolling starts.

(7) Line 7 is found by subtracting line 4 from line 6 and represents the c_l distribution on the down aileron side after maximum rolling velocity has been reached.

(8) Line 8 gives the effective angles of attack found by dividing the c_l values in line 7 by the m_o values in line 3.

(9) Line 9 is found by subtracting line 8 from line 5 and represents the angle between the relative wind at each section and the direction of flight.

(10) Line 10 is found by multiplying line 7 by line 9 and represents the induced-drag-coefficient distribution for the wing with aileron down

after maximum rolling velocity has been reached.

(11) Line 11 is found by adding line 4 to line 6 and represents the c_l distribution on the down aileron side immediately after fully reversing the ailerons at maximum rolling velocity. If it is assumed that the right aileron is down at the start, this line gives the values for the left wing after reversing the ailerons.

(12) The c_{d_i} distribution for this same wing after the ailerons are reversed is found in line 14 by means of lines 12 and 13 in the same way as was done above for the other wing before reversing the ailerons.

(13) The absolute angles of attack for the wing with aileron up before rolling starts is copied into line 15 from Table XII, line 45.

(14) Line 16 gives the c_l distribution for the wing with aileron up, as given in Table XII, line 19, before rolling starts.

(15) Line 17 is found by adding line 4 to line 16 and represents the c_l distribution on the up aileron side after maximum rolling velocity has been attained.

(16) The c_{d_i} distribution is computed in the same way as before in lines 18, 19 and 20.

(17) Line 21 is found by subtracting line 4 from line 16, and represents the c_l distribution on the up aileron side immediately after fully reversing the ailerons at maximum rolling velocity. If it is assumed that the right aileron is down at the start, this line gives the values for the right wing after reversal of the ailerons.

(18) The c_{d_i} values are found for this condition as for the others.

7.55 <u>Yawing Moment</u> - Lines (25-31) have been added to Table XIII for the purpose of computing the yawing-moment coefficient. This is done by the usual integration, except that, in this case, one-half the differences in c_{d_i} values on the two wings for each condition are used; because the distribution of c_{d_i} is unsymmetrical but not antisymmetrical. The value of C_n' for the wing was found to be 0.00872 immediately before reversing the ailerons and .00122 immediately after reversal. In Paragraph 7.436 the value before rolling started was found to be .00375. This shows that the change in C_n' produced by a complete reversal of the ailerons is twice that produced by full displacement before rolling starts, as it should be:

$$.00872 - .00122 = .00750 = 2 \times .00375$$

57882 O—38——15

This neglects, of course, the yawing produced by the increase in profile drag of a displaced aileron.

7.56 Plot of Results - The values of c_l and c_{d_i} for the full span wing before and after the ailerons are reversed at maximum rolling velocity are plotted in Figure 7-6.

TABLE V A
SOLUTION OF A_n COEFFICIENTS
ANTI-SYMMETRICAL

$\frac{p'b}{2V} = 1.0$

	1	2	3a	3b	3c	4a	4b	4c
1	$(C_2 - C_6)A_4$	-.1549	---	.00960	.01028	----	-.00149	-.00159
2	$(C_4 - C_8)A_6$	-.2328	---	.01190	.01208	----	-.00277	-.00281
3	$(C_6 - C_{10})A_8$	-.0383	---	.00225	.00231	----	-.00009	-.00009
4	$(C_8 - C_{12})A_{10}$	-.0123	---	.00087	.00089	----	-.00001	-.00001
5	1 + 2 + 3 + 4					----	-.00436	-.00450
6	$2B_2$ - 5	$2B_2$ = 1.0				1.0	1.00436	1.00450
7	A_2 = 6 / $2P_2$	$2P_2$ = 3.5977				.2780	.27917	.27921
8	$(C_2 - C_6)A_2$	-.1549	.2780	.27917	.27921	-.04306	-.04324	-.04325
9	$(C_2 - C_{10})A_6$	-.1932	----	.01190	.01208	----	-.00230	-.00233
10	$(C_4 - C_{12})A_8$	-.2451	----	.00225	.00231	----	-.00055	-.00057
11	$(C_6 - C_{14})A_{10}$	-.0240	----	.00087	.00089	----	-.00002	-.00002
12	8 + 9 + 10 + 11					-.04306	-.04611	-.04617
13	$2B_4$ - 12	$2B_4$ = 0				.04306	.04611	.04617
14	A_4 = 13 / $2P_4$	$2P_4$ = 4.4853				.0096	.01028	.01029
15	$(C_4 - C_8)A_2$	-.2328	.2780	.27917	.27921	-.06472	-.06499	-.06500
16	$(C_2 - C_{10})A_4$	-.1932	.0096	.01028	.01029	-.00185	-.00199	-.00199
17	$(C_2 - C_{14})A_8$	-.1789	----	.00225	.00231	----	-.00040	-.00041
18	$(C_4 - C_{16})A_{10}$	-.2348	----	.00087	.00089	----	-.00020	-.00021
19	15 + 16 + 17 + 18					-.06657	-.06758	-.06761
20	$2B_6$ - 19	$2B_6$ = 0				.06657	.06758	.06761
21	A_6 = 20 / $2P_6$	$2P_6$ = 5.5934				.01190	.01208	.01209
22	$(C_6 - C_{10})A_2$	-.0383	.2780	.27917	.27921	-.01065	-.01069	-.01069
23	$(C_4 - C_{12})A_4$	-.2451	.0096	.01028	.01029	-.00235	-.00252	-.00252
24	$(C_2 - C_{14})A_6$	-.1789	.01190	.01208	.01209	-.00213	-.00216	-.00216
25	$(C_2 - C_{18})A_{10}$	-.1680	----	.00087	.00089	----	-.00015	-.00015
26	22 + 23 + 24 + 25					-.01513	-.01552	-.01552
27	$2B_8$ - 26	$2B_6$ = 0				.01513	.01552	.01552
28	A_8 = 27 / $2P_8$	$2P_8$ = 6.7241				.00225	.00231	.00231
29	$(C_8 - C_{12})A_2$	-.0123	.2780	.27917	.27921	-.00342	-.00343	-.00343
30	$(C_6 - C_{14})A_4$	-.0240	.0096	.01028	.01029	-.00023	-.00025	-.00025
31	$(C_4 - C_{16})A_6$	-.2348	.01190	.01208	.01209	-.00279	-.00284	-.00284
32	$(C_2 - C_{18})A_8$	-.1680	.00225	.00231	.00231	-.00038	-.00039	-.00039
33	29 + 30 + 31 + 32					-.00682	-.00691	-.00691
34	$2B_{10}$ - 33	$2B_{10}$ = 0				.00682	.00691	.00691
35	A_{10} = 34 / $2P_{10}$	$2P_{10}$ = 7.8075				.00087	.00089	.00089

TABLE VI A

SEMI-WING LIFT AND DRAG COEFFICIENTS

ANTI-SYMMETRICAL

$\frac{p'b}{2V} = 1.0$

1	2	3	4	5
ODD CIRCULATION CONSTANTS FOR NO TWIST & $C_L = 1.0$				$3A_1A_2 =$
				$5A_2A_3 =$
				$7A_3A_4 =$
				$9A_4A_5 =$
$A_1 =$	$A_2 =$ .27921	$2(A_2)^2 =$	$\frac{2}{3}A_2 =$ .18614	$11A_5A_6 =$
$A_3 =$	$A_4 =$ .01029	$4(A_4)^2 =$	$-\frac{4}{15}A_4 =$ -.00274	$13A_6A_7 =$
$A_5 =$	$A_6 =$ .01209	$6(A_6)^2 =$	$\frac{6}{35}A_6 =$ .00207	$15A_7A_8 =$
$A_7 =$	$A_8 =$ .00231	$8(A_8)^2 =$	$-\frac{8}{63}A_8 =$ -.00029	$17A_8A_9 =$
$A_9 =$	$A_{10} =$ .00089	$10(A_{10})^2 =$	$\frac{10}{99}A_{10} =$ .00009	$19A_9A_{10} =$
		$\Sigma nA_n^2 =$	$\Sigma =$ .18527	$\Sigma(2n+1)A_nA_{n+1} =$

LIFT COEFFICIENT OF SEMI-WING, $C_{L_b} = 4A\,U_0 \Sigma$ COL. 4 $= 4 \times 7.582 \times .2801 \times .18527 = 1.5738$

$C_{D_{i_b}} = \pi A U_0^2 \sum nA_n^2 =$

ROLLING MOMENT COEFFICIENT, $C_l' = \frac{\text{ROLLING MOMENT}}{qbS} = \frac{\pi}{4} A U_0 A_2 = .7854 \times 7.582 \times .2801 \times .27921$
$= .46571$

YAWING MOMENT COEFFICIENT $\frac{C_n'}{C_L} = \frac{\text{YAWING MOMENT}}{qbS} = \frac{\pi A}{4} U_0^2 \Sigma (2n+1) A_n A_{n+1}$

$$\bar{y}_{L_b} = \left[\frac{\pi \quad A_2}{8 \Sigma \text{ COL. } 4}\right]\frac{b}{2} = \frac{3.1416 \times .27921}{8 \times .18527} \times \frac{952}{2} = .59182 \times 476 = 281.70$$

TABLE VII A

COMPUTATION OF LOAD DISTRIBUTIONS

ANTI-SYMMETRICAL

$\frac{p'b}{2V} = 1.0$

1	a	b	1	2	3	4	5	6	7	8	9
2	TABLE I LINE 3	θ	81	72	63	54	45	36	27	18	9
3		SIN 2θ	.3090	.5878	.8090	.9511	1.0000	.9511	.8090	.5878	.3090
4		SIN 4θ	-.5878	-.9511	-.9511	-.5878	0	.5878	.9511	.9511	.5878
5		SIN 6θ	.8090	.9511	.3090	-.5878	-1.0000	-.5878	.3090	.9511	.8090
6		SIN 8θ	-.9511	-.5878	.5878	.9511	0	-.9511	-.5878	.5878	.9511
7		SIN 10θ	1.0000	0	-1.0000	0	1.0000	0	-1.0000	0	1.0000
8	$A_2 = .27921$	A_2 SIN 2θ	.0863	.1641	.2259	.2656	.2792	.2656	.2259	.1641	.0063
9	$A_4 = .01029$	A_4 SIN 4θ	-.0060	-.0098	-.0098	-.0060	0	.0060	.0098	.0098	.0060
10	$A_6 = .01209$	A_6 SIN 6θ	.0098	.0015	.0037	-.0071	-.0121	-.0071	.0037	.0115	.0098
11	$A_8 = .00231$	A_8 SIN 8θ	-.0022	-.0014	.0014	.0022	0	-.0022	-.0014	.0014	.0022
12	$A_{10} = .00089$	A_{10} SIN 10θ	.0009	0	-.0009	0	.0009	0	-.0009	0	.0009
13	8 + 9 + 10 + 11 + 12	ΣA_n SIN $n\theta$	.0888	.1644	.2203	.2547	.2680	.2623	.2371	.1868	.1052
14	TABLE I [11₀ x 13]	$\frac{m_s C_s}{C}$	6.236	7.054	8.058	9.276	10.722	12.368	14.112	16.187	27.553
15	13 x 14	c_{l_b}	.5538	1.1597	1.7752	2.3626	2.8735	3.2441	3.3460	3.0237	2.8986
16	TABLE I LINE 11	m_o									
17	TABLE I LINE 6	α_a									
18	15 / 16	α_o									
19	17 - 18	$\frac{W}{V}$									
20	19 x 15	$c_{d_{i_b}}$									
21	TABLE I LINE 9	C	171.03	151.20	132.36	114.98	99.48	86.24	75.58	65.89	38.71
22	21 x 15	Cc_{l_b}	94.72	175.35	234.97	271.65	285.86	279.77	252.89	199.23	112.05
23	22 x 3	Cc_{l_b} SIN 2θ	29.27	103.07	190.09	258.37	285.86	266.09	204.59	117.11	34.62
24		SIN θ	.9877	.9511	.8910	.8090	.7071	.5878	.4540	.3090	.1564
25	20 x 21	$Cc_{d_{i_b}}$									
26	25 x 24	$Cc_{d_{i_b}}$ SIN θ									

LINE 26: $\frac{\pi}{20}\frac{b}{S}\Sigma Cc_{d_{i_b}} \sin\theta = \frac{\pi}{20 C_{AVE}}(\Sigma 26) = C_{D_{i_b}}$

LINE 23: $C_l' = \frac{\text{ROLLING MOM.}}{q b S} = \frac{\pi}{4} A u_o A_2 = \frac{\pi}{80}\frac{b}{S}\Sigma Cc_{l_b} \sin 2\theta = \frac{\pi}{80 C_{AVE}}(\Sigma 23)$

$= .00031276 \times 1489.07 = .46572$

TABLE XIII

[C]OMBINATION OF DAMPING DISTRIBUTIONS FOR ROLL WITH NORMAL DISTRIBUTIONS FOR

$\alpha = 13.9°, \delta_F = 0°, \& \ \delta_A = -30°, +6°$

1	a	b	$\frac{p'b}{2V}$	$F\delta_A$	0	1	2	3	4	5	6	7	8	9
2	TABLE VII A LINE 15	C_{ℓ_b}	1.0	–	0	.5538	1.1597	1.7752	2.3626	2.8735	3.2441	3.3460	3.0237	2.8986
3	TABLE I LINE 11	m_o	—	—	5.574	5.581	5.590	5.601	5.615	5.632	5.650	5.670	5.688	5.701
4	$\frac{p'b}{2V} \times$ (2)	C_{ℓ_b}	.06850	—	0	.0379	.0794	.1216	.1618	.1968	.2222	.2292	.2071	.1986
5	TABLE XII LINE 41	α_a	0	.230	.2618	.2618	.2618	.2618	.2618	.29967	.30883	.30883	.30917	.29135
6	TABLE XII LINE 18	C_ℓ	0	.230	1.0453	1.1492	1.1980	1.2304	1.2977	1.3812	1.4066	1.3230	1.1227	1.0500
7	(6) - (4)	$C_\ell - C_{\ell_b}$	-.06850	.230	1.0453	1.1113	1.1186	1.1088	1.1359	1.1844	1.1844	1.0938	.9156	.8514
8	(7) / (3)	α_o	-.06850	.230	.1875	.1991	.2001	.1980	.2023	.2103	.2096	.1929	.1610	.1493
9	(5) - (8)	W/V	-.06850	.230	.0743	.0627	.0617	.0638	.0595	.0894	.0992	.1159	.1482	.1421
10	(7) x (9)	C_{d_i}	-.06850	.230	.0777	.0697	.0690	.0707	.0676	.1059	.1175	.1268	.1357	.1210
11	(6) + (4)	$C_\ell + C_{\ell_b}$	+.06850	.230	1.0453	1.1871	1.2774	1.3520	1.4595	1.5780	1.6288	1.5522	1.3298	1.2486
12	(11) / (3)	α_o	+.06850	.230	.1875	.2127	.2285	.2414	.2599	.2802	.2883	.2738	.2338	.2190
13	(5) - (12)	W/V	+.06850	.230	.0743	.0491	.0333	.0204	.0019	.0195	.0205	.0350	.0754	.0724
14	(11) x (13)	C_{d_i}	+.06850	.230	.0777	.0583	.0425	.0276	.0028	.0308	.0334	.0543	.1003	.0904
15	TABLE XII LINE 45	α_a	0	-.738	.2618	.2618	.2618	.2618	.2618	.14025	.11087	.11087	.10977	.16697
[16]	TABLE XII LINE 19	C_ℓ	0	-.738	1.0453	1.1234	1.1794	1.1924	1.1041	.9122	.7150	.6048	.5433	.5510
17	(16) + (4)	$C_\ell + C_{\ell_b}$	+.06850	-.738	1.0453	1.1613	1.2588	1.3140	1.2659	1.1090	.9372	.8340	.7504	.7496
18	(17) / (3)	α_o	+.06850	-.738	.1875	.2081	.2252	.2346	.2254	.1969	.1659	.1471	.1319	.1315
19	(15) - (18)	W/V	+.06850	-.738	.0743	.0537	.0366	.0272	.0364	-.0566	-.0550	-.0362	-.0221	+.0355
20	(19) x (17)	C_{d_i}	+.06850	-.738	.0777	.0624	.0461	.0357	.0461	-.0628	-.0515	-.0302	-.0166	+.0266
21	(16) - (4)	$C_\ell - C_{\ell_b}$	-.06850	-.738	1.0453	1.0855	1.1000	1.0708	.9423	.7154	.4928	.3756	.3362	.3524
22	(21) / (3)	α_o	-.06850	-.738	.1875	.1945	.1968	.1912	.1678	.1270	.0872	.0662	.0591	.0618
23	(15) - (22)	W/V	-.06850	-.738	.0743	.0673	.0650	.0706	.0940	.0133	.0237	.0447	.0507	.1052
24	(23) x (21)	C_{d_i}	-.06850	-.738	.0777	.0731	.0715	.0756	.0886	.0095	.0117	.0168	.0170	.0371
25		SIN 2θ			0	.3090	.5878	.8090	.9511	1.0000	.9511	.8090	.5878	.3090
26	TABLE I LINE 9	C			191.35	171.03	151.20	132.36	114.98	99.48	86.24	75.58	65.89	38.71
27	(25) x (26)	C×SIN2θ			0	52.848	88.875	107.08	109.36	99.48	82.023	61.144	38.730	11.961
28	(10) - (20)				0	.0073	.0229	.0350	.0215	.1687	.1690	.1570	.1523	.0944
29	(27) x (28)				0	.386	2.035	3.748	2.351	16.782	13.862	9.600	5.899	1.129
30	(24) - (14)				0	.0148	.0290	.0480	.0858	-.0213	-.0217	-.0375	-.0833	-.0533
31	(27) x (30)				0	.782	2.577	5.140	9.383	-2.119	-1.780	-2.293	-3.226	-.638

LINE 29, $C'_n = \frac{\pi}{80 C_{AVE}} \frac{1}{2} \sum (29) = \frac{.00031276}{2} \times (55.792) = .00872$

[LI]NE 31, $C'_n = \frac{\pi}{80 C_{AVE}} \frac{1}{2} \sum (31) = \frac{.00031276}{2} \times (7.826) = .00122$

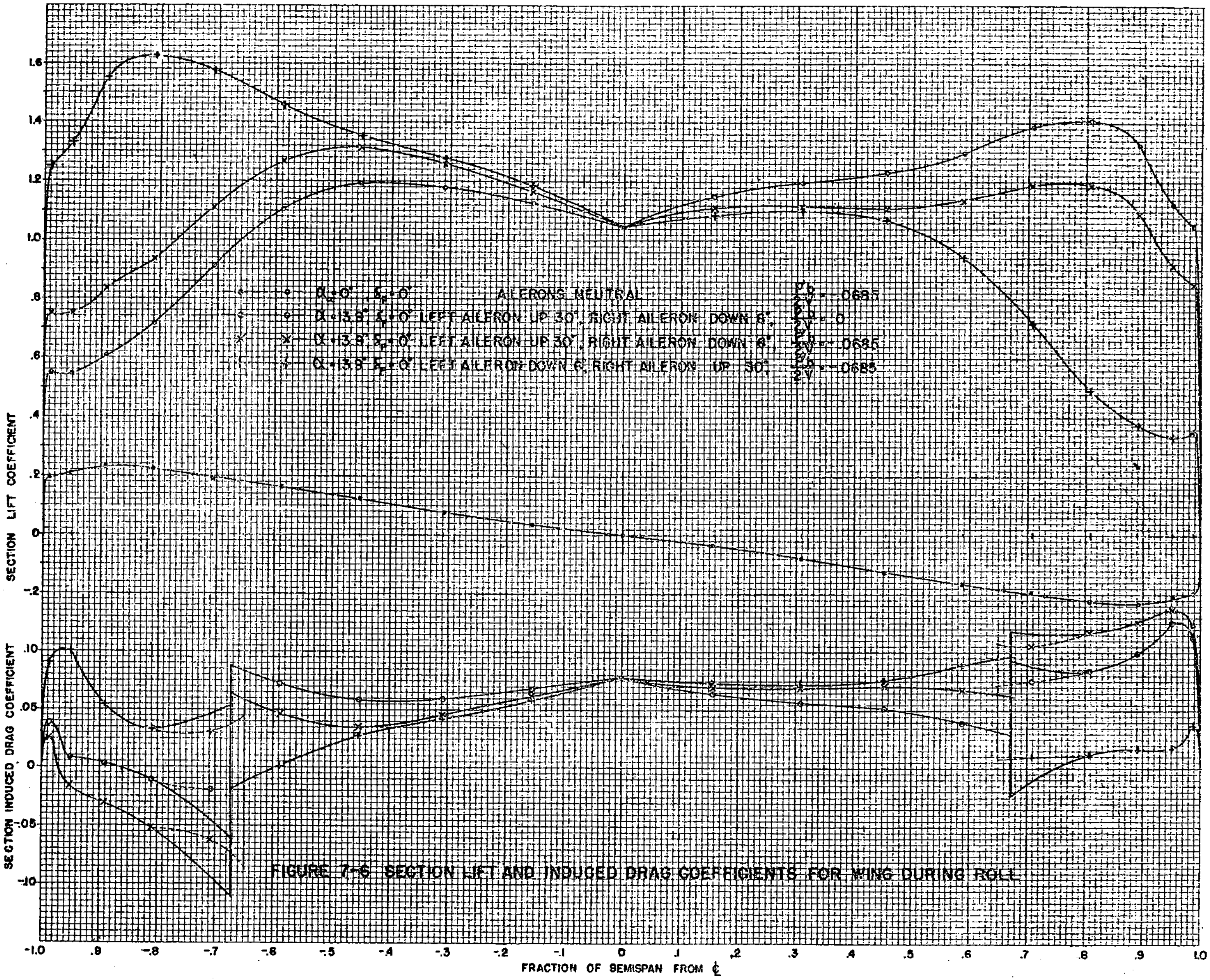

FIGURE 7-6 SECTION LIFT AND INDUCED DRAG COEFFICIENTS FOR WING DURING ROLL