THE VISUAL PERCEPTION OF SPATIAL EXTENT

FEDERAL AVIATION AGENCY
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ABSTRACT

This study was concerned with the manner in which perceived depth and perceived frontoparallel size varied with physical distance and hence with each other. An equation expressing the relation between perceived frontoparallel size and physical depth was developed and applied to frontoparallel size judgments determined with four observers under two viewing conditions. Using the same equation and an expression of the size-distance invariance hypothesis, an additional equation was developed which related perceived and physical depth. The additional equation, when applied to judgments of perceived depth from the same observers under the same viewing conditions, produced results not in agreement with those expected from the size-distance invariance hypothesis. This is interpreted as evidence against the validity of the size-distance invariance hypothesis in its usual form. The results from the apparent depth judgments also were applied to the problem of the nonveridicality of the perceptual bisection of depth intervals.

INTRODUCTION

Many studies have been concerned with the problem of the relation between physical and perceived space. This problem can be conveniently but arbitrarily divided into (1) the perception of vertical or horizontal extents (the perception of frontoparallel size) and (2) the perception of depth extents. Frontoparallel and depth extents have sometimes been considered to be perceptually interrelated with an expression of the interrelation given by the size-distance invariance hypothesis. The purpose of the present study is to investigate the manner in which both perceived frontoparallel size and perceived depth can vary with observation distance and hence with each other.

The problem of this study can be illustrated with the aid of Fig. 1. In Fig. 1, the frontoparallel extents \( S_x, S_y, \) and \( S_z \) are placed at distances \( D_x, D_y, \) and \( D_z \), respectively, from the observer. The perceived frontoparallel extents (perceived sizes) associated with the widths \( S_x, S_y, \) and \( S_z \) will be called \( S'_x, S'_y, \) and \( S'_z \), respectively, with the respective visual angles labeled \( \theta_x, \theta_y, \) and \( \theta_z \). The perceived depths (perceived relative depths) associated by the observer with the relative depth intervals \( d_x \) and \( d_y \) will be called \( d'_x \) and \( d'_y \), respectively.

* The data of this study were collected at the U. S. Army Medical Research Laboratory, Fort Knox, Kentucky.

The perceived distances (perceived absolute distances) which the observer associates with $D_n$, $D_o$, and $D_s$ will be called $D'_n$, $D'_o$, and $D'_s$, respectively. The problem of how the perceived frontoparallel size $S'$ varies as a function of $D$ is the problem which has usually been investigated under the topic of "size constancy." The problem of how the perceived absolute distance $D'$ or the perceived relative depth $d'$ varies as a function of $D$ is the problem which has usually been investigated under the topic of "depth constancy."

**The Size-Distance Invariance Hypothesis**

The specification of the relation between perceived frontoparallel size and perceived depth by means of the size-distance invariance hypothesis is given by

$$S' = K \cdot \theta \cdot D'$$  \hspace{1cm} (1)

where $K$ is a constant such that

$$K = \frac{S'}{\theta \cdot D'} = \frac{S'}{\theta \cdot D'_s} \text{ etc.}$$ \hspace{1cm} (2)

In Eq. (1), $S'$ is the perceived size of any object whose visual (retinal) size is $\theta$, and which is located at any physical distance $D$, from the observer. The term $D'$ is the perceived absolute distance associated by the observer with $D$. In Eq. (2), $S'$ is the perceived size of a particular object (Object e) whose visual (retinal) size is $\theta$, and which is located at the particular physical distance $D$, from the observer with $D'$ being the perceived absolute distance associated with $D$. In the equations used in this study, the subscript "v" will refer to a variable quantity while the subscripts e, f, and g (see Fig. 1) will refer to specific values. As is illustrated in Eq. (2), whenever a particular subscript such as e is used, any other particular subscript such as f could have been substituted for it throughout the expression.

**An Equation of Size Constancy**

An equation permitting the quantitative expression of perceived frontoparallel size $S'$ as a function of physical distance $D$, can be developed from the two following hypothetical situations.

**Situation I.** An object of constant frontoparallel size $S$ is presented at various distances $D_n$. Suppose that the perceived frontoparallel size of this object $S'$ remains constant. This is an example of perfect frontoparallel size constancy and can be expressed as

$$S' = S.$$ \hspace{1cm} (3)

Eq. (3) states that for a constant value of $S$, the perceived size $S'$ at any distance $D$, is equal to its perceived size $S'$ at the particular distance $D_n$. Suppose, however, that the perceived size of $S$ decreases proportionately to the distance $D_n$. This is an example of zero amount of frontoparallel size constancy and is expressed as

$$S' = S' \cdot (D_n/D_s).$$ \hspace{1cm} (4)

Eq. (4) states that the perceived size $S'$ of $S$ at any distance $D$, is equal to its perceived size $S'$ at the particular distance $D_n$, multiplied by the ratio of the two distances.

Eqs. (3) and (4) are special cases of the more general equation

$$S' = S' \cdot (D_n/D_s)^n.$$ \hspace{1cm} (5)

In Eq. (5), when $n = 1$, Eq. (3) results and when $n = 0$, Eq. (4) results. Thus, $n$ is an expression of the amount of frontoparallel size constancy present. When $n = 0$, zero amount of frontoparallel size constancy is present.

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1 For a different statement of the problem of depth constancy see R. Over, Am. J. Psychol. 74, 308 (1961).

2 A relation similar to Eq. (5) has been developed for use in an equation concerned with the perception of three-dimensional shape (W. C. Gogel, J. Psychol. 50, 179 (1960)).
When \( n = 1 \), perfect frontoparallel size constancy is present. When \( n > 1 \), over-frontoparallel size constancy is present, i.e., the perceived size of a physically constant \( S \) increases as the distance \( D \) increases.

**Situation II.** An object of constant visual angle \( \theta \) is presented at various distances \( D \). It follows in a development similar to that used for Eq. (5) that for perfect frontoparallel size constancy,

\[
S' = S' (D/D),
\]

while for zero amount of frontoparallel size constancy

\[
S' = S'(D/D),
\]

or, in general,

\[
S' = S' (D/D)^n.
\]

Eqs. (5) and (8) are special cases of the more general equation

\[
S' = K \theta D,\]

where

\[
\theta, \text{ in radians} = \frac{S}{D},
\]

and \( K \) is a constant such that

\[
K = \frac{S'}{\theta D} = \frac{S'}{\theta D}, \text{ etc.}
\]

Eq. (9) specifies the relationship between the perceived frontoparallel size \((S')\) of an object and its physical distance \((D)\) for any value of visual angle \( \theta \), as a function of the amount of frontoparallel size constancy \((n)\).
Again, perfect and zero frontoparallel size constancy are expressed by \( n = 1 \) and \( n = 0 \), respectively, while values of \( n \) between 0 and 1, for example, indicate that some amount of, but not perfect, frontoparallel size constancy is present.

Eq. (9) can also be derived directly from the Thouless Index.\(^\text{4}\) The Thouless Index expressed in symbols which are consistent with Eq. (9) is as follows:

\[
T = \text{Thouless Index} = \frac{\log S' - \log \theta}{\log S - \log \theta} \quad (12)
\]

where

\[
S' = S'/S, \quad \theta = \theta'/\theta, \text{ and } S = S'/S. \quad (13)
\]

From Eq. (12),

\[
(S/\theta)' = S'/\theta. \quad (14)
\]

Using Eqs. (10), (11), and (13), Eq. (14) becomes

\[
S' = K_\theta D'_\theta. \quad (15)
\]

Thus, comparing Eqs. (9) and (15),

\[
T = n. \quad (16)
\]

An Equation of Depth Constancy

Combining Eqs. (1) and (9), the relation between perceived absolute distance \( D' \) and physical distance \( D \) is

\[
D' = K_\theta D. \quad (17)
\]

where

\[
K_\theta = \frac{D'_\theta}{D'_\theta} = \frac{D'_\theta}{D'_\theta} \text{ etc.} \quad (18)
\]

Eqs. (9) and (17) can be written as

\[
\log S' = n \log D + \log K_\theta. \quad (19)
\]

and

\[
\log D' = n \log D + \log K_\theta. \quad (20)
\]

respectively. Also from Eqs. (2), (11), and (18),

\[
K_\theta = K_\theta/K_\theta. \quad (21)
\]

If the size-distance invariance hypothesis as expressed by Eq. (1) is correct, \( n \) should be the same in Eqs. (19) and (20), and Eq. (21) should be valid. In the present study, the hypothesized equality of \( n \) between Eqs. (19) and (20) was tested by experimentally determining the function relating physical distance to both perceived size (Eq. 19) and perceived distance (Eq. 20) for two different conditions of observation.

APPARATUS

The Visual Alley

Perceived frontoparallel size and perceived depth were measured in an alley 1020 cm long and 128 cm wide. The floor of the alley consisted of either (1) a uniform black cloth, or (2) a checkerboard pattern of light and dark gray rectangles (each 41 cm long and 23 cm wide). The right wall of the alley was covered with tan cloth. Tan curtains formed the left wall, and the end of the alley was covered by black velvet. Schematic drawings of the alley are shown in Fig. 2. A baffle, 28.5 cm high, covered with tan cloth, extended along the left side of the alley, 33 cm from the left wall. Ten white (9 cm) squares (not shown in Fig. 2) vertically presented in frontoparallel planes were distributed in depth on the floor of the alley, with five of these along the baffle and five along the right side of the alley. The center of the alley was clear of white squares permitting the placement of the lettered objects shown in Fig. 2. The alley was illuminated by overhead fluorescent lights. The alley and every object in the alley was viewed by the observer with his right eye only (the left eye-piece at the viewing position was occluded). The eye of the observer at the viewing position was 23.8 cm above the floor of the alley.

The Measurement of Perceived Frontoparallel Size

To measure the perception of frontoparallel size along the alley, red rectangular objects (Objects $h$, $i$, $j$, $l$, and $m$, of Fig. 2A) were placed one at a time at different distances along the alley and the observer adjusted the lateral distance between two white rods (Objects $a$ and $b$) until this distance seemed the same as the width of the particular rectangle at the particular distance. The two white rods (6 mm in diameter and 10 cm high) were vertically presented on the floor of the alley at a constant distance of 117.5 cm from the observer. The right rod was stationary while the left rod could be moved laterally by the experimenter. A small black barrier (2.5 cm high) which extended across the alley at the base of the rods prevented the observer from seeing the slot on the alley floor in which the right rod was movable. The red rectangles subtended a constant visual angle (1° 47' 24'') and had the following sizes and positions in the alley:

Object $h$ was 7.6 cm wide by 1.7 cm high at 243.8 cm from the observer.
Object $i$ was 11.4 cm wide by 2.5 cm high at 365.7 cm from the observer.
Object $j$ was 15.2 cm wide by 3.4 cm high at 457.6 cm from the observer.
Object $l$ was 22.8 cm wide by 5.1 cm high at 731.4 cm from the observer.
Object $m$ was 30.4 cm wide by 6.8 cm high at 975.2 cm from the observer.

The Measurement of Perceived Depth

To measure perceived depth in the alley, Objects $a$, $b$, $h$, $i$, $j$, $l$, and $m$ were removed and Objects $p$, $q$, and $w$ were used instead (Fig. 2B). Object $p$ was placed at a constant distance from the observer (171.5 cm) and the observer adjusted a depth interval (usually between Objects $q$ and $w$) at different distances from himself to duplicate the perceived distance from himself to Object $p$. 

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Figure 2. A top view diagram illustrating the physical conditions involved in the size and depth judgments.
Objects $p$ and $q$ were red rectangles 15 cm high by 5 cm wide which were vertically positioned on the floor of the alley. Object $w$ was a red 7.6 cm square placed in a fron-toparallel position with the bottom of the square always resting on the floor of the alley. Object $w$ was attached by a glass rod to a cart (invisible to the observer) which moved on a track behind the baffle. By turning a knob at the viewing position, the observer could move the red square directly toward or away from himself in depth.

**PROCEDURE**

**The Fron-toparallel Size Adjustments**

The observer signaled to the experimenter to move Object $a$ (Fig. 2A) laterally to the right or left until the distance between the inner edges of the rods visually appeared equal to the width of Rectangles $h$, $i$, $j$, $l$, or $m$. The adjustment was repeated four times after which a different rectangle at a different distance was presented. The distances in centimeters adjusted between the rods measured the perceived width $S'$ of the particular rectangle. The order of presenting the rectangles was randomly determined for each observer.

**The Depth Adjustments**

The observer moved Object $w$ (the red square) back and forth in depth until the distance of $w$ behind $p$ visually seemed equal to the distance of $p$ from himself. This was done three times. The average of the three adjusted distances was calculated by the experimenter and $q$ was placed at this distance. The observer then moved $w$ behind $q$ until the visually perceived depth between $w$ and $q$ ($d_{w'q}$) seemed to duplicate the visually perceived distance of $p$ from himself. Again, this adjustment was made three times. The new average position of $w$ was found, $q$ was moved to this new distance and the process was repeated. This process resulted in a series of depth intervals ($d_{w'i}$) each of which appeared to the observer to be equal to the distance of $p$ from himself and thus equal to each other.

**Observers and Orders**

Four men were used in the experiment as observers. Previously these observers had experience in making size and distance judgments in experimental situations. Each observer had a visual acuity in his right eye (corrected if necessary) of at least 20/22.

The depth and fron-toparallel size adjustments always were completed first with the checkerboard pattern and following this with the uniform black covering on the floor of the alley.

**RESULTS**

**The Fron-toparallel Size Adjustments**

The data from the experiment relevant to Eq. (19) are shown in Fig. 3. Since the red rectangles subtended a constant visual angle, $\theta$ in Eqs. (9) and (19) is constant. The ordinate of Fig. 3 gives the logarithms of the average results in centimeters from each observer for the fron-toparallel size adjustments for both the checkerboard pattern and the uniform black floor. The abscissa is the logarithm of the physical distance in centimeters of the fron-toparallel targets from the observer. The data points of Fig. 3 are reasonably linear. Using the method of least squares, a straight line of best fit was computed for each of the distributions of Fig. 3. From Eq. (19), the slope of the straight line of best fit is $n$ with $\log K_\theta$ the ordinate intercept. The resulting values of $n$ and of $K_\theta$ (with $K_\theta$ determined for the constant value of $\theta$ expressed in radians) are given in Table I. As might be expected, there was greater fron-toparallel size constancy when the checkerboard pattern rather than the black cloth floor was used. In all cases the amount of fron-toparallel size constancy was between zero constancy ($n = 0$) and perfect constancy ($n = 1$).

**The Depth Adjustments**

In the previous equations, $D'$ is a perceived absolute distance (an apparent distance from the observer to the object). This is to be distinguished from $d'$ which is a perception of
TABLE I

Values of $n$, $K_s$, and $K_i$ as Determined from the Slopes and Intersection Values* of the Straight Lines of Best Fit for the Data of Fig. 3 and Fig. 4.

<table>
<thead>
<tr>
<th></th>
<th>Checkerboard Floor</th>
<th></th>
<th>Uniform Black Floor</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>From Size</td>
<td>From Depth</td>
<td>$K_s/K_i$</td>
<td>From Size</td>
</tr>
<tr>
<td>Obs.</td>
<td>Judgments</td>
<td>Judgments</td>
<td></td>
<td>Judgments</td>
</tr>
<tr>
<td>A</td>
<td>$0.50$</td>
<td>$21.25$</td>
<td>$0.80$</td>
<td>$2.07$</td>
</tr>
<tr>
<td>B</td>
<td>$0.67$</td>
<td>$6.89$</td>
<td>$0.91$</td>
<td>$0.95$</td>
</tr>
<tr>
<td>C</td>
<td>$0.61$</td>
<td>$12.83$</td>
<td>$0.91$</td>
<td>$1.03$</td>
</tr>
<tr>
<td>D</td>
<td>$0.86$</td>
<td>$1.96$</td>
<td>$0.93$</td>
<td>$0.91$</td>
</tr>
</tbody>
</table>

* For convenience, values of $K_i$ in this table have been arbitrarily multiplied by 100.

Figure 3. The relation between the logarithm of the perceived size in centimeters of the rectangles and the logarithm of their physical distance in centimeters.
relative distance (an apparent depth between objects). The successively perceived relative depths in the present experiment were all adjusted to be equal to the perceived absolute distance to Object $p$ and hence equal to each other. It follows that the perceived absolute distance throughout the visual field can be expressed in units of the perceived absolute distance to the nearest object (Object $p$). Relevant to Eq. (20), the logarithms of the resulting perceived absolute distances $D'$ (with the perceived absolute distance to $p$ set equal to unity) and the associated physical distances $D$ in centimeters required to produce the perceived absolute distances are shown in Fig. 4. The data points of Fig. 4 are reasonably linear. Using the method of least squares, a straight line of best fit was determined and $n$ and $K_n$ were computed for each of the curves of Fig. 4. The results are shown in Table I. In all cases but one, the amount of depth constancy (as shown by the value of $n$) was between zero constancy ($n = 0$) and perfect constancy ($n = 1$).

![Figure 4. The relation between the logarithm of perceived depth (expressed in units of the nearest Object $p$) and the logarithm of the physical distance in centimeters required to produce that perceived depth.](image-url)
The average results for each observer from the depth adjustments, using the checkerboard pattern and the uniform black floor, is given in Fig. 5. The average adjusted depth interval in centimeters which appeared to the observer to be equal to the distance from himself to Object p is given on the ordinate with the physical distance in centimeters from the observer to the least distant part of this interval shown on the abscissa.

**DISCUSSION**

From the essentially linear graphs of Fig. 3, it follows that a single value of n for a particular observer and a particular floor type (checkerboard or uniform black floor) appropriately describes the frontoparallel adjustments. A similar conclusion applies to the depth adjustments shown in Fig. 4. But, in every instance (see Table I) the value of n from the same observer and same floor type is greater with the depth than with the frontoparallel size adjustments. According to the size-distance invariance hypothesis, however, n should be the same for the depth adjustments as for the frontoparallel size adjustments. The differences in n for the two types of judgments in Table I therefore provides evidence against the validity of the size-distance invariance hypothesis.

Using Eq. (21), K, was determined from K, and K, with the results shown in Table I. From Eqs. (2), (11), (18), and (21), differences between the K, values calculated from the results from the checkerboard floor and from the uniform black floor would be expected to occur as a consequence of differences in n. Furthermore, since n differed for the two floor types, K, in Table I would be expected to vary as a function of the unit used in the specification of D. The differences in the calculated values of K, in Table I reflect both the differences in n and the size of the unit used in the specification of D. Differences in K, in this experiment therefore do not constitute a direct or independent evaluation of the validity of the size-distance invariance hypothesis.

Eq. (9) is convenient for describing size constancy data. If, as this study indicates, n is generally a constant for the entire visual field (or for a large portion of the visual field) for a particular observer, the usefulness of Eq. (9) will be increased. Eq. (17) provides an equally convenient description of depth constancy data. When n = 1 (perfect depth constancy) perceived and physical depth are proportional. When n = 0 (zero depth constancy) perceived depth does not increase even though physical depth increases. From Fig. 1, it is clear that

\[
D' = d' + D
\]  

(22)

and therefore, Eq. (17) can be expressed in terms of d' instead of D' as follows:

\[
d' = K_s(D_i - D_i'), \text{ etc.} 
\]  

(23)

In some studies, it might be more convenient to use Eq. (23) rather than Eq. (17) in summarizing data from depth judgments.

In a recent study testing the size-distance invariance hypothesis, Ueno\(^*\) has developed equations similar to Eqs. (9) and (17) of the present study from Stevens' power law\(^*\) which states that psychological magnitude is a power function of stimulus magnitude. Using the psychological method of transposition of Oyama\(^*\), Ueno has compared the value of the exponent in the power function involving perceived size with that involving perceived distance. Exponents computed from the two types of judgments were not always similar with large differences sometimes occurring for monocular and reduced conditions of observation. Both the study by Ueno and the present study indicate, contrary to the size-distance invariance hypothesis, that the power function relating perceived size and physical distance does not always involve the same value of exponent as that relating perceived distance and physical

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\(^*\) S. S. Stevens, Am. Psychologist 17, 29 (1962).

distance. These studies together with previous evidence indicate that the size-distance invariance hypothesis in its usual form is not always valid.

Fig. 5 is useful in analyzing errors which can occur in the perception of distance. The first point on all the graphs of Fig. 5 has an ordinate value of 171.5 cm which is the physical distance of Object p from the observer. The observer's perception of the distance of p is a perception of absolute distance. All other points on the graphs represent the results from judgments of relative distances, i.e., results from perceptions of the depth between two objects. The ordinate value of the second point for three out of four observers is less than 171.5 cm. This means that for these observers the perceived depth per unit of physical depth was usually greater in the first relative depth judgment than in the judgment of the absolute distance of p. For the following two relative depth judgments, however, the perceived depth per unit of physical depth usually decreased with the final relative judgment showing a decrease in this ratio for the uniform black floor but not for the checkerboard floor. From these data, it is possible to infer the results which would have occurred if the observer had been

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*See (a) F. P. Kilpatrick and W. H. Itelson, Psychol. Rev. 60, 223 (1953); (b) W. Epstein, J. Park, and A. Casey, Psychol. Bull. 59, 491 (1961); and (c) W. C. Gogel, Vision Research 3, 106 (1963).*
given the task of adjusting a movable object to perceptually bisect the distance from himself to a stationary object. This task would produce two perceptually equal intervals with the nearer interval being a judgment of absolute distance (a judgment of the distance from the observer to the movable object) and the farther depth interval being a judgment of relative distance (a judgment of the depth between the movable and the stationary object). From the present study, for near portions of the alley, the observer would usually underestimate the nearer (absolute) distance with respect to the farther (relative) distance. However, if a depth between two visible stationary objects (a relative depth) were perceptually bisected by a movable third object, the reverse sometimes would occur. The physical depth between the nearest object and the movable object sometimes would be overestimated relative to the physical depth between the movable object and the farthest object. These inferences from the data of the present study might help explain some of the discrepancies in the results sometimes encountered in studies involving the perceptual bisection of a depth interval."

The reason for the increase in perceived depth per unit of physical depth from the checkerboard but not from the uniform black floor at the far distances is not clear. Perhaps some complex effects are to be expected from judgments involving depth near the terminating wall of the alley.

In many respects it is unfortunate that the size-distance invariance hypothesis of Eq. (1) probably is not valid. If Eq. (1) had been valid, for a constant value of \( n \), Eqs. (1), (9), and (17) would have together resulted in a simple predictive system of considerable parsimony. A few measures of perceived extent for a particular observer would have permitted the prediction of perceived extent at any orientation (frontoparallel, depth, or a combination of both) throughout the visual field. A size-distance invariance hypothesis can be determined from Eqs. (9) and (17) without assuming that \( n \) is the same in the two equations. However, the resulting equation would be more complicated and less parsimonious than Eq. (1).

**ACKNOWLEDGMENTS**

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