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TREATMENT OF UNCERTAINTY IN FLIGHT SAFETY ANALYSIS
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TREATMENT OF UNCERTAINTY IN FLIGHT SAFETY ANALYSIS

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ABSTRACT

When analyzing public risks posed by launch and reentry operations, a key element is the consideration of uncertainty. This paper addresses the treatment of different kinds of data uncertainty, which includes incertitude (the exact value is unknown) and variability (the value depends on the situation). "Variability" refers to factors that are not known well in advance of initiating the launch or reentry operation but *will be known or could be known* with significantly lower uncertainty at the time the commitment is made to initiate the operation. The range of variability usually decreases as the operation approaches, an important consideration for the analysis process. This paper presents a practical process for analysis that accounts appropriately for variability and other types of uncertainty, so that a final decision is made that properly accounts for only for the best estimate of the uncertainty at that time the operation is initiated.

1 INTRODUCTION

The recent promulgation of new Federal Aviation Administration (FAA) regulations for commercial launch and re-entry (14 CFR 450) [1], explicitly require accounting for all known sources of uncertainty in launch and reentry safety assessments [2, 3]. The regulation requires that a valid flight safety analysis explicitly account for all known uncertainties.[1] Further, the regulations specifically require that a flight safety analysis distinguish between variability and random (i.e. aleatory) uncertainty in the normal trajectory data (§450.117). Further, the regulations quire that the risk criteria are satisfied at the time the operator initiates launch or re-entry flight, either through a countdown analysis or a prior analysis that accounts for all foreseeable conditions (§450.135, §450.137). In U.S. jurisprudence, "foreseeable" events are those which a "reasonable person would be able to predict or expect."[2] If data could be obtained without unreasonable effort to inform a risk prediction, then that data should be used (or otherwise accounted for). For this paper, we call such data "obtainable," to avoid the potential loophole that data could be considered unavailable because it was not present at the analysis site when an analysis is performed.

1.1 Incertitude vs. Variability

The distinction between incertitude and variability is critical for risk-based decision making. The core principle is that acceptability of an operation should be based on the conditions at the time the final decision is made. This decision is the latest time at which there is no risk to the public if the operation is aborted. For launch and re-entry operations, this usually occurs only seconds before engine ignition. At this time, many of the unknowns in mission planning could now be known. Thus, the risk decision should be based on an analysis that includes the best obtainable information. A reasonably prudent person making a life-and-death decision would gather all relevant obtainable data or otherwise account as best as possible for the potential situations. Of course, some uncertainty will always remain, both due to measurement limitations and limits on predicting future events. But uncertainty does not cover unknowns that could be known immediately prior to the decision to initiate a launch or reentry operation. Uncertainty also exists in models, as another aspect of incertitude, but it is not the focus of this paper.

As an example of uncertainty and variability, consider a launch from a mobile platform, such as a ship, as illustrated in Figure 1. From the planned launch location, a set of trajectories could be simulated that represent uncertainties in guidance and performance, the normal

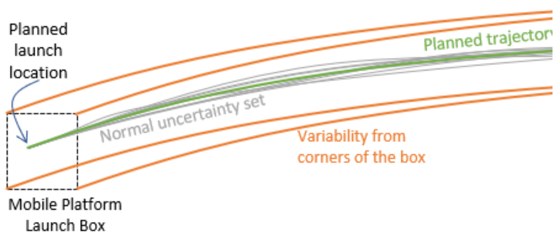

Figure 1. Example of Uncertainty and Variability

1 For example, 14 CFR 450.115(b) requires that "An operator's flight safety analysis method must have a level of fidelity sufficient to (1) demonstrate that any risk to the public satisfies the safety criteria of § 450.101, including the use of mitigations, accounting for all known sources of uncertainty."

2 https://definitions.uslegal.com/f/foreseeable/, retrieved 2021-Sep-17.

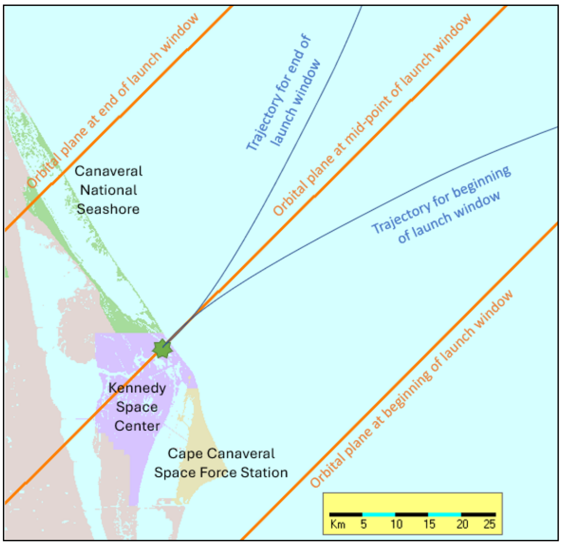

Figure 2. Trajectory variation across launch window (notional)

uncertainty set. The planned trajectory would be near the mean of this set. But due to currents and winds, the ship will be in a different location. A launch rule would normally require that it be within a specified launch box. The variability of the launch point then leads to various trajectories, and the trajectories initiating from the corners are illustrated in the figure.

As a more subtle example of variability, some missions use different trajectories depending on when in the launch window the mission occurs in order to rendezvous, as illustrated in Figure 2. Of course, the launch time is known at the time the commit decision is made, and thus the actual planned trajectory is known. At the time the commit decision is made the variability of planned trajectories should not be treated as uncertain; only the uncertainty in the trajectory for the designated launch time should be accounted for in the analysis used to determine if the public risks meet the regulatory criteria.

1.2 Analysis Considerations

Risk analysis is performed at various times in the mission planning and execution process, so a core question is how to perform analyses that appropriately account for variability at the time the analysis is performed. Early on in the process, a risk analysis informs the range of acceptable missions (e.g. an azimuth range). This also may be used to perform long-term planning, such as determining fence lines (for keep-out areas), developing coordination agreements with adjacent agencies, and obtaining insurance. Several months to several weeks ahead of time, a specific mission plan is developed which identifies placement of operational assets and mission rules, establishes the range of acceptable atmospheric conditions and determines flight safety limits. A few weeks ahead of time, mitigations are established, such as the final hazard areas. In the days leading up to the mission through the countdown, evaluation of the final planned operation is performed to determine acceptability.

1.3 Goal

The goal is that the analyses produce results that allow decisions to be made with high confidence. It is not necessary to obtain answers with a specific fidelity, only enough fidelity necessary to facility an informed decision. It is not possible to evaluate every possible condition ahead of the mission since some parameters are continuous variables. It is usually sufficient to classify the potential outcomes as green/yellow/red, where:

- Green – clearly meets risk acceptability requirements
- Yellow – refinement may or may not show meeting requirements
- Red – clearly does not meet requirements

A simple categorization such as this is most helpful for decision-makers. A yellow result could lead to further analysis or pursuing other options. Of course, the final risk results need to be green for the operation to commence.

2 VARIABILITIES

There are two aspects to considering variabilities, the data that could vary and the importance of each variation.

2.1 Sources of Variabilities

As a framework for considering variability, consider five categories of variability: mission plan, trajectory, population, failure probability, and atmospheric effects. There may be others, and there is some overlap, as further described below. Also, some data may have been developed only in preliminary form (such as using debris data from a similar vehicle) early in mission planning For each category the range of possibilities typically becomes narrower as time progresses towards the mission.

2.1.1 Mission plan

The mission plan is the earliest to be narrowed, as other variabilities depend on it. This includes:

- Mission objective, such as target orbit,
- Payload, especially payload weight,
- Vehicle configuration, such as use of kick motors or strap-ons, and
- Launch or landing site.

These may be decided without a safety analysis (e,g, based on past experience), but sometimes risk acceptability is an important factor in these decisions. It is important to evaluate these in the overall framework of variability, as there may be interactions with other factors that affect the analysis result.

2.1.2 Trajectory

Even once the mission plan is determined, there are several reasons that the trajectory varies in ways that will be known better at the time of commit:

- For re-entries, from which orbital node,
- The time of ignition (as discussed previously),
- For captive-carry launches, the position and velocity at release,
- For ship, rail, etc. launches or landings, the position (and sometimes orientation and angular velocity) of the vessel/vehicle,
- For interceptor missions, the trajectory of the target, and
- Atmospheric effects, especially for unguided and open loop phases of flight.

2.1.3 Population

The number and location of people is a very important factor for risk assessment and has significant variability. Many factors that affect the location of people (both geographically and within shelters). Season, time of day, time of week, other site operations, spectators, weather, and events all affect the population distribution. Far ahead of the mission, the timing and thus the population, may have a large potential range, but by the launch countdown, the population can be known quite well—last minute surveys are sometimes even performed for specific locations that are particularly important to the risk result (such as a beach or other recreational area near the launch point).

2.1.4 Atmosphere effects on hazards

Hazards are dependent on the environment. Debris impact locations may be strongly affected by the winds. Blast wave focusing is driven by the wind and temperature profile. Toxics are affected by the winds and the density profile. Early in the analysis process, the entire range of atmospheric conditions might still be possible, but then the month and time of day are often identified. Statistical variations are appropriate at this time, but then in the mission countdown, forecast data is available which can be an accurate prediction.

2.1.5 Failure probability

The failure probability is a variability because other missions occur between the initial analysis and the subject mission. These events typically update the failure probability estimate for the mission. Especially if there are missions of the subject vehicle, a string of successes could result in a significantly lower probability of failure, and even one failure can significantly increase the probability of failure.

2.2 Importance of each variability

There are two questions for each of these effects. The first is: what portions of flight are affected? Many of these effects are only important for the portion of flight near the launch or landing area (such as the captive carry release state vector and spectators). Others have little effect on the launch or landing but are important for downrange overflight (such as launch time effects on trajectory). The second is: how significant is the effect of the variability on risk results? The significance is as compared to other variabilities—the ones with the largest effect are much more important to study than the minor ones. However, the significance changes as a function of time until the operation, because the ranges of many variabilities become smaller at different rates. For example, wind variability is quite large when the time of year of an operation is not known, then is somewhat smaller a month or two out (before forecasts have any predictive accuracy), and then are very small 24 hours ahead of time. On the other hand, a ship launch region may be established at the end of mission planning, but the actual ship position (which depends on sea conditions) is still variable until commit decision (with reducing variability in the countdown).

3 UNCERTAINTIES

As with variability, both the sources of uncertainty and their importance need to be determined.

3.1 Sources of uncertainties

Irreducible uncertainties exist in the trajectory, population, failure response and breakup, hazard propagation, and probability of failure. Evaluation of uncertainty is a critical factor for evaluating the sufficiency of fidelity of the input data and analysis process [4], [5].

3.1.1 Trajectory

Trajectory uncertainty is due to unknowns in the vehicle properties and atmospheric conditions. Vehicle property uncertainty may be due to measurement, modeling, and/or manufacturing uncertainty. Physical vehicle properties include mass and aerodynamic properties and engine performance. The vehicle guidance, navigation, and control systems also introduce uncertainty, from the sensor noise to the response of the control surfaces. Many of these uncertainties become smaller during the maturation of a vehicle as data from previous flights refines the values. The air density and wind profiles, and their gradients also lead to trajectory uncertainty. The trajectory uncertainty is typically the most important uncertainty for planned events. It may also significantly contribute to impact dispersions from failure events (especially on-trajectory failures).

3.1.2 Population

Population data includes uncertainty because the location of every person cannot be precisely known—and they will move during the flight. They may even respond to debris falling.[3] In addition to their position, whether they are inside or outside, and the type of structure, has uncertainty. Over large regions, this uncertainty is typically unimportant, as the total number of people is more significant. However, for areas near the launch or landing site, if the uncertainty in population undermines confidence in compliance with the public risk criteria, then a countdown survey is appropriate to reduce this.

3.1.3 Failure response

There are many uncertainties in the failure response and breakup. A very significant one is the trajectory that results from a failure that does not immediately breakup up the vehicle, and then the subsequent likelihood and time of breakup. Breakup event dynamics are quite uncertainty resulting in a range of possible debris outcomes. Likewise, the explosive yield from an impact of propellant has significant uncertainty due to the impact conditions and mixing. Toxic release rates and quantities also have a significant uncertainty. These uncertainties are difficult to reduce and are often large contributions to the uncertainty in both collective and individual risk results.

3.1.4 Atmospheric effects on hazard propagation

The uncertainty in the atmospheric conditions also affects the propagation of debris, blast waves, and toxic species. The importance of these depends on the hazard and the location. For debris, in regions outside of the launch/landing area, usually monthly variability is treated as an uncertainty.[4] Near the launch and landing area, unless there is a significant weather front in the area, 24-hour forecasts in the US can be quite good, such that uncertainty is nearly irrelevant for debris. In the launch area, blast analyses and toxic analyses are more sensitive to local weather conditions. If far-field blast overpressure is a significant risk contributor, obtaining accurate data to characterize wind and temperature data is critical. If toxic hazards are significant, a radiometric wind profiler is appropriate in order to minimize the uncertainty. For toxic hazards outside the launch/landing area, typically a conservatively large casualty area is used to account for uncertainty in wind conditions at impact.

3.1.5 Probability of failure, especially allocation

A complete probability of failure analysis provide an estimate of uncertainty both in the total failure probability and in the allocation of probability between flight phases and failure modes. This may be the most significant uncertainty for debris risk, especially regarding the casualty expectation for a mission as the trajectory and failure response uncertainties are typically somewhat averaged out in the computation of E_C. This uncertainty is larger for novel vehicles where there are fewer similar past missions.

3.2 Significance

The FAA requires that significant uncertainties be considered. The rule does not intend for applicants to characterize the influence of all random uncertainties or variability, but only those with a significant influence on the potential impact locations for hazardous debris. For normal trajectory analysis, the FAA considers "a significant influence" to include any parametric uncertainties that affect the crossrange IIP location or downrange IIP rate by at least one percent relative to the combined uncertainty of all parameters. IIP location and rate are used because they represent a convenient surrogate for the potential impact location and dispersion of hazardous debris.

To determine which parameters are significant for the trajectory, first, a baseline for uncertainty should be established. This is accomplished by first:

1. Identifying several parameters for which the uncertainty is likely to be most significant (e.g. thrust magnitude and direction uncertainty);
2. Computing dispersed trajectories accounting for those uncertainties together; and
3. Measuring the statistics (mean, standard deviation) of both the cross-range IIP and the downrange IIP rate.

Then other potential sources of uncertainty should be examined. This is accomplished by:

1. Computing additional sets of dispersed trajectories with both the baseline uncertainties and with the additional uncertainties (usually logically connected subsets, e.g. lift coefficients)

[3]Spectators responded to falling debris following the recent Firefly Alpha failure: https://www.youtube.com/watch?v=wDkSDF4vQ-o.

[4]It is possible to use a global three-dimension forecast in this case, and further research is suggested to evaluate the effects on hazard areas, especially for planned events. This is likely to have little effect on casualty expectation.

2. Measuring the statistics of both the cross-range IIP and the downrange IIP rate. The mean of the baseline should be used, not the revised mean, when computing statistics.
3. Comparing the statistics of the two metrics to the baseline case, and if the statistics do not change by more than 1 percent, then these uncertainties in the subset need not be considered. They should be compared at the 95 percent or greater confidence level at a minimum (e.g. two sigma for a normal distribution).

A similar approach should be used to assess other uncertainties. The metrics are 1) the relative change in casualty expectation (mission and conditional) at the acceptability thresholds 2) the change in the required hazard areas (based on individual risk).

4 ANALYSIS PROCESS

This paper recommends a process for analysis to evaluate variability in a pragmatic way. There are five phases to the process, which is repeated at different times during the mission planning process, as illustrated in Figure 3. The first step is to identify the purpose of the analysis, that is, what decision(s) is it intended to inform? The analysis then occurs, as discussed in sections 4.1to 4.3 below. And then the analysis results used to make decision(s).

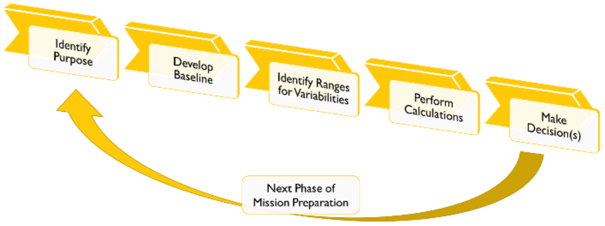

Figure 3. Variability Analysis Process

The different purposes for any analysis include:

- Trade Studies: How much does safety constrain different mission options?
- Availability: What is the likelihood that weather (or other) will preclude the mission on the day of operation?
- Mitigation Planning: What mitigations are needed to encompass all possible conditions on the day of operation?
- Launch Countdown: Are analyses ready to demonstrate compliance with safety criteria at the time of decision?

4.1 Develop baseline results

A first step is to perform an analysis for a reference mission. The reference trajectory should be near the median of the anticipated range. A risk analysis should be performed (for debris, far-field overpressure, and toxics, as appropriate), accounting only for significant uncertainties that would be present in the final analysis. If there are multiple distinct trajectories (such as re-entries from different orbital nodes, or comparisons of different launch or landing sites), then a reference trajectory for each should be analyzed. The resulting risk values (expected casualties, probability of casualty) for each reference mission are then used, because the uncertainties are irreducible. This provides a baseline of casualty expectation and probability of casualty to compare each variability.

4.2 Identify Ranges for Variabilities

Then, each different variability should be analyzed individually to determine the relative significance. The selection of parameters to use for each variability is usually not statistically representative; rather the objective is to span the range of potential risk results. Thus, the analyst should consider each variability and select the ones that are expected to produce significantly different results. For example, for a ship-based launch, the exterior of the acceptable launch region should be examined with sufficient intermediate values to identify the range. The results should also be examined to identify if it is possible that intermediate variability samples would be more extreme. It also may be helpful to sample the variabilities in more detail to determine which regions of a parameter have different outcomes (see section 1.3).

While for some variabilities, such as a ship launch box, it is straightforward to identify sampling, for others it is

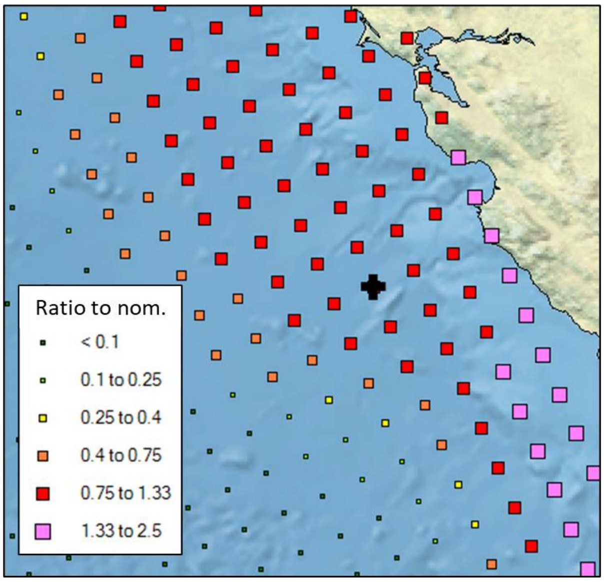

Figure 4. Example Drop Point Variability Result

more challenging. Two examples of more challenging aspects are wind and probability of failure.

When wind is being examined as a variability, actual historical wind data should be used as input. In the United States, the NOAA/ESRL Radiosonde Database[5] is a useful source, providing a very large number of historical profiles. Many of these are very similar; what is typically of interest is to investigate different types of winds. One way to categorize wind effects for debris risk is to bin the wind profiles based on wind power (see appendix) and select a few wind profiles from each bin for variability study. For toxic risk, the wind power can be used as well, but typically only the lower 100-200 m of the atmosphere are important. For FFBO, categorization of the sonic velocity profile, defined as the vector sum of the speed of sound and the wind speed as a function of altitude, in the direction of local population centers, is most important [6].

A second challenge is to assess the uncertainty in the allocation of failure probability. Usually there is less uncertainty in the total probability than the probability of each mode or within each phase of flight. Thus, a distribution where the uncertainty in the different modes (or phases) is related (i.e. must total to a certain value). An example of such distribution is the multinominal Dirichlet distribution [8].

4.3 Timeline

During the preparation for a mission, there are a number of different times when flight safety analyses may be performed.

4.3.1 Trade studies

Safety analysis is often an important factor in major program decisions, such as which facility to use or whether a flight safety system is necessary. For this case, unknowns are applied to understand the range of variability.

As an example, the drop location for a mission was examined to determine the maximum conditional casualty expectation, as shown in Figure 4. The points show the ratio of the CE_C for a drop each location compared to the CE_C nominal drop point (shown with the plus symbol). The variabilities were considered in different ways:

- The failure response impact probability dispersion was conservatively approximated, and the casualty area derived from similar vehicles.
- Atmospheric variation was a very small effect, as the wind effects are far smaller than the failure response dispersion in the vicinity of the population centers.
- The variability due to sheltering (due to time of day of the operation) was also varied and provided to the decision-maker.
- The failure probability was not relevant as the metric was a conditional risk for a specific failure mode.

This result provided decision makers with the data to determine the benefit of moving the drop point, as compared to other mission requirements.

4.3.2 Availability study

For availability studies, the key data is the atmosphere and the risk metric is usually collective casualty expectation. Typically, only risks for people in the near-site area are sensitive to atmospheric effects, and usually the trajectory variability does not significantly affect risk in this area. Therefore the risks from far-off-site areas should be determined separately, and added to the near-site risk (sometimes in matrix form). A challenge is that population data may also have significant variability, and thus it may be necessary to develop different population scenarios as well. Debris, toxics, and FFBO are sensitive in different ways to atmospheric effects, as discussed above, so those should be considered separately (but it is possible that important effects are correlated, so ideally the same set of atmospheric profiles should be used). Thus, this may be an extensive analysis as there are many variables to examine, but it may provide significant value to reduce the chances of weather-based launch postponements.

4.3.3 Mitigation planning

There are two primary aspects of mitigation: abort rules and hazard areas. Usually by this time, the mission plan and failure probability are established, but the potential variability in atmospheric effects, population, and the trajectory need to be considered. Flight abort rules should normally be considered first, to ensure that collective population and critical asset risks are acceptable. This needs to be considered across the range of trajectories and atmospheric profiles. Since hazard areas depend on the abort rules and usually need significant advance planning, this normally must occur before atmospheric forecasts are sufficiently accurate. Usually for hazard area calculations, it is reasonable to identify atmospheric and trajectories that define the extreme values. Since these are usually based on individual risk (*if* a person is present), the population variability does not matter..

4.3.4 Countdown analysis

In the mission countdown, usually the variability in the atmospheric forecast has been eliminated. There still

[5]https://ruc.noaa.gov/raobs/

remains variability in the initiating time (and state vector for mobile platforms), which can affect both the trajectory and the atmospheric data. Therefore, risks from initiating times should all be evaluated with the latest atmospheric data, unless it has been demonstrated that the different trajectories have a negligible effect on risk in the region where atmospheric data is important. The population data, especially due to spectators, may still be being updated, and this can be an important consideration in mission acceptability. It is therefore helpful to determine the casualty expectation *per person* at the locations of interest for each variability. An notional example of a calculation with this approach is shown in **Error! Reference source not found.**. The downrange risk varies with the launch time because the trajectory moves, whereas the launch area risk changes due to changes in the weather forecast. The count of the people in each of the three locations of interest would be updated during the mission. The 75% confidence bound is assumed to be 20% higher in the far-field and 50% higher in the near-site and at each location.

Table 1. Example Countdown Variability Calculation

	Timeframe in launch window		
	Early	**Middle**	**Late**
Far-field EC	70	60	50
Near-site baseline EC	5	5	5
Location 1 EC/person	0.20	0.25	0.40
Location 2 EC/person	0.12	0.13	0.14
Location 3 EC/person	0.05	0.04	0.04
Location 1 Count	8		
Location 2 Count	25		
Location 3 Count	45		
Total EC	82	72	64
75% Upper Bound EC	107	95	87

5 SUMMARY

The distinction between variability and uncertainty is critical for logically consistent approaches to risk analysis. A decision-maker should always be provided results that use the best obtainable information, along with the uncertainty associated with those results. Thus, this paper has provided three key elements to allow risk analysts to properly account for these concepts. First, the distinction between uncertainty and variability has been clarified. Second, the primary sources of the two have been presented and described. And third, the application of the concepts to analyses at different phases of mission preparation provides a pragmatic guide for analysts.

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APPENDIX: WIND POWER

Wind power is a vector measure of the total strength of a wind profile across different altitudes. To derive this metric, let us assume that an object is falling at terminal velocity with a horizontal velocity equal to the wind. (The effects of the initial velocity have been eliminated by drag.)

The horizontal distance traveled by the object due to wind can be found by multiplying the velocity of the wind, $v_w(t)$, by the time, $\Delta d = v_w(t)\Delta t$. In integral form, this is

$$\Delta \boldsymbol{d} = \oint \boldsymbol{v}_{\boldsymbol{w}}(t)dt,$$

where integration is performed along the path of the object from some altitude to impact. In the vertical direction the debris is traveling at terminal velocity, v_{term}, therefore $dt = \frac{da}{v_{\text{term}}}$, which simplifies the integration limits to be from the initial altitude, a_0, to the altitude of impact. This gives

$$\boldsymbol{\Delta d} = \int_{a_0}^{a_{\text{impact}}} \frac{\boldsymbol{v_w}(a)}{v_{\text{term}}(a)} da,$$

Terminal velocity is only changes during the interval as a function of altitude, as it is given by:

$$v_{\text{term}} = \sqrt{2\frac{\beta_m g(a)}{\rho_{air}(a)}}$$

where *g(a)* is gravitational acceleration, $\rho_{air}(a)$ is air density, and $\beta_m = \frac{m}{C_D A}$ is the ballistic coefficient of the object, respectively. By substitution and rearranging, the "wind power" is obtained:

$$P_W = \sqrt{2\beta_m}\boldsymbol{\Delta d} = \int_{a_0}^{a_{impact}} \boldsymbol{v_w}(a)\sqrt{\frac{\rho_{air}(a)}{g(a)}} da.$$

This measure is independent of all parameters of the object. The integral can be computed numerically based on wind data separately for two orthogonal components (e.g. North-South and East-West).

DISCLAIMER

The opinions presented in this paper are those of the authors and do not necessarily reflect the views of the Federal Aviation Administration or the United States Government.