Describing Orbits

4.1.4

In This Section You'll Learn to...

- ☞ Define the classic orbital elements (COEs) used to describe the size, shape, and orientation of an orbit and the location of a spacecraft in that orbit
- ☞ Determine the COEs given the position, $\vec{R}$, and velocity, $\vec{V}$, of a spacecraft at one point in its orbit
- ☞ Explain and use orbital ground tracks

I n the last chapter we looked at the restricted two-body problem and developed an equation of motion to describe in strictly mathematical terms, how spacecraft move through space. But many times it's not enough to generate a list of numbers that give a spacecraft's position and velocity in inertial space. Often, we want to visualize its orbit with respect to points on Earth. For example, we may want to know when a remote-sensing spacecraft will be over a flood-damaged area (Figure 4.1.4-1).

Figure 4.1.4-1. **Mississippi River Flooding.** Here we show an Earth Observation System view of the river flooding at St. Louis, Missouri, in 1993. *(Courtesy of NASA/Goddard Space Flight Center)*

Space Mission Architecture. This chapter deals with the Trajectories and Orbits segment of the Space Mission Architecture introduced in Figure 4.1.-20.

In this chapter, we'll explore two important tools that help us "see" spacecraft motion—the classic orbital elements (COEs) and ground tracks. Once you get the hang of it, you'll be able to use these COEs to visualize how the orbit looks in space. Ground tracks will allow you to determine when certain parts of the Earth pass into a spacecraft's field of view, and when an observer on Earth can see the spacecraft.

4.1.1.1 Orbital Elements

In This Section You'll Learn to...

☞ Define the classic orbital elements (COEs)

☞ Use the COEs to describe the size, shape, and orientation of an orbit and the location of a spacecraft in that orbit

☞ Explain when particular COEs are undefined and which alternate elements we must use in their place

If you're flying an airplane and the ground controllers call you on the radio to ask where you are and where you're going, you must tell them six things: your airplane's

- Latitude
- Longitude
- Altitude
- Horizontal velocity
- Heading (i.e. north, south, etc.)
- Vertical velocity (ascending or descending)

Knowing these things, controllers can then predict your future position.

Space operators do something similar, except they don't ask where the spacecraft is; instead, they use radar at tracking sites to measure it's current position, $\vec{R}$, and velocity, $\vec{V}$. As we'll see in Chapter 8, this information helps them predict the spacecraft's future position and velocity. Notice that position, $\vec{R}$, and velocity, $\vec{V}$, are vectors with three components each. Unfortunately, unlike latitude and longitude used for aircraft, $\vec{R}$ and $\vec{V}$ aren't very useful in visualizing a spacecraft's orbit.
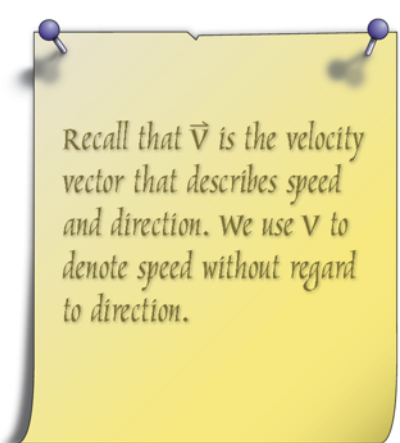
For example, suppose you're given this current position and velocity for a spacecraft

$$\vec{R} = 10,000 \ \hat{I} + 8000 \ \hat{J} - 7000 \ \hat{K} \ \ \text{km}$$

$$\vec{V} = 4.4 \ \hat{I} + 3.1 \ \hat{J} - 2.7 \ \hat{K} \ \ \text{km/s}$$

What could you tell about the orbit's size and shape or the spacecraft's position?

With the tools you've learned, about the only thing you could do is plot $\vec{R}$ and $\vec{V}$ in a 3-dimensional coordinate system and try to visualize the orbit that way. Fortunately, there's an easier way. Hundreds of years ago, Johannes Kepler developed a method for describing orbits that allows us to visualize their size, shape, and orientation, as well as the spacecraft's position within them. Because we still need six quantities to describe an orbit and a spacecraft's place in it, Kepler defined six orbital elements. We call these the *classic orbital elements (COEs)*, and we'll use

Recall that $\vec{V}$ is the velocity vector that describes speed and direction. We use V to denote speed without regard to direction.

them to tell us the four things we want to know, as summarized in the COEs checklist on the left. In the rest of this section, we'll go through each of the four things on the checklist and learn which COE describe the given properly. As a preview, we'll learn

- Orbital size, uses the semimajor axis, a
- Orbital shape, is defined by eccentricity, e
- Orientation of the orbital plane in space, uses
  - inclination, i
  - right ascension of the ascending node, Ω
- Orientation of the orbit within the plane is defined by argument of perigee, ω and finally
- Spacecraft's location in the orbit is represented by true anomaly, ν

Let's go through these elements to see what each one contributes to our understanding of orbits and check them off one at a time on our COE checklist.

Defining the Classic Orbital Elements (COEs)

Let's start with orbital *size*. In Chapter 4 we related the size of an orbit to its specific mechanical energy using the relationship

$$\varepsilon = -\frac{\mu}{2a}$$ (4.1.4-1)

where

ε = specific mechanical energy (km^2/s^2)

μ = gravitational parameter of the central body (km^3/s^2)

a = semimajor axis (km)

The semimajor axis, a, describes half the distance across the orbit's major (long) axis, as shown in Figure 4.1.4-2, and we use it as our first COE.
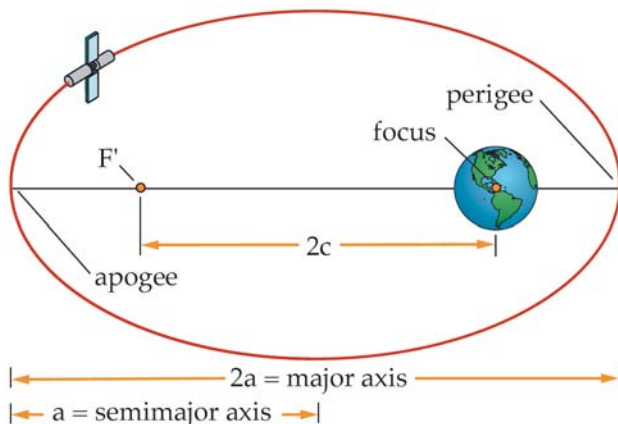

Figure 4.1.4-2. Semimajor Axis. The semimajor axis, a, is one half the distance across the long axis of an ellipse. The distance between the foci (F and F') of the ellipse is 2c.

Classic Orbital Elements (COEs) Checklist

✔ Orbit's size

✔ Orbit's shape

○ Orbit's orientation
  • Orbital plane in space
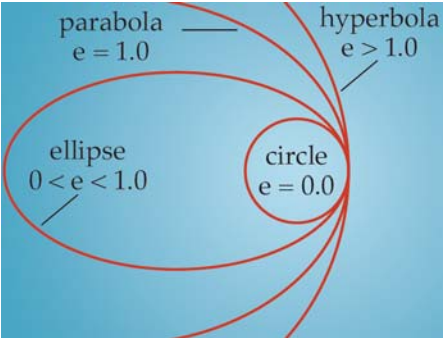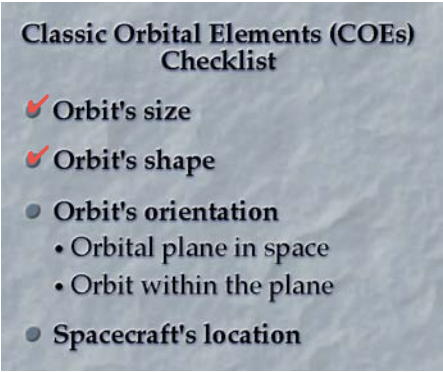 • Orbit within the plane

○ Spacecraft's location

With the orbit's size accounted for, the next thing we want to know is its shape. In Chapter 4, we described the "out of roundness" of a conic section in terms of its eccentricity, e. *Eccentricity* specifies the shape of an orbit by looking at the ratio of the distance between the two foci and the length of the major axis.

$$e = \frac{2c}{2a} \qquad (4.1.4\text{-}2)$$

Table 4.1.4-1 summarizes the relationship between an orbit's shape and its eccentricity and Figure 4.1.4-3 illustrates this relationship.

Table 4.1.4-1. Relationship Between Conic Section and Eccentricity.

Conic Section	Eccentricity
Circle	e = 0
Ellipse	0 < e < 1
Parabola	e = 1
Hyperbola	e > 1

Figure 4.1.4-3. Eccentricity. Eccentricity defines an orbit's shape.

Now we have two pieces of our orbital puzzle: its size, a, and its shape, e. Next we tackle its orientation in space. Previously we learned that because specific angular momentum is constant, an orbital plane is stationary in inertial space. To describe its orientation, we refer to an inertial coordinate system—the geocentric-equatorial coordinate system, shown in Figure 4.1.4-4. (In the following discussion, we describe angles between key vectors, so make sure you know how to perform dot products and how to change from degrees to radians.)

The first angle we use to describe the orientation of an orbit with respect to our coordinate system is inclination, i. *Inclination* describes the tilt of the orbital plane with respect to the fundamental plane (the equatorial plane in this case). We could describe this tilt as the angle between the two planes, but this is harder to do mathematically. Instead, we define inclination as the angle between two vectors: one perpendicular to the orbital plane, $\hat{h}$ (the specific angular momentum vector), and one perpendicular to the fundamental plane, $\hat{K}$, as shown in Figure 4.1.4-5. Inclination has a range of values from 0° to 180°.
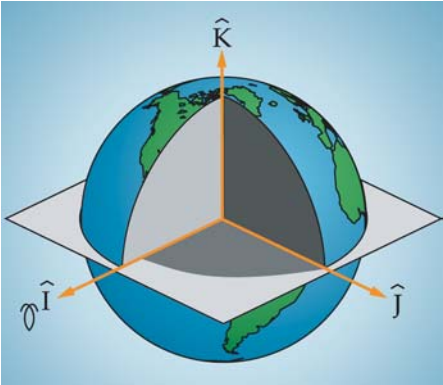

Figure 4.1.4-4. The Geocentric-equatorial Coordinate System. We use the geocentric-equatorial coordinate system to reference all orbital elements. The fundamental plane is Earth's equatorial plane, the principal direction ($\hat{I}$) points in the vernal equinox direction, ♈, the $\hat{K}$ unit vector points to the North Pole, and $\hat{J}$ completes the right-hand rule.

We use inclination to define several different kinds of orbits. For example, an Earth orbit with an inclination of 0° or 180° is an *equatorial orbit*, because it always stays over the equator. If the orbit has i = 90°, we call it a *polar orbit* because it travels over the North and South Poles. We also use the value of inclination to distinguish between two major classes of orbits. If $0° \le i < 90°$, the spacecraft is moving with Earth's rotation (in an easterly direction), and the spacecraft is in a *direct orbit* or *prograde orbit*. If $90° < i \le 180°$, the spacecraft is moving opposite from Earth's rotation (in a westerly direction), so it's in an *indirect orbit* or *retrograde orbit*. Table 4.1.4-2 summarizes these orbits.

Thus, inclination is the third COE. It specifies the *tilt* of the orbital plane with respect to the fundamental plane and helps us understand an orbit's orientation with respect to the equator.

The fourth COE is another angle, *right ascension of the ascending node, Ω*, used to describe orbital orientation with respect to the principal direction, $\hat{I}$. Before you give up on this complex-sounding term, let's look at each of its pieces. First of all, what is "right ascension?" It's similar to longitude except its reference point is the vernal equinox and it doesn't rotate with Earth. So, right ascension of the ascending node is an angle we measure along the equator, starting at the $\hat{I}$ direction.

Now let's look at the other part of this new angle's name, "ascending node" (or a node of any kind)? As we just described, the orbital plane normally tilts (is inclined) with respect to the fundamental plane (unless i = 0° or 180°). From plane geometry, you may remember that the intersection of two planes forms a line. In our case, the intersection of the orbital plane and the fundamental plane is the *line of nodes*. The two points at which the orbit crosses the equatorial plane are the nodes. The node where the spacecraft goes from below the equator (Southern Hemisphere) to above the equator (Northern Hemisphere) is the *ascending node*. Similarly, when the spacecraft crosses the equator heading south, it passes through the *descending node*. See Table 4.1.4-2.
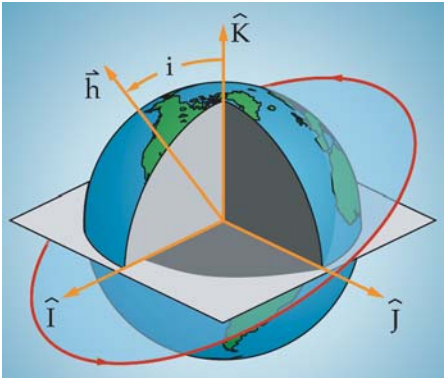

Figure 4.1.4-5. Inclination. Inclination, i, describes the tilt of the orbital plane with respect to the equator. The angle between the two planes is the same as the angle between $\hat{K}$ (which is perpendicular to the equator) and $\hat{h}$ (which is perpendicular to the orbital plane).

Table 4.1.4-2. Types of Orbits and Their Inclination.

Inclination	Orbital Type	Diagram
0° or 180°	Equatorial	
90°	Polar	i = 90°
0° ≤ i < 90°	Direct or Prograde (moves in the direction of Earth's rotation)	ascending node
90° < i ≤ 180°	Indirect or Retrograde (moves against the direction of Earth's rotation)	ascending node

Now let's put "right ascension" and "ascending node" together. The right ascension of the ascending node describes the orbital plane's orientation with respect to the principal direction. That is, how is the orbital plane rotated in space? We use the vernal equinox direction or $\hat{I}$ (an inertial reference) as the starting point and measure eastward along the equator to the ascending node. Thus, the right ascension of the ascending node, Ω, is the angle from the principal direction, $\hat{I}$, to the ascending node. It acts like a celestial map reference to give us the *swivel* of the orbit, helping us to better understand its orientation in space. Figure 4.1.4-6 illustrates the right ascension of the ascending node. Its range of values is $0° \leq \Omega < 360°$. That's now 4 out of 6 on our COE checklist.
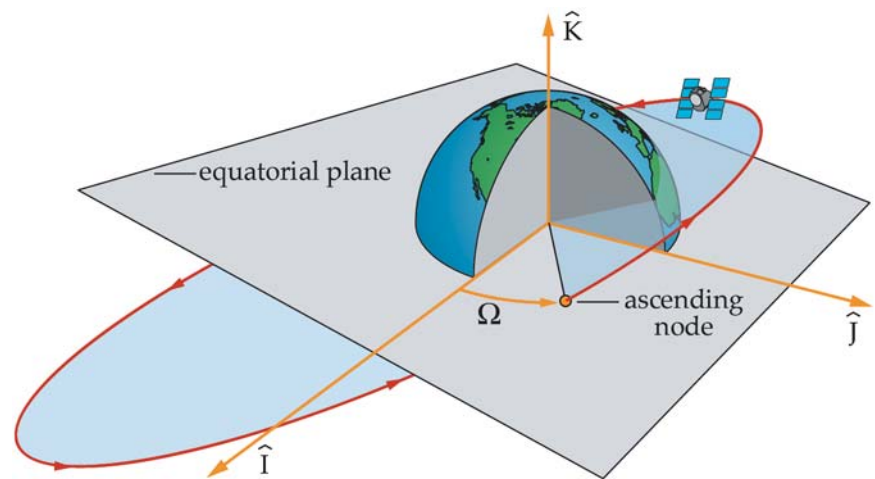

Figure 4.1.4-6. Right Ascension of the Ascending Node, Ω. This angle describes the swivel of the orbital plane with respect to the principal direction. It is the angle along the equator between the principal direction, $\hat{I}$, and the point where the orbital plane crosses the equator from south to north (ascending node), measured eastward.

Classic Orbital Elements (COEs) Checklist

✔ Orbit's size

✔ Orbit's shape

✔ Orbit's orientation

  • Orbital plane in space
    – Inclination, i
    – Right ascension of the ascending node, Ω

  • Orbit within the plane
    – Argument of perigee, ω

○ Spacecraft's location

Let's recap where we are. We now know the orbit's *size*, a, its *shape*, e, its *tilt*, i, and its *swivel*, Ω. But we don't know how the orbit is oriented within the plane. For example, for an elliptical orbit, we may want to know whether perigee (point closest to Earth) is in the Northern or Southern Hemisphere. This is important if we want to take high-resolution pictures of a particular point. So, for this fifth orbital element, we measure the angle along the orbital path between the ascending node and perigee and call it *argument of perigee, ω*. To remove any ambiguities, we always measure this angle in the direction of spacecraft motion.

Where does this unusual sounding term "argument of perigee" come from? To begin with, perigee is an easily identifiable point on the orbit to reference. But why "argument"? Because we're "making clear" (from Latin) where perigee is. So our fifth COE, argument of perigee, ω, is the angle measured in the direction of the spacecraft's motion from the ascending node to perigee. It gives us the orientation of the orbit within the orbital plane, as shown in Figure 4.1.4-7. The range on argument of perigee is $0° \leq \omega < 360°$. That's 5 down and 1 to go on our COE checklist.
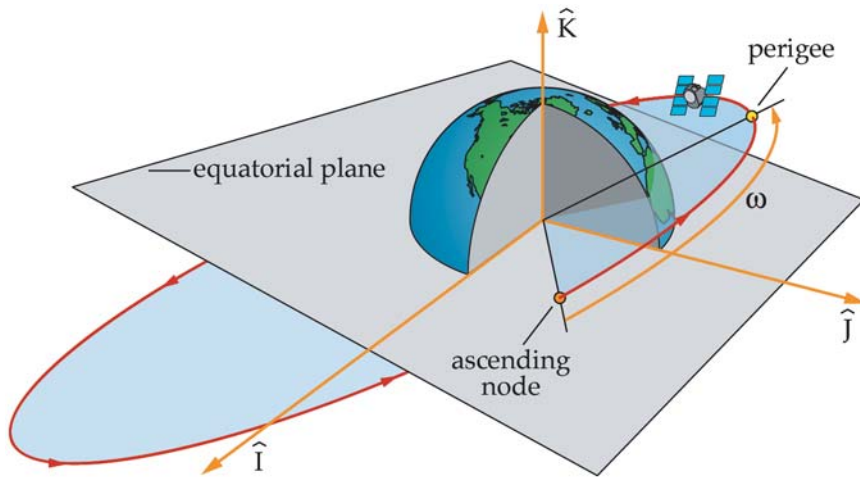
Figure 4.1.4-7. Argument of Perigee, ω. This angle describes the orientation of an orbit within its orbital plane. It is the angle between the ascending node and perigee, measured in the direction of the spacecraft's motion.

After specifying the size and shape of the orbit, along with its orientation (tilt and swivel), we still need to find a spacecraft's location within the orbit. As we've already seen in Chapter 4, we can find this using the true anomaly. *True anomaly, ν*, is the angle along the orbital path from perigee to the spacecraft's position vector, $\vec{R}$. Similar to the argument of perigee, we measure true anomaly in the direction of the spacecraft's motion. Figure 4.1.4-8 shows true anomaly. Its range of values is $0° \leq \nu < 360°$.

True anomaly, ν, tells us the location of the spacecraft in its orbit. Of all the COEs, only true anomaly changes with time (while our two-body assumptions hold) as the spacecraft moves in its orbit.

Now that you've seen all six of the COEs, we can show four of them together in Figure 4.1.4-9 (we can show size and shape only indirectly in the way we draw the orbit). Table 4.1.4-3 summarizes all six. That completes our COE checklist. We've shown all you need to know about describing an orbit and locating a spacecraft within it.
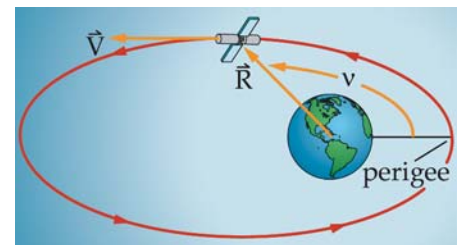

Figure 4.1.4-8. True Anomaly. True anomaly, ν, specifies the location of a spacecraft within the orbit. It is the angle between perigee and the spacecraft's position vector measured in the direction of the spacecraft's motion. Of all the COEs, only true anomaly changes with time (as long as our

Classic Orbital Elements (COEs) Checklist

✔ **Orbit's size**

✔ **Orbit's shape**

✔ **Orbit's orientation**

- Orbital plane in space
  - Inclination, i
  - Right ascension of the ascending node, Ω
- Orbit within the plane
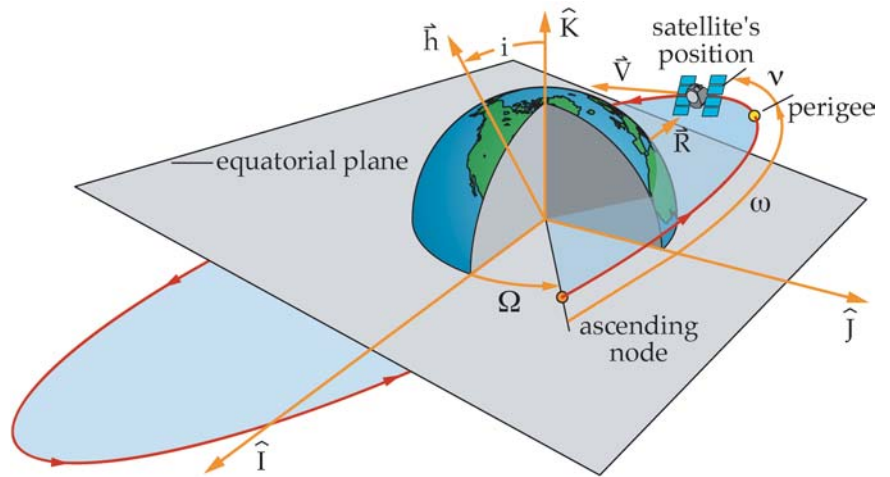 - Argument of perigee, ω

✔ **Spacecraft's location**

Figure 4.1.4-9. Classic Orbital Elements (COEs). Here we show four of the six COEs. We use the COEs to visualize an orbit and locate a spacecraft in it. The other two COEs, semimajor axis, a, and eccentricity, e, specify the size and shape of an orbit.

Table 4.1.4-3.Summary of Classic Orbital Elements.

Element	Name	Description	Range of Values	Undefined
a	Semimajor axis	Size	Depends on the conic section	Never
e	Eccentricity	Shape	$e = 0$: circle $0 < e < 1$: ellipse	Never
i	Inclination	Tilt, angle from $\hat{K}$ unit vector to specific angular momentum vector $\vec{h}$	$0 \leq i \leq 180°$	Never
Ω	Right ascension of the ascending node	Swivel, angle from vernal equinox to ascending node	$0 \leq \Omega < 360°$	When $i = 0$ or $180°$ (equatorial orbit)
ω	Argument of perigee	Angle from ascending node to perigee	$0 \leq \omega < 360°$	When $i = 0$ or $180°$ (equatorial orbit) or $e = 0$ (circular orbit)
ν	True anomaly	Angle from perigee to the spacecraft's position	$0 \leq \nu < 360°$	When $e = 0$ (circular orbit)

By now you may wonder what all these COEs are good for! Let's look at an example to see how they can help us visualize an orbit. Suppose a communication satellite has the following COEs

- Semimajor axis, a = 50,000 km

- Eccentricity, e = 0.4

- Inclination, i = 45°

- Right ascension of the ascending node, Ω = 50°

- Argument of perigee, ω = 110°

- True anomaly, ν = 170°

To begin with, as in Figure 4.1.4-10, we can sketch the size and shape of the orbit given the semimajor axis and the eccentricity. The eccentricity of 0.4 indicates an elliptical orbit (it's between 0 and 1). The semimajor axis of 50,000 km tells us how large to draw the orbit.

Now that we see the orbit in two dimensions, we can use the other COEs to visualize how it's oriented in three dimensions. Because the inclination angle is 45°, we know the orbital plane tilts 45° from the equator. We can also describe inclination as the angle between the specific angular momentum vector, $\hat{h}$, and $\hat{K}$ in the geocentric-equatorial coordinate system. So we can sketch the crossing of the two planes in three dimensions as you see in Figure 4.1.4-11.

Next, to find the swivel of the orbital plane with respect to the principal direction, we use the right ascension of the ascending node, Ω. After locating the principal direction in the equatorial plane, $\hat{I}$, we swivel the orbital plane by positioning the ascending node 50° east of the $\hat{I}$ vector. What we know so far gives us the picture of the orbit in Figure 4.1.4-12.
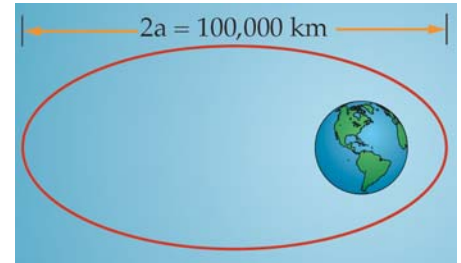

Figure 4.1.4-10. Orbital Size and Shape. Here we show the approximate size and shape of an orbit with a semimajor axis of 50,000 km and an eccentricity of 0.4.
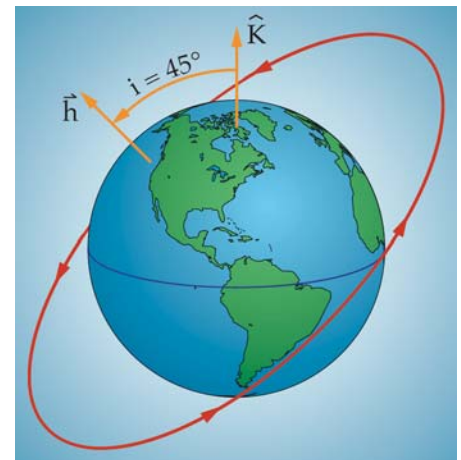

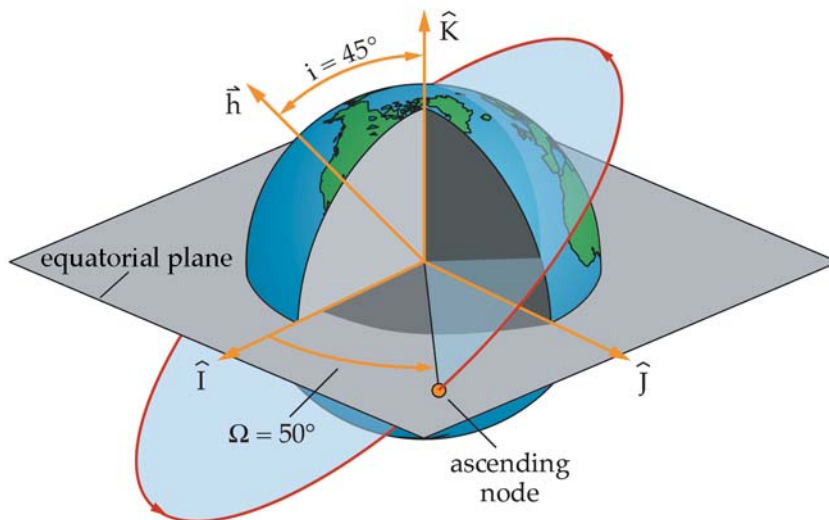**Figure 4.1.4-11. Inclination.** This orbit has an inclination of 45°.

Figure 4.1.4-12. Our Orbit So Far. Here we show an orbit with an inclination, i, of 45° and a right ascension of the ascending node, Ω, of 50°.

So, we've completely specified the orbit's size and shape, as well as the orientation of the orbital plane in space. But we still don't know how the orbit is oriented within the plane. Argument of perigee, ω, comes next. To locate perigee within the orbital plane, we rotate perigee 110° from the ascending node, in the direction of spacecraft motion. Figure 4.1.4-13 shows how to orient the orbit in the orbital plane.
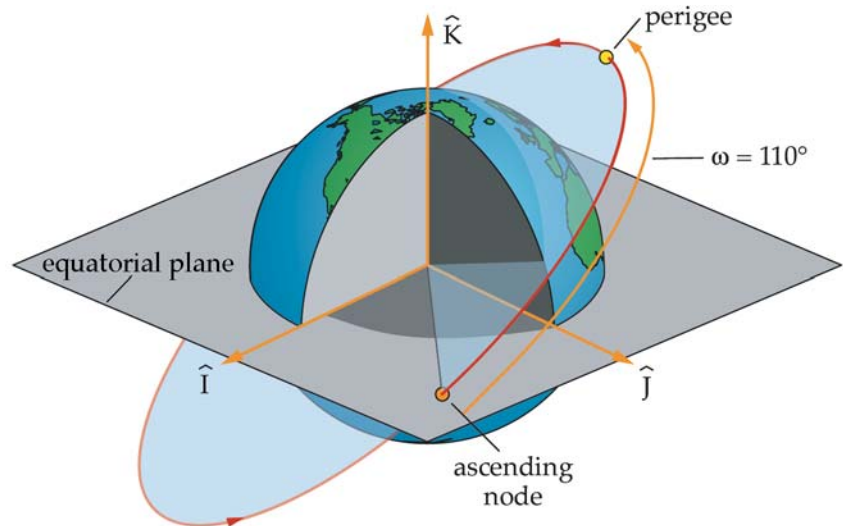

Figure 4.1.4-13. Argument of Perigee for the Example. We rotate perigee 110° from the ascending node to determine the argument of perigee, ω, is 110°.

Finally, we locate our communication satellite within the orbit. Using the value of true anomaly, ν, we measure 170° in the direction of spacecraft motion from perigee to the spacecraft's position. And there it is in Figure 4.1.4-14!
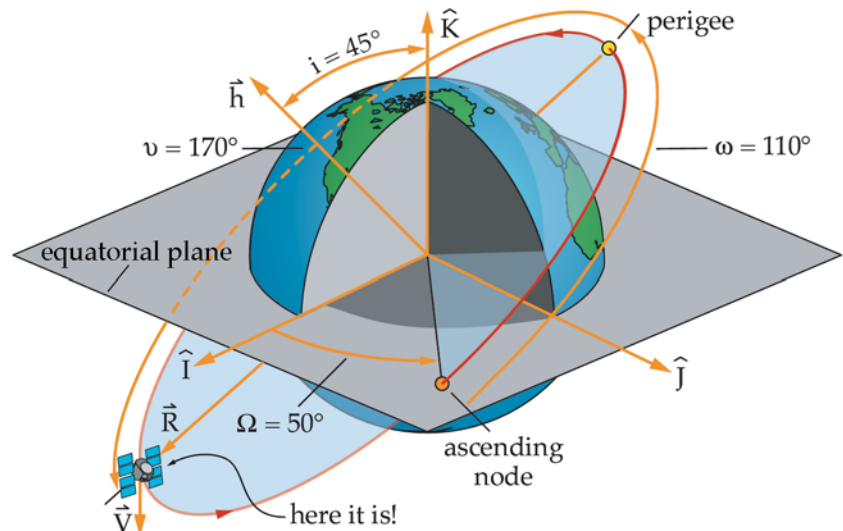

Figure 4.1.4-14. Finding the Satellite. Here we show the position of a satellite with the following COEs: a = 50,000 km; i = 45°; Ω = 50°; ω = 110°; ν = 170°.

As we already know, various missions require different orbits, as described by their COEs. Table 4.1.4-4 shows various types of missions and their typical orbits. A *geostationary orbit* is a circular orbit with a period of about 24 hours and inclination of 0°. Geostationary orbits are particularly useful for communication satellites because a spacecraft in this orbit appears motionless to an Earth-based observer, such as a fixed ground station for a cable TV company. *Geosynchronous orbits* are inclined orbits with a period of about 24 hours. A *semi-synchronous orbit* has a period of 12 hours. *Sun-synchronous orbits* are retrograde, low-Earth orbits (LEO) typically inclined 95° to 105° and often used for remote-sensing missions because they pass over nearly every point on Earth's surface. A *Molniya orbit* is a semi-synchronous, eccentric orbit used for some specific communication missions.

Table 4.1.4-4.Orbital Elements for Various Missions.

Mission	Orbital Type	Semimajor Axis (Altitude)	Period	Inclination	Other
• Communication • Early warning • Nuclear detection	Geostationary	42,158 km (35,780 km)	~24 hr	~0°	$e \cong 0$
• Remote sensing	Sun-synchronous	~6500 – 7300 km (~150 – 900 km)	~90 min	~95°	$e \cong 0$
• Navigation – GPS	Semi-synchronous	26,610 km (20,232 km)	12 hr	55°	$e \cong 0$
• Space Shuttle	Low-Earth orbit	~6700 km (~300 km)	~90 min	28.5°, 39°, 51°, or 57°	$e \cong 0$
• Communication/ intelligence	Molniya	26,571 km (R_p = 7971 km; R_a = 45,170 km)	12 hr	63.4°	ω = 270° e = 0.7

Alternate Orbital Elements

Now that we've shown how to find all the classic orbital elements (COEs), we're ready to share some bad news—they're not always defined! For example, a *circular orbit* has no perigee. In this case, we have no argument of perigee, ω, or true anomaly, v, because both use perigee as a reference. To correct this deficiency, we bring in an alternate orbital element to replace these two missing angles. In general, whenever we face a peculiar orbit with one or more of the COEs undefined, we work backward from the spacecraft's position vector (the one thing that's always defined) to the next quantity that is defined. For our circular-orbit example, instead of using true anomaly to define position, we use the first alternate element—the argument of latitude, u. We measure *argument of latitude, u*, along the orbital path from the ascending node to the spacecraft's position in the direction of the spacecraft's motion.
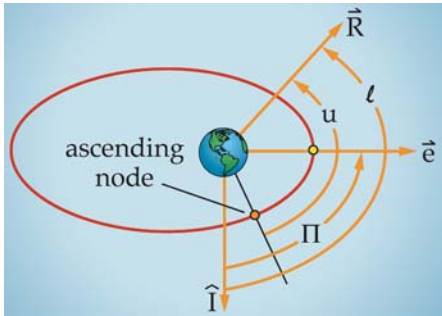
Figure 4.1.4-15. Alternate Orbital Elements. We use the alternate orbital elements when one or more of the classic orbital elements are undefined. u is the argument of latitude. Π is the longitude of

Another special situation that requires an alternate element is an equatorial orbit (i = 0° or 180°). In this case, the line of intersection between the equator and the orbital plane is missing (the line of nodes), so the ascending node doesn't exist. This time the right ascension of the ascending node, Ω, and the argument of perigee, ω, are undefined. We replace them with another alternate element, the *longitude of perigee, Π*— the angle measured from the principal direction, $\hat{I}$, to perigee in the direction of the spacecraft's motion.

Finally, a *circular equatorial orbit* has neither perigee *nor* ascending node, so the right ascension of the ascending node, Ω, the argument of perigee, ω, and true anomaly, ν, are all undefined! Instead, we use a final alternate element to replace all of them—the *true longitude, l*. We measure this angle from the principal direction, $\hat{I}$, to the spacecraft's position vector, $\vec{R}$, in the direction of the spacecraft's motion. Figure 4.1.4-15 and Table 4.1.4-5 summarize these alternate orbital elements.

Table 4.1.4-5. Alternate Orbital Elements.

Element	Name	Description	Range of Values	When to Use
u	Argument of latitude	Angle from ascending node to the spacecraft's position	$0° \leq u < 360°$	Use when there is no perigee (e = 0)
Π	Longitude of perigee	Angle from the principal direction to perigee	$0° \leq \Pi < 360°$	Use when equatorial (i = 0 or 180°) because there is no ascending node
l	True longitude	Angle from the principal direction to the spacecraft's position	$0° \leq l < 360°$	Use when there is no perigee and ascending node (e = 0 and i = 0 or 180°)

Section Review

Key Concepts

➤ To specify a spacecraft's orbit in space, you need to know four things about it
- Orbit's Size
- Orbit's Shape
- Orbit's Orientation
- Spacecraft's Location

➤ The six classic orbital elements (COEs) specify these four pieces of information
- Semimajor axis, a—one-half the distance across the long axis of an ellipse. It specifies the orbit's size and relates to an orbit's energy.
- Eccentricity, e—specifies the shape of an orbit by telling what type of conic section it is
- Inclination, i—specifies the orientation or tilt of an orbital plane with respect to a fundamental plane, such as the equator
- Right ascension of the ascending node, Ω—specifies the *orientation* or *swivel* of an orbital plane with respect to the principal direction, $\hat{I}$
- Argument of perigee, ω—specifies the orientation of an orbit within the plane
- True anomaly, ν—specifies a spacecraft's location within its orbital plane

➤ Whenever one or more COEs are undefined, you must use the alternate orbital elements

4.1.4.2 Spacecraft Ground Tracks

In This Section You'll Learn to...

- ☞ Explain why spacecraft ground tracks look the way they do
- ☞ Use ground tracks to describe why certain types of missions use specific types of orbits
- ☞ Use ground tracks to determine the inclination and period for direct orbits

The six classic orbital elements (COEs) allow us to visualize an orbit from space. Now let's beam back to Earth to see orbits from our perspective on the ground.

Many spacecraft users need to know what part of Earth their spacecraft is passing over at any given time. For instance, remote-sensing satellites must be over precise locations to get the coverage they need. As we'll see, we can learn a lot about a spacecraft's orbit and mission by examining the track it makes along Earth.

To understand ground tracks, imagine you're driving from San Francisco to Omaha. To get there, you go east out of San Francisco on Interstate 80 for a couple thousand miles. If you have a road map of the western United States, you can trace your route on the map by drawing a meandering line along I-80, as shown in Figure 4.1.4-16. This is your ground track from San Francisco to Omaha.

Now imagine you're taking the same trip in an airplane. You can trace your air route on the same map, but because you don't need roads, this ground track is nearly a straight line.

A spacecraft's ground track is similar to these examples. It's a trace of the spacecraft's path over Earth's surface. But it's more complicated because the spacecraft goes all the way around (more than 40,000 km or 25,000 mi.) during each orbit and Earth spins on its axis at more than 1600 km/hr (1000 m.p.h.) at the equator at the same time, as we show in Figure 4.1.4-17.

So what does a ground track look like? To make things easy, let's start by pretending Earth doesn't rotate. (Try not to get dizzy—we'll turn the rotation back on soon.) Picture an orbit above this non-rotating Earth. The ground track follows a great circle route around Earth. A *great circle* is any circle that "slices through" the center of a sphere. For example, lines of longitude, as shown in Figure 4.1.4-18, are great circles, because they slice through Earth's center, but lines of latitude are not great circles (except for 0° latitude at the equator), because they don't. An orbital trace must be a great circle because the spacecraft is in orbit around Earth's center; thus, the orbital plane also passes through Earth's center.

When we stretch Earth onto a flat-map projection (called a Mercator projection), the ground track looks a little different. To visualize how this flattening affects the ground-track shape, imagine Earth as a soda can. A
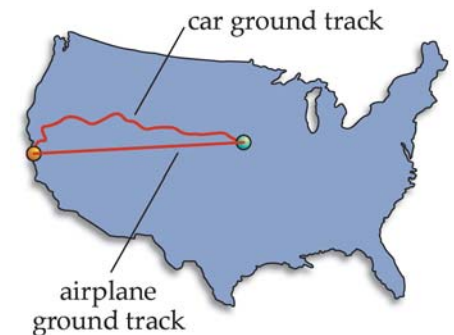

Figure 4.1.4-16. Car and Airplane Ground Tracks. Ground tracks for a trip by car and air from San Francisco to Omaha.
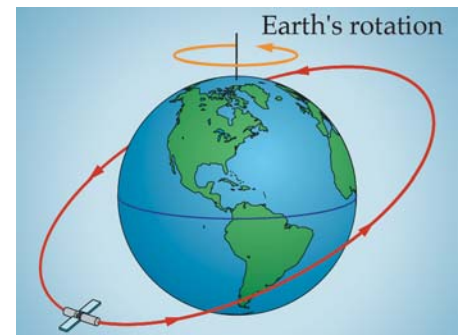

Figure 4.1.4-17. Earth and Spacecraft Motion. The Earth spins on its axis at nearly 1600 km/hr (1000 m.p.h.) at the equator, while a spacecraft orbits above it.
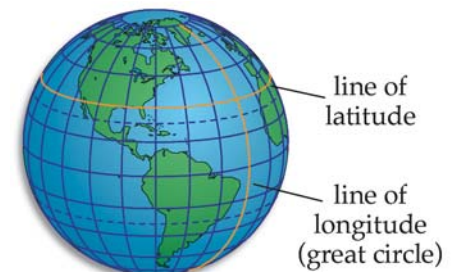

Figure 4.1.4-18. Great Circles. A great circle is any circle around a sphere which bisects it (cuts it exactly in half). Lines of longitude are great circles whereas lines of latitude (except for the equator) are not.
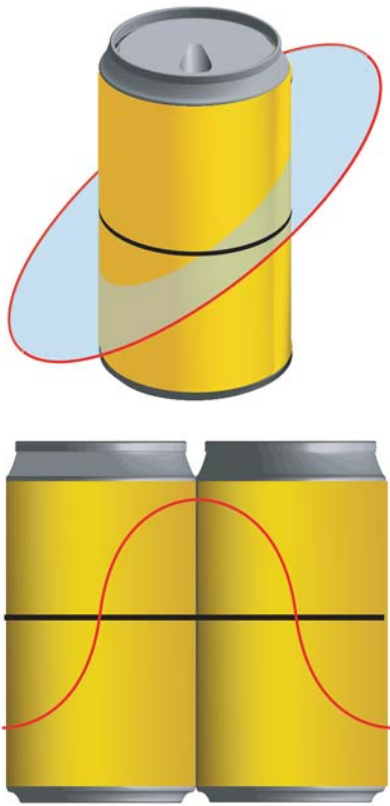
trace of the orbit on the soda can is shown in Figure 4.1.4-19. It looks like a circle slicing through the center of the can. But what if we were to flatten the can and look at the orbital trace, as shown in Figure 4.1.4-19? It looks like a sine wave!

Now imagine yourself on the ground watching the spacecraft pass overhead. Because we stopped Earth from rotating, the ground track will always stay the same, and the spacecraft will continue to pass overhead orbit after orbit, as shown in Figure 4.1.4-20. Even if we change the size and shape of the orbit, the ground track will look the same.
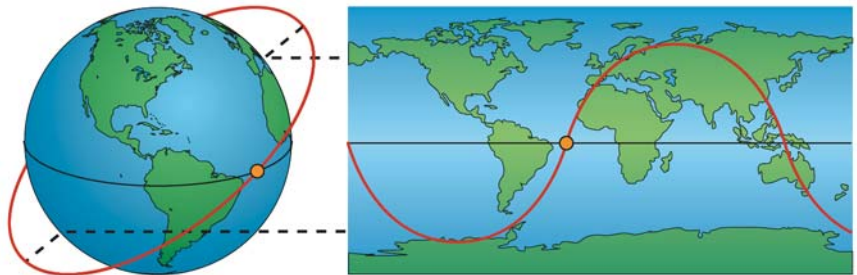

Figure 4.1.4-19. Orbiting around a Soda Can. Imagine an orbit around a soda can. It draws a circle around the can. When we flatten the can, the line looks like a sine wave.

Figure 4.1.4-20. An Orbit's Ground Track for a Non-Rotating Earth. For a non-rotating Earth, the ground track of an orbit will continuously repeat.

But suppose we start Earth rotating again. What happens? The spacecraft passes overhead on one orbit but appears to pass to the *west* of you on the next orbit. How can this be? Because the orbital plane is immovable in inertial space, the spacecraft stays in the same orbit. But you're fixed to Earth and as it rotates to the east, you move away from the orbit, making it look as if the spacecraft moved, as seen in Figure 4.1.4-21. Each ground track traces a path on Earth farther to the west than the previous one.
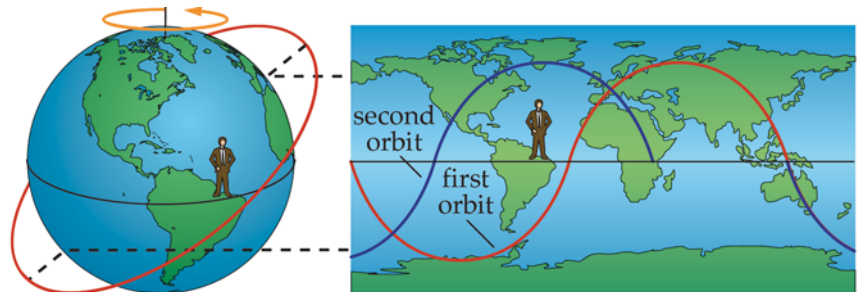

Figure 4.1.4-21. A Normal Spacecraft Ground Track. As Earth rotates, successive ground tracks appear to shift to the west from an Earth-based observer's viewpoint.

Can we learn something about the orbit from all of this? Sure! Because Earth rotates at a fixed rate of about 15° per hr (360° in 24 hrs = 15°/hr) or 0.25° per minute, we can use this rotation as a "clock" to tell us the orbit's period. By measuring how much the orbit's ground track moves to the west from one orbit to the next, and we can establish a new parameter, *node displacement, ΔN*. We measure ΔN along the equator from one ascending node to the next and define it to be positive in the direction of the spacecraft's motion. Thus, the nodal displacement to the west during one orbit is the difference between 360° and ΔN.

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We can put this ground track shift to work in finding the orbital period because the nodal displacement is simply Earth's rotation rate times the period of the orbit. For example, suppose the period of an orbit were two hours. Earth would rotate 30° (2 hr × 15°/hr) during one orbital revolution, producing a nodal displacement of 330° (360° − 30°). In terms of ΔN, we find the period from

$$\text{Period (hours)} = \frac{360° - \Delta N}{15°/\text{hr}} \text{ (for direct orbits)} \qquad (4.1.4\text{-}3)$$

[*Note:* As is, this equation applies only to direct orbits with a period less than 24 hours. For other orbits, the same concept applies but the equation changes. We'll only consider direct orbit ground tracks with periods less than 24 hours, so this equation will suffice.] If we can determine the period, we can also determine the orbit's semimajor axis using the equation below.

$$P = 2\pi\sqrt{\frac{a^3}{\mu}} \qquad (4.1.4\text{-}4)$$

where

P = period (s)

π = 3.14159. . .(unitless)

a = semimajor axis (km)

μ = gravitational parameter (km^3/s^2) = 3.986 × 10^5 km^3/s^2 for Earth

So, by finding ΔN from the ground track, we can find the period and then the semimajor axis. For example, in the ground track in Figure 4.1.4-22, ΔN is 315°. We find the orbital period using Equation (4.1.4-3) and the semimajor axis using Equation (4.1.4-4). But we must be careful to watch the units when using these equations.
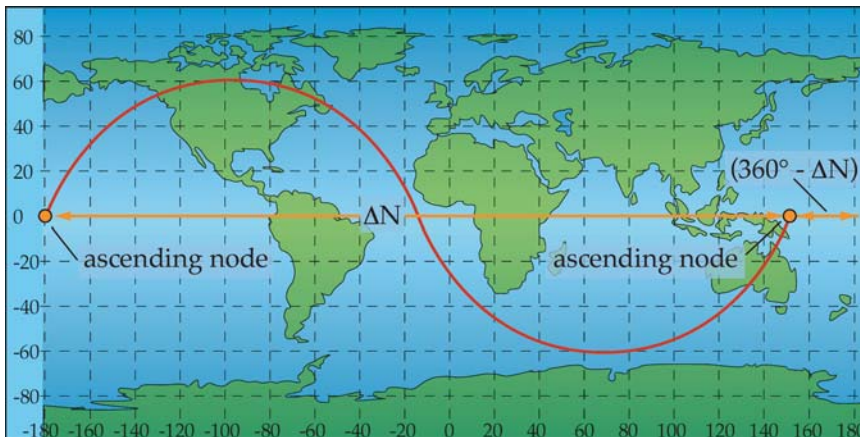

Figure 4.1.4-22. Ascending Node Shift Due to the Rotating Earth. We measure ΔN along the equator from one ascending node to the next. It is positive in the direction of spacecraft motion. Thus, 360° − ΔN represents the amount Earth rotates during one orbit.

As the orbit's size increases, the semimajor axis gets bigger, so ΔN gets smaller. This happens because the spacecraft takes longer to make one revolution as Earth rotates beneath it (the bigger the semimajor axis, a, the longer the period). As the orbit gets bigger, the ΔN gets smaller, so the ground track appears to compress or "scrunch" together. Recall, we define a geosynchronous orbit as one with a period of approximately 24 hours. For such an orbit, the ΔN is 0°. This means the spacecraft's period matches Earth's rotational period. Thus, the orbit appears to retrace itself and form a figure 8, as shown in Figure 4.1.4-23, orbit D. If the orbit lies in the equatorial plane (has an inclination of 0°), the ground track will be just a dot on the equator, similar to orbit E, in Figure 4.1.4-23. A spacecraft with a period of 24 hours and an inclination of 0° is in a geostationary orbit. This name means the spacecraft appears stationary to Earth-based observers, making these orbits very useful for communication satellites. Once we point the receiving antenna at the satellite, we don't have to move the antenna as Earth rotates.
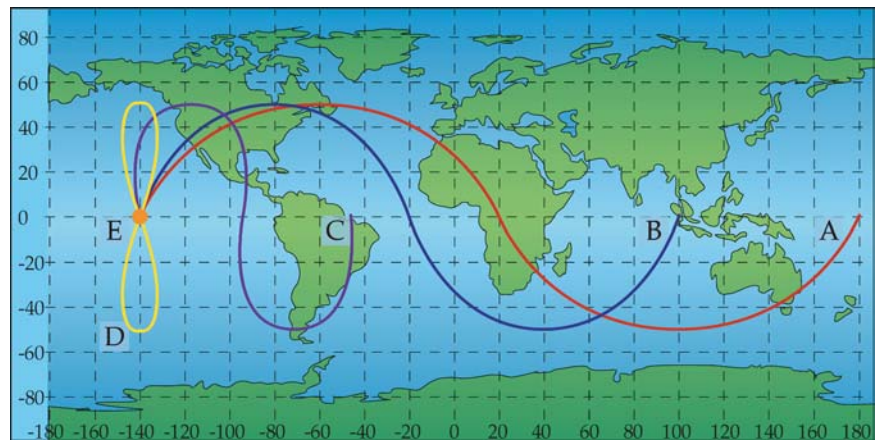

Figure 4.1.4-23. Orbital Ground Tracks. Orbit A has a period of 2.67 hours. Orbit B has a period of 8 hours. Orbit C has a period of 18 hours. Orbit D has a period of 24 hours. Orbit E has a period of 24 hours.
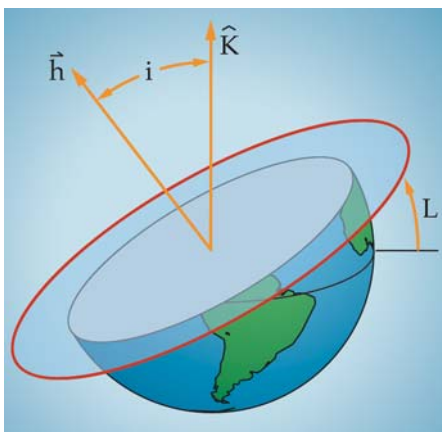

Figure 4.1.4-24. Inclination Equals Highest Latitude, L. Because inclination relates the angle between the orbital plane and the equatorial plane, the highest latitude reached by a spacecraft equals its inclination (for direct orbits).

Besides using the ground track to determine an orbit's semimajor axis, we can also find its inclination. Imagine a spacecraft in a 50° inclined orbit. From our definition of inclination, we know in this case the angle between the equatorial plane and the orbital plane is 50°. What's the highest latitude the spacecraft will pass over directly? 50°! The highest latitude any spacecraft passes over equals its inclination. Let's see why.

Remember that latitude is the Earth-centered angle measured from the equator north or south to the point in question. But the orbital plane also passes through Earth's center, and the angle it forms with the equatorial plane is its inclination, as we show in Figure 4.1.4-24. Thus, for direct (prograde) orbits, when a spacecraft reaches its northernmost point, the point on Earth directly below it lies on the latitude line equal to the orbit's inclination.

In this way, we can use the ground track to tell us the orbit's inclination.

- For a direct orbit (0 < i < 90°), we find the northernmost or southernmost point on the ground track and read its latitude. This "maximum latitude" equals the orbit's inclination.
- For a retrograde orbit (90 < i < 180°), we subtract the maximum latitude from 180° to get the inclination

The Earth coverage a spacecraft's mission requires affects how we select the orbit's inclination. For example, if a remote-sensing spacecraft needs to view the entire surface during the mission, it needs a near polar inclination of about 90°. In Figure 4.1.4-25 we see several spacecraft ground tracks with the same period but with varying inclinations.
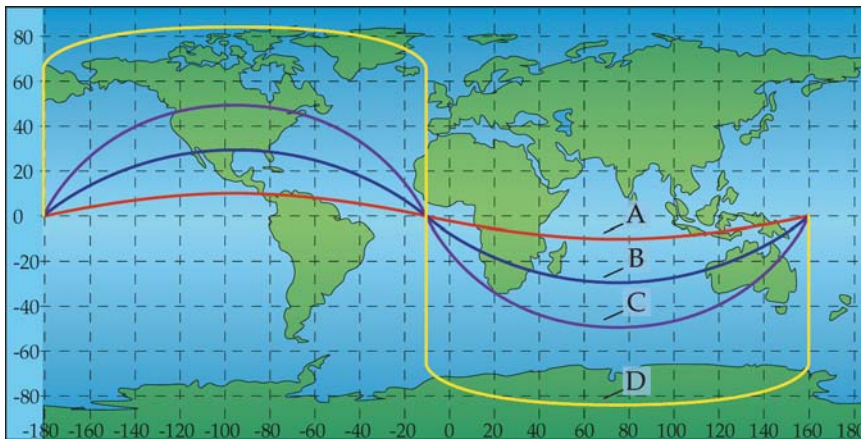

Figure 4.1.4-25. Changing Inclination. All four ground tracks represent orbits with a period of 4 hours. We can find the inclination of these orbits by looking at the highest latitude reached. Orbit A has an inclination of 10°. Orbit B has an inclination of 30°. Orbit C has an inclination of 50°. Orbit D has an inclination of 85°. (Note that Orbit D appears distorted, because ground distances elongate near the poles on a Mercator projection map.)

So far we've looked only at circular orbits. Now let's look at how eccentricity and the location of perigee affect the shape of the ground track. If an orbit is circular, its ground track is symmetrical. If an orbit is elliptical, its ground track is lopsided. That is, it will not look the same above and below the equator. Remember, a spacecraft moves fastest at perigee, so it travels farthest along its path near perigee, making the ground track look spread out. But, near apogee it's going slower, so the ground track is more scrunched. We show this effect in the two ground tracks in Figure 4.1.4-26. Orbit A has perigee in the Northern Hemisphere; Orbit B has perigee in the Southern Hemisphere.
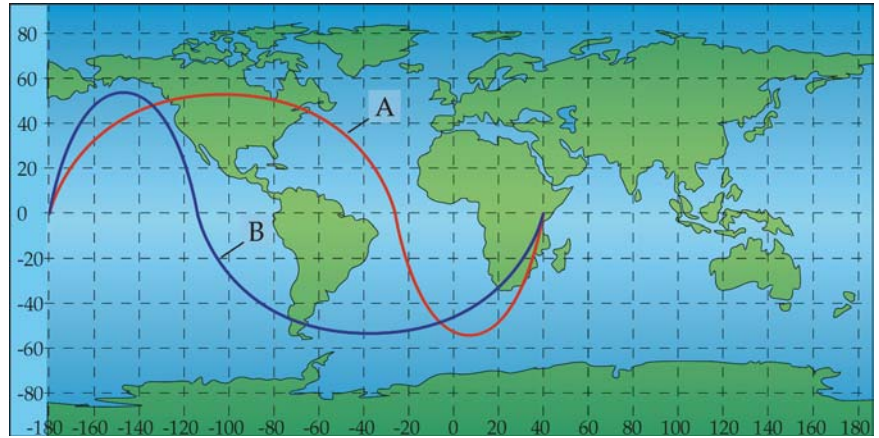
Figure 4.1.4-26. Changing Perigee Location. Both ground tracks represent orbits with periods of 11.3 hours and inclinations of 50°. Both orbits are highly eccentric. Orbit A has perigee over the Northern Hemisphere. Orbit B has perigee over the Southern Hemisphere. If the mission objective is to get high-resolution photographs of locations in the United States, then orbit A has perigee properly positioned.

Section Review

Key Concepts

➤ A ground track is the path a spacecraft traces on Earth's surface as it orbits. Because a spacecraft orbits around Earth's center, the orbital plane slices through the center, so the ground track is a great circle.

➤ When the spherically-shaped Earth is spread out on a two-dimensional, Mercator-projection map, the orbital ground track resembles a sine wave for orbits with periods less than 24 hours

➤ Because orbital planes are fixed in inertial space and Earth rotates beneath them, ground tracks appear to shift westward during successive orbits

➤ From a ground track, you can find several orbital parameters

- Orbital period—by measuring the westward shift of the ground track

- Inclination of a spacecraft's orbit—by looking at the highest latitude reached on the ground track (for direct orbits)

- Approximate eccentricity of the orbit—nearly circular orbits appear symmetrical, whereas eccentric orbits appear lopsided

- Location of perigee—by looking at the point where the ground track is spread out the most

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Bate, Roger R., Donald D. Mueller, and Jerry E. White. *Fundamentals of Astrodynamics.* New York, NY: Dover Publications, 1971.

Wertz, James R. and Wiley J. Larson. *Space Mission Analysis and Design.* Third edition. Dordrecht, Netherlands: Kluwer Academic Publishers, 1999.