**Supplemental Application Guidance for Unguided Suborbital Launch Vehicles**

**Attachment 1 - Trajectory Analysis**

Draft - 08/15/98

**General**

1. An applicant shall conduct a trajectory analysis in accordance with the definitions, input, methodology, and output prescribed below.
2. The launch vehicle's ability to fly a perfect trajectory (i.e. zero error) is undermined by quantifiable perturbing forces that cause drag impact point offsets in the uprange, downrange, and cross-range directions. The perturbing forces are categorized by their individual sources and are normally associated with launch vehicle performance variations. Hence, each source is referred to as a key dispersion parameter. These key parameters are crucial to determining the three-sigma vehicle dispersion, defined below.

**Definitions**

1. Crossrange - The distance measured along a line whose direction is either 90 degrees clockwise (right crossrange) or 90 degrees counter-clockwise (left crossrange) to the projection of the launch vehicle velocity vector azimuth into a horizontal plane. This plane is tangent to the ellipsoidal earth model at the intersection point of a line and the earth's surface where the line is normal with the earth's surface and passes through the launch vehicle's current earth centered position.
2. Downrange - The distance measured along a line whose direction is parallel to the projection of the launch vehicle velocity vector azimuth into a horizontal plane. This plane is tangent to the ellipsoidal earth model at the intersection point of a line and the earth's surface where the line is normal with the earth's surface and passes through the launch vehicle's current earth centered position.
3. Drag Impact Point (DIP) - The drag impact point is defined at the intersection of a launch vehicle stage's or other impacting component's predicted ballistic trajectory with the earth's surface. This method of trajectory prediction includes the effects of atmospheric influences as a function of drag forces and Mach number.
4. Maximum Range Trajectory - The maximum range trajectory is an optimized nominal "type" trajectory extended to fuel exhaustion of each stage. This trajectory is computed at the flight elevation angle that produces the maximum downrange DIP at any given time after lift-off.
5. Nominal Trajectory - The nominal trajectory is the trajectory that the vehicle will fly if all vehicle aerodynamic parameters are exactly as expected, if all vehicle internal and external systems perform exactly as planned, and there are no external perturbing influences (e.g. winds) other than atmospheric drag and gravity.
6. Normal Flight - Normal flight refers to all possible trajectories of a properly performing launch vehicle whose Drag Impact Point (DIP) location does not deviate from the nominal location more than three standard deviations (three-sigma) in the uprange, downrange, left-crossrange, or right crossrange directions.
7. Three-Sigma Vehicle Dispersion - Three-sigma dispersions define the expected uprange, downrange, and crossrange limits of normality for the launch vehicle. Impact dispersion of a launch vehicle is the statistical deviation of the actual impact point from the predicted nominal impact point. It is used to calculate the probability of impacting within a given distance of the nominal impact point. The dispersion distance is in terms of a standard deviation value (referred to as sigma). Theoretical dispersion is used when insufficient launches have occurred to adequately define flight dispersion with a high degree of confidence, and is determined by varying each of the parameters that affect impact range or azimuth. Each parameter is varied by its three sigma value, and then used to determine its individual effect on the vehicle's impact dispersion. The square root of the sum of the squares of the individual impact dispersions provides the total 3 sigma impact dispersion of the vehicle. Assuming a normal distribution, this represents the area in which 99.7% of all impacts will occur.
8. Uprange - The distance measured along a line whose direction is 180 degrees to the projection of the launch vehicle velocity vector azimuth into a horizontal plane. This plane is tangent to the ellipsoidal earth model at the intersection point of a line and the earth's surface where the line is normal with the earth's surface and passes through the launch vehicle's current earth centered position.

**Input**

The trajectory analysis requires inputs necessary to produce a 6-degree-of-freedom trajectory. Trajectory data computed with commercially available software products shall consider the following parameters in the trajectory computations:

1. Launcher Data: geodetic latitude and longitude; height above sea level; position (location) errors; and launch azimuth and elevation.
2. Reference Ellipsoidal Earth Model: name, semi-major axis, semi-minor axis, eccentricity, flattening parameter, gravitational parameter, rotation angular velocity, gravitational harmonic constants (J2,J3,J4), and mass of earth.
3. Vehicle characteristics (per stage): nozzle exit area, distance from nose-tip to nozzle exit, reference drag area, reference diameter, thrust vs. time, propellant weight vs. time, coefficient of drag vs. mach number, distance from nose-tip to center of gravity vs. time, yaw moment of inertia vs. time, pitch moment of inertia vs. time, pitch damping coefficient vs. mach number, aerodynamic damping coefficient vs. mach number, normal force coefficient vs. mach number, distance from nose-tip to center of pressure vs. mach number, axial force coefficient vs. mach number, roll rate vs. time, static stability margin vs. time, gross mass, burnout mass, vacuum thrust, vacuum ISP, stage dimensions, weight of each spent stage.
4. Launch events: stage ignition times, stage burn times, and stage separation times, referenced to ignition time of first stage (T-0).
5. Atmosphere: density vs. altitude, pressure vs. altitude, speed of sound vs. altitude, temperature vs. altitude.
6. Wind errors: error in measurement of wind direction vs. altitude and wind magnitude vs. altitude, wind forecast error (i.e. error due to time delay from wind measurement to launch).

**Methodology - Nominal No-Wind Trajectory**

To generate the nominal no-wind trajectory, the following procedure is acceptable.

1. Identify individual launch vehicle key performance parameters. Typical key performance parameters include thrust, weight, aerodynamic drag, staging times, stage separation-force, launcher height, launcher elevation, and launcher azimuth. The applicant is responsible for including all key performance parameters in the trajectory analysis computations.
2. Run a no-wind trajectory simulation using a 6-Degree-Of-Freedom (6-DOF) model. Generally, the six degrees-of-freedom are launch vehicle position translation along three axes of an orthogonal earth centered coordinate system, and launch vehicle orientation in roll, pitch and yaw. The 6-DOF program computes the translations and orientations in response to internal and external forces and moments. The following assumptions may be incorporated in the 6-DOF program with accompanying justification: (1) the air-frame is a rigid body, (2) the air-frame has a plane of symmetry coinciding with the vertical plane of reference, (3) the vehicle has aerodynamic symmetry in roll (4) the air-frame has six degrees-of-freedom, and (5) the aerodynamic forces and moments are assumed to be functions of mach number and are linear with small flow incidence angles of attack.
3. At each staging event time tabulate the geodetic latitude and longitude of the nominal DIP location for each impacting vehicle stage or component.

**Methodology - Maximum Range Trajectory**

This trajectory provides the maximum possible downrange DIP for each launch vehicle stage or impacting body. The applicant shall use the nominal trajectory methodology above modified to optimize the launch vehicle performance and flight profile to create the desired conditions for maximum downrange DIP. For liquid systems, this shall include a fuel exhaustion trajectory.

**Methodology - Trajectory Dispersions**

To generate the three-sigma dispersions in terms of DIP distance from the nominal DIP location, the following procedure is suggested.

1. Identify individual key dispersion parameters that contribute to the dispersion of the vehicle's DIP. These should include thrust misalignment, thrust variation, weight variation, fin misalignment, impulse variation, aerodynamic drag variation, staging timing variation, stage separation-force variation, uncompensated wind, launcher elevation error, launcher azimuth error, launcher tip-off, and launcher location error. The applicant is responsible for including all key dispersion parameters in the trajectory analysis computations.
2. Estimate the values of the three-sigma variations for each of the individual key parameters.
3. Run a series of no-wind trajectory simulations using a 6-Degree-Of-Freedom (6-DOF) model where only one three-sigma value of a key parameter is introduced into each trajectory simulation.
4. For a one stage launch vehicle, tabulate the individual three-sigma downrange, uprange, left-crossrange, and right-crossrange DIP deviations, measured from the nominal DIP location, accumulated by each of the key parameters over the entire stage action-time. Calculate the square root of the sum of the squares (RSS) of all the individual DIP deviations in the downrange, uprange, left-crossrange, and right-crossrange directions. The RSS value, at each staging time, for each direction is the three-sigma downrange, uprange, and crossrange DIP deviation.
5. For a two stage vehicle, impact dispersions can be determined using the following two step process:
   1. Compute first stage impacts resulting from first stage deviations and root sum square their difference from the nominal first stage impact to determine first stage dispersion.
   2. Compute second stage dispersion by:
      1. Simulating first stage deviations with nominal second stage performance to determine the difference from nominal second stage impact.
      2. Simulating nominal first stage performance and injecting second stage performance deviations to determine the difference from nominal second stage impact.
      3. Root sum square the impact deviations determined in 1 and 2 above to determine second stage dispersion.

Note: Applicants wishing to use a different procedure (such as a Monte Carlo trajectory analysis), shall provide a description of the method used to generate the nominal trajectory, maximum range trajectory, and three-sigma dispersions and the rationale for why it produces equivalent results.

**Output**

The data items listed below are the minimum requirements. Additional requirements may be established in the application review process.

1. The applicant shall provide a brief discussion of the process that was used for the trajectory simulation.
2. The applicant shall provide a brief discussion of all assumptions and procedures used in deriving each of the key parameters and their standard deviations.
3. Launch point origin data: name, geodetic latitude (+N), longitude (+E), geodetic height, launch azimuth measured clockwise from True North.
4. Name of reference ellipsoid earth model: If other than WGS-84, provide the following additional data: semi-major axis, semi-minor axis, eccentricity, flattening parameter, gravitational parameter, rotation angular velocity, gravitational harmonic constants (J2, J3, J4), and mass of earth.
5. The applicant may find it necessary to convert latitude and longitude coordinates between different ellipsoidal earth models in order to complete the trajectory analysis. If a conversion is necessary, the applicant shall provide the equations for geodetic datum conversions and a sample calculation for converting the geodetic latitude and longitude coordinates between the models employed.
6. The applicant shall provide a tabular listing of each performance and key parameter used in the trajectory computations, each key parameter's plus/minus three-sigma variation, and the one-, two-, or three-sigma DIP displacement from the nominal no-wind DIP caused by each key parameter.
7. Vehicle performance data. The applicant shall provide a graphical and tabular presentation of the nominal and maximum range trajectories from launch until impact of the final stage. Time steps for the graphical presentation shall be at even intervals, not to exceed one second increments during thrusting flight, and for times corresponding to ignition, thrust termination or burnout, and separation of each stage (or impacting body). If stage action times are less than four seconds, time intervals should be reduced to 0.2 seconds or less. The graphical presentation shall depict the total launch vehicle velocity vs. time, present-position ground-range vs. time, and altitude above the reference ellipsoid vs. time. The tabular presentation shall provide the time, altitude above the reference ellipsoid, present position ground range, and total launch vehicle velocity for ignition, burnout, separation, booster apogee, and booster impact of each stage (or impacting body).
8. Vehicle impact data. The applicant shall provide a graphical and tabular presentation of the launch vehicle's DIP for the nominal trajectory, the maximum impact range boundary, and the three-sigma drag impact point dispersion areas. The graphical presentation shall include an overall depiction showing: (1) the DIP for the nominal trajectory, (2) a circle whose radius is equal to the farthest downrange impact point range from the maximum range trajectory, and (3) the three-sigma drag impact dispersions for each stage (or impacting body). The tabular presentation shall include the geodetic latitude (positive North of the equator) and longitude (positive east of the Greenwich Meridian) of each point describing the nominal DIP positions, the maximum range circle, and the three-sigma impact dispersion area boundaries. The coordinates shall be rounded to the fourth decimal point. Each stage's dispersion area shall be described by a minimum of 20 coordinate pairs.